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# Military Attack Helicopter Selection Using Distance Function Measures in Multiple Criteria Decision Making Analysis

C. Ardil



***Abstract*****—**This paper aims to select the best military attack helicopter to purchase by the Armed Forces and provide greater reconnaissance and offensive combat capability in military operations. For this purpose, a multiple criteria decision analysis method integrated with the variance weight procedure was applied to the military attack helicopter selection problem.

A real military aviation case problem is conducted to support the Armed Forces decision-making process and contributes to the better performance of the Armed Forces. Application of the methodology resulted in ranking lists for ordering and prioritizing attack helicopters, providing transparency and simplicity to the decision-making process.

Nine military attack helicopter models were analyzed in the light of strategic, tactical, and operational criteria, considering attack helicopters. The selected military attack helicopter would be used for fire support and reconnaissance activities required by the Armed Forces operation.

This study makes a valuable contribution to the problem of military attack helicopter selection, as it represents a state-of-the-art application of the MCDMA method to contribute to the solution of a real problem of the Armed Forces. The methodology presented in this paper can be used to solve real problems of a wide variety, especially strategic, tactical and operational, and is, therefore, a very useful method for decision making.

***Keywords*****—**Aircraft selection, military attack helicopter selection, attack helicopter fleet planning, MCDMA, multiple criteria analysis, multiple criteria decision making analysis, distance function measure.

## I. Introduction

THE national defense strategy aims to ensure the security of the country both in peacetime and in crises, establishing guidelines for the proper preparation and training of the Armed Forces. Even in times of peace, the Armed Forces must be equipped and trained to guarantee its sovereignty and strategic interests, supporting its foreign policy and positions in global forums. The national defense strategy guides, the strategic planning of the Armed Forces, whose compliance imposes the availability of the Armed Forces able to act in line with the political, strategic, and economic magnitude of the country in the global scenario.

According to the objectives and guidelines established in the national defense policy and the national defense strategy, condition the preparation and use of the Armed Forces, the Air Force is responsible for the employment of the AirPower. The AirPower comprises the ability to use the land, the air, and the sea, at the disposal of a force with expeditionary property, in a permanent condition of prompt employment, ensuring the projection of power over the land.

This force is characterized by the Armed Forces, in a position to fulfill missions related to the basic tasks of the AirPower. Among the necessary means for carrying out air missions, the use of attack helicopter aircraft is highlighted, suitable for reconnaissance and air fire support activities, due to their large number of weapons and ability to engage air and ground targets.

This function is currently performed by military helicopter aircraft of smaller size and with low firepower, which could be replaced by more modern means, specific for this purpose, thus ensuring better performance in the execution of such activities. Therefore, it will be of great value to the Air Force to acquire new helicopter aircraft with the capabilities to provide the necessary fire support to military operations, capable of carrying out advanced aerial reconnaissance and offensive air support, especially in the fight against armored vehicles and enemy troops on the ground.

In this sense, the national defense strategy provides, the acquisition of attack helicopter aircraft, acquired with commercial, industrial and technological compensation. Considering this need, due to the quantity, diversity, and complexity of existing models, the task of selecting a more appropriate aircraft to the Air Force’s needs, aiming to provide support to the operations performed by the Air Force, is not simple at all.

In this context, the expression multiple criteria decision making analysis (MCDMA) is used as an umbrella term to describe a set of formal approaches that attempt to explicitly take multiple criteria into account to help stakeholders and groups explore important decisions. The multiple criteria decision-making analysis process generally involves a choice between several alternatives. Efficiency in decision-making consists in choosing the alternative that offers the best results possible. The viable alternatives selected for goal attainment and evaluation are compared according to criteria and under the influence of attributes. In this context, MCDMA methods are very useful to support the decision-making process because they consider not only technical issues but also value judgments, evaluate alternatives to solve real problems, and present a high level of multidisciplinary. Therefore, these methods ensure greater accuracy and reliability in the decision-making process.

Despite the diversity of MCDMA approaches, methods, and techniques, the basic ingredients of MCDMA are a finite or infinite set of actions (alternatives, solutions, courses of

C.Ardil is with the National Aviation Academy, Baku, Azerbaijan. http://orcid.org/0000-0003-2457-7261

action, etc.), at least two criteria, and at least one decision-maker. Given these basic elements, MCDMA is a quantitative tool that helps make decisions, mainly in terms of choosing, ranking, or sorting the actions.

The existing helicopter models used in the main Air Force can be analyzed in the light of various criteria, whether qualitative or quantitative, such as speed, armaments, autonomy, load capacity, maneuverability, systems, and aggregate technologies [1-16]. Regarding the applications of the MCDMA model in military problems, it stands out: scoring and classification of military systems, ordering and evaluating weapon systems, selecting the best location for the installation of a military base, selecting the best advanced military training aircraft for the Air Force, positioning of the surveillance system within national security, evaluating airworthiness criteria for military aircraft, selecting ground vehicles for the provision of military units intended for multinational operations, obtaining the classification of the threat of military targets for risk management for obsolescence in the Armed Forces, the adoption of a combination of methodologies enables the identification of the variables and a rational analysis of the information, evaluating the effectiveness of air combat of military aircraft.

Selected military training aircraft for the Spanish Air Force, through hybrid modeling composed of AHP, TOPSIS, and Fuzzy Logic was investigated [17]. Application of Fuzzy Reference Ideal Method (FRIM) to the military advanced training aircraft selection was conducted [18].

The Interval Type-2 Fuzzy AHP (IT2FAHP) and Interval Type-2 Fuzzy (IT2FTOPSIS) methods to choose the most suitable aircraft to be acquired were used [19].

Attack helicopters were evaluated by the integration of ordinal evaluation into the cardinal procedure from the PROMETHEE method, thus, enabling the realization of qualitative and quantitative data [20].

The selection of the best helicopter to be acquired by the Brazilian Navy, enabling greater logistical and combat capacity in marine operations was considered. For this purpose, the AHP-TOPSIS-2N, a hybrid multicriteria method composed by the Analytic Hierarchy Process (AHP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), and two normalization procedures (2N) were applied [21].

Based on the applications of MCDA for aircraft selection, the important criteria to evaluate attack helicopters can be defined from the literature: In the domain of MCDMA methods, the MCDMA model is originated to deal with the military helicopter aircraft selection problem. Therefore, among the various MCDMA methods available, this compensatory model, with the advantage of combining a concept of hierarchy with weights associated with the concept of checking how much an alternative is closer and farther from an ideal alternative.

Other reasons to apply the MCDMA method include the fact that the analyzed problem is expected to have more criteria than alternatives and the criteria are numerically well defined.

This paper aims to support the decision-making process in a real military problem, for choosing the best attack helicopter aircraft to be acquired by the Air Force. This choice fills a gap in the Air Force's strategic, tactical and operational capabilities, as Air Force does not have military helicopter aircraft suitable for attack missions. Therefore, the relevance of this study consists in contributing to increasing the country's defense and sovereignty capacity.

The contemporary models of military attack helicopters used by Armed Forces from developed countries with proven effectiveness in combat were analyzed. For this, the MCDMA method was applied to evaluate the military helicopter aircraft considering evaluation criteria. The alternatives and criteria were analyzed considering the literature acknowledgment of military operations with attack helicopter aircraft.

This paper is structured into four sections. This introduction describes the objectives of the research and the literature review. Section 2 provides the methodology, while section 3 presents the case study. Section 4 presents the results and concludes the research.

## II. Methodology

Mathematical modeling of MCDMA runs through main steps, summarized as follows: structuring (identification of decision objective, criteria, and alternatives), measurement (designation of weights for the criteria and scores for the alternatives), and synthesis of the results obtained. The MCDMA model consists of multiple criteria decision-making analysis techniques adopted in complex scenarios, characterized by multiple and conflicting objectives using entropy that is a measure of uncertainty.

To understand the method, it is necessary a prior understanding of the two techniques that compose it. The MCDMA model is a multiple criteria methodology that aims to select or choose the best alternatives in a process that considers different evaluation criteria. The proposed method allows the evaluation of both quantitative and qualitative criteria. It is a compensatory method, indicated mainly for problems with a set of alternatives and criteria, considering the discrimination of results and cognitive effort in the evaluation. Also, the concepts of compensatory decision rules are in accord with military culture, which facilitates the analysis by the decision-makers.

The MCDMA model is a comprehensive tool developed for constructing decision models and establishing decision priorities concerning a finite set of alternatives. Evaluations are made using a measure of uncertainty, that represents the relative measure of one alternative over another concerning a given criterion. The MCDMA model orders the alternatives according to the proximity of the reference ideal solution (RIS). The best alternative is the one that is closer to the RIS and the farthest from the nadir ideal solution (NIS). For the application of the MCDMA method, the procedural steps are described below:

Step 1. Establish the decision matrix.

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_i \end{pmatrix} \begin{pmatrix} \overset{g_1}{x_{11}} & \cdots & \overset{g_j}{x_{1j}} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} \end{pmatrix}_{ixj} \quad (1)$$

Step 2. Obtain the normalized decision matrix.

Benefit criteria

$$p_{ij} = \frac{x_{ij}}{\max x_{ij}} \quad (2)$$

Cost criteria

$$p_{ij} = \frac{\min x_{ij}}{x_{ij}} \quad (3)$$

where $i = 1,...,I;\ j = 1,...,J$ .

Step 3. Calculate the importance weight of each attribute. By applying the variance method, the weights of each criterion are obtained.

$$\sigma_J^2 = \frac{1}{m}\sum_{i=1}^{m}\left(x_{ij} - \overline{x_{ij}}\right)^2 \quad (4)$$

$$\omega_j = \frac{\sigma_J^2}{\sum_{j=1}^{n}\sigma_J^2} \quad (5)$$

where

$$\begin{cases} \omega_j > 0 \\ \sum_{l=1}^{J}\omega_j = 1 \\ j = 1,2,...,J \end{cases} \quad (6)$$

where $\sigma_J^2$ is the variance of the attribute $g_j$ , $x_{ij}$ is the average value of the set of data, $\omega_j$ is the degree of importance weight of attribute $g_j$ , $I$ is the number of alternatives, and $J$ is the number of attributes.

Step 4. Construct the weighted normalized decision matrix.

$$v_{ij} = \omega_j p_{ij} \quad (7)$$

Step 5: Calculate the global score of each alternative.

$$\zeta_i = \sum_{j=1}^{J}\omega_j p_{ij} \quad (8)$$

Step 5. Obtain the reference ideal solution (RIS) and nadir ideal solution (NIS) vectors.

$$a^+ = \left\{v_1^+,...,v_n^+\right\} = \left\{(\max_i v_{ij} \mid j \in B), (\min_i v_{ij} \mid j \in C)\right\} \quad (9)$$

$$a^- = \left\{v_1^-,...,v_n^-\right\} = \left\{(\min_i v_{ij} \mid j \in B), (\max_i v_{ij} \mid j \in C)\right\} \quad (10)$$

where $B$ is the benefit criteria, and $C$ *is* the cost criteria.

Step 6. Determine the $L_1$, $L_2$, and $L_\infty$ norm distance function measures of each alternative from the reference and nadir ideal solutions.

a) Calculation of the $L_1$ norm distance function measure of each alternative from the reference ideal solution is calculated:

$$\xi_{i,L_1}^+ = \sum_{j=1}^{J}| v_{ij} - v_j^+ | \quad (11)$$

Similarly, the distance from the anti-ideal solution is calculated:

$$\xi_{i,L_1}^- = \sum_{j=1}^{J}| v_{ij} - v_j^- | \quad (12)$$

b) Calculation of the $L_2$ norm distance function measure of each alternative from the reference ideal solution is calculated:

$$\xi_{i,L_2}^+ = \left[\sum_{j=1}^{J}(v_{ij} - v_j^+)^2\right]^{1/2} \quad (13)$$

Similarly, the distance from the anti-ideal solution is calculated:

$$\xi_{i,L_2}^- = \left[\sum_{j=1}^{J}(v_{ij} - v_j^-)^2\right]^{1/2} \quad (14)$$

c) Calculation of the $L_\infty$ norm distance function measure of each alternative from the reference ideal solution is calculated:

$$\xi_{i,,L_\infty}^+ = \max_i\left(| v_{ij} - v_j^+ |\right) \quad (15)$$

Similarly, the distance from the anti-ideal solution is calculated:

$$\xi_{i,L_\infty}^- = \max_i\left(| v_{ij} - v_j^- |\right) \quad (16)$$

Step 7. Calculation of proximity coefficient to the reference ideal alternative.

$$\xi_i = \left[(\xi_i^+ - \min \xi_i^+)^2 + (\zeta_i^- - \max \xi_i^-)^2\right]^{1/2} \quad (17)$$

Step 8. Rank all alternatives according to the proximity coefficient, select the best one.

The proposed methodology adds to the concept of hierarchy with the importance weights of attributes associated with the concept of controlling how close or far an alternative is from a reference ideal alternative.

Table 1. Military Attack Helicopter Decision Criteria

| Decision Criteria | Definition |
|---|---|
| Maximum speed ( $g_1$ ) | Maximum speed (km/h) is the highest speed an attack helicopter can achieve, and it is required for reconnaissance, response, and fire support of troops in critical situations. |
| Main gun ( $g_2$ ) | Main cannon (mm) is a weapon used in close combat, consisting of air operations of fixed-wing and rotating aircraft against enemy targets that are friendly close forces. |
| Maximum takeoff weight (MTOW) ( $g_3$ ) | Maximum takeoff weight (MTOW) (kg) is the maximum weight at which the aircraft is certified for take-off due to structural or other limits. |
| Service ceiling ( $g_4$ ) | Service ceiling (km) is the maximum height at which a particular type of helicopter aircraft can sustain a specified rate of climb. |

## III. Application

The design of the attack helicopter has matured since its launch. A modern attack helicopter has evolved into a universal combat platform that follows the same mainline design principles and elements. A modern, specialized attack helicopter is most likely to include all of the following elements: tandem seat (usual gunner at the front, pilot at the rear), narrow fuselage, little or no cargo space, armored crew compartment using 20 to 30 mm guns, shaped charge guided anti-tank missiles (ATGMs), unguided smaller diameter rockets (usually high explosive fragmentation) and nose-mounted sensor suite. As unmanned aerial vehicles become more and more prominent, many integrate elements of short-range air defense, such as sidewinder missiles on the tips of the weapons' pylons or stinger missiles at hardpoints, as with standard designs.

For the applicability of the analysis, nine military attack helicopter models with proven combat effectiveness, which are used in the Air Forces with wide experience and acknowledgment in air operations, were evaluated.

The only information found in helicopter manufacturers' manuals was analyzed. Attack helicopters are a financially and developmentally expensive and burdensome luxury, so only leading countries can develop and implement their designs. As a result, the absolute majority of available attack helicopters make up this list as the options are pretty limited, even global.

The optimization direction of the selected evaluation attributes is taken as maximization in the decision-making process as shown in Table 1. The decision matrix for the military attack helicopter alternatives is given in Table 2.

Table 2. Decision matrix for military attack helicopter alternatives

| Optimization | | max | max | max | max |
|---|---|---|---|---|---|
| Alternatives | | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
| ATAK T129 | $a_1$ | 281 | 20 | 5056 | 6100 |
| ATAK T629 | $a_2$ | 318 | 30 | 11000 | 6100 |
| AH-1Z VIPER | $a_3$ | 370 | 20 | 8391 | 6100 |
| Mi-28NE | $a_4$ | 280 | 30 | 12100 | 5700 |
| Mi-35M | $a_5$ | 300 | 23 | 11500 | 5700 |
| Ka-52 Alligator | $a_6$ | 300 | 30 | 10800 | 5500 |
| Denel Rooivalk | $a_7$ | 309 | 20 | 8750 | 6100 |
| TIGER HAD | $a_8$ | 271 | 30 | 6600 | 4000 |
| AH-64E APACHE | $a_9$ | 293 | 30 | 10432 | 6100 |

The solution steps of the proposed methodology are given as flows:

Step 1. Identify the alternatives, and attributes and normalize the decision matrix using equations from (1) to (3). The normalized decision matrix is given in Table 3.

Table 3. Normalized decision matrix

| Optimization | | max | max | max | max |
|---|---|---|---|---|---|
| Alternatives | | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
| ATAK T129 | $a_1$ | 0,76 | 0,67 | 0,42 | 1 |
| ATAK T629 | $a_2$ | 0,86 | 1 | 0,91 | 1 |
| AH-1Z VIPER | $a_3$ | 1 | 0,67 | 0,69 | 1 |
| Mi-28NE | $a_4$ | 0,76 | 1 | 1 | 0,93 |
| Mi-35M | $a_5$ | 0,81 | 0,77 | 0,95 | 0,93 |
| Ka-52 Alligator | $a_6$ | 0,81 | 1 | 0,89 | 0,90 |
| Denel Rooivalk | $a_7$ | 0,84 | 0,67 | 0,72 | 1 |
| TIGER HAD | $a_8$ | 0,73 | 1 | 0,55 | 0,66 |
| AH-64E APACHE | $a_9$ | 0,79 | 1 | 0,86 | 1 |

Step 2. Calculate the importance weights of the attributes using equations from (4) to (6). The calculated objective criteria weights are given in Table 4.

Table 4. Calculated variances and criteria weights of the attributes

| Optimization | max | max | max | max |
|---|---|---|---|---|
| Alternatives | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
| $\sigma_J^2$ | 0,0056 | 0,0243 | 0,0345 | 0,0111 |
| $\omega_j$ (Variance weight) | 0,0742 | 0,3218 | 0,4566 | 0,1474 |
| $\omega_j$ (Mean weight) | 0,25 | 0,25 | 0,25 | 0,25 |

Step 3. Calculate the mean weighted ( $\omega_j = 1/n$ ) (MN) normalized matrix and global scores of alternatives using equations from (7) to (8). The calculated mean weighted

normalized matrix and global scores of alternatives are given in Table 5.

Table 5. Mean weighted normalized decision matrix (MW)

| Optimization | | max | max | max | max |
|---|---|---|---|---|---|
| Alternatives | | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
| ATAK T129 | $a_1$ | 0,19 | 0,17 | 0,10 | 0,25 |
| ATAK T629 | $a_2$ | 0,21 | 0,25 | 0,23 | 0,25 |
| AH-1Z VIPER | $a_3$ | 0,25 | 0,17 | 0,17 | 0,25 |
| Mi-28NE | $a_4$ | 0,19 | 0,25 | 0,25 | 0,23 |
| Mi-35M | $a_5$ | 0,20 | 0,19 | 0,24 | 0,23 |
| Ka-52 Alligator | $a_6$ | 0,20 | 0,25 | 0,22 | 0,23 |
| Denel Rooivalk | $a_7$ | 0,21 | 0,17 | 0,18 | 0,25 |
| TIGER HAD | $a_8$ | 0,18 | 0,25 | 0,14 | 0,16 |
| AH-64E APACHE | $a_9$ | 0,20 | 0,25 | 0,22 | 0,25 |

The ranking of alternatives based on the global scores of the unweighted normalized decision matrix (UW) and the mean weighted normalized decision matrix (MW) is given in Table 6.

Table 6. Ranking of alternatives based on the unweighted normalized decision matrix (UW), and the mean weighted normalized decision matrix (MW)

| Alternatives | | $\zeta_i^{UW}$ | Rank | $\zeta_i^{MW}$ | Rank |
|---|---|---|---|---|---|
| ATAK T129 | $a_1$ | 0,0527 | 9 | 0,7110 | 9 |
| ATAK T629 | $a_2$ | 0,0699 | 1 | 0,9421 | 1 |
| AH-1Z VIPER | $a_3$ | 0,0623 | 6 | 0,8400 | 6 |
| Mi-28NE | $a_4$ | 0,0684 | 2 | 0,9228 | 2 |
| Mi-35M | $a_5$ | 0,0642 | 5 | 0,8656 | 5 |
| Ka-52 Alligator | $a_6$ | 0,0668 | 4 | 0,9013 | 4 |
| Denel Rooivalk | $a_7$ | 0,0598 | 7 | 0,8062 | 7 |
| TIGER HAD | $a_8$ | 0,0544 | 8 | 0,7334 | 8 |
| AH-64E APACHE | $a_9$ | 0,0678 | 3 | 0,9135 | 3 |

Step 4. Determine the reference ideal solution (RIS) and nadir ideal solution (NIS) vectors using equations (9) and (10) as shown in Table 7.

Table 7. The reference ideal solution (RIS) and nadir ideal solution (NIS) vectors

| Optimization | max | max | max | max |
|---|---|---|---|---|
| Attributes | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
| $a^+$ | 1 | 1 | 1 | 1 |
| $a^-$ | 0 | 0 | 0 | 0 |

Step 5. Calculate the $L_n$ norm distance function measures of each alternative from the ideal solutions using equations from (11) to (17) as shown in Table 8, Table 9, and Table 10.

Table 8. Ranking of alternatives based on the mean weighted normalized decision matrix (MW) using the $L_1$ norm distance function measure

| Alternatives | | $\xi^+_{i,L_1}$ | $\xi^-_{i,L_1}$ | $\zeta_{i,L_1}$ | Rank |
|---|---|---|---|---|---|
| ATAK T129 | $a_1$ | 3,2890 | 0,7110 | 0,3269 | 9 |
| ATAK T629 | $a_2$ | 3,0579 | 0,9421 | 0,0000 | 1 |
| AH-1Z VIPER | $a_3$ | 3,1600 | 0,8400 | 0,1444 | 6 |
| Mi-28NE | $a_4$ | 3,0772 | 0,9228 | 0,0274 | 2 |
| Mi-35M | $a_5$ | 3,1344 | 0,8656 | 0,1083 | 5 |
| Ka-52 Alligator | $a_6$ | 3,0987 | 0,9013 | 0,0578 | 4 |
| Denel Rooivalk | $a_7$ | 3,1938 | 0,8062 | 0,1922 | 7 |
| TIGER HAD | $a_8$ | 3,2666 | 0,7334 | 0,2952 | 8 |
| AH-64E APACHE | $a_9$ | 3,0865 | 0,9135 | 0,0405 | 3 |

Table 9. Ranking of alternatives based on the mean weighted normalized decision matrix (MW) using the $L_2$ norm distance function measure

| Alternatives | | $\xi^+_{i,L_2}$ | $\xi^-_{i,L_2}$ | $\zeta_{i,L_2}$ | Rank |
|---|---|---|---|---|---|
| ATAK T129 | $a_1$ | 3,2890 | 0,7110 | 0,3269 | 9 |
| ATAK T629 | $a_2$ | 3,0579 | 0,9421 | 0,0000 | 1 |
| AH-1Z VIPER | $a_3$ | 3,1600 | 0,8400 | 0,1444 | 6 |
| Mi-28NE | $a_4$ | 3,0772 | 0,9228 | 0,0274 | 2 |
| Mi-35M | $a_5$ | 3,1344 | 0,8656 | 0,1083 | 5 |
| Ka-52 Alligator | $a_6$ | 3,0987 | 0,9013 | 0,0578 | 4 |
| Denel Rooivalk | $a_7$ | 3,1938 | 0,8062 | 0,1922 | 7 |
| TIGER HAD | $a_8$ | 3,2666 | 0,7334 | 0,2952 | 8 |
| AH-64E APACHE | $a_9$ | 3,0865 | 0,9135 | 0,0405 | 3 |

Table 10. Ranking of alternatives based on the mean weighted normalized decision matrix (MW) using the $L_\infty$ norm distance function measure

| Alternatives | | $\xi^+_{i,L_\infty}$ | $\xi^-_{i,L_\infty}$ | $\zeta_{i,L_\infty}$ | Rank |
|---|---|---|---|---|---|
| ATAK T129 | $a_1$ | 0,8955 | 2,5E-01 | 0,1104 | 9 |
| ATAK T629 | $a_2$ | 0,7851 | 2,5E-01 | 0,0000 | 1 |
| AH-1Z VIPER | $a_3$ | 0,8333 | 2,5E-01 | 0,0482 | 6 |
| Mi-28NE | $a_4$ | 0,8108 | 2,5E-01 | 0,0257 | 4 |
| Mi-35M | $a_5$ | 0,8083 | 2,4E-01 | 0,0263 | 5 |
| Ka-52 Alligator | $a_6$ | 0,7973 | 2,5E-01 | 0,0122 | 2 |
| Denel Rooivalk | $a_7$ | 0,8333 | 2,5E-01 | 0,0482 | 7 |
| TIGER HAD | $a_8$ | 0,8636 | 2,5E-01 | 0,0785 | 8 |
| AH-64E APACHE | $a_9$ | 0,8020 | 2,5E-01 | 0,0169 | 3 |

Step 6. Calculate the variance weighted ($\omega_j$) (VW) normalized matrix and global scores of alternatives using equations from (7) to (8). The calculated variance weighted normalized matrix and global scores of alternatives are given in Table 11.

Table 11. Variance weighted normalized decision matrix (VW)

| Optimization | | max | max | max | max |
|---|---|---|---|---|---|
| Alternatives | | $g_1$ | $g_2$ | $g_3$ | $g_4$ |
| ATAK T129 | $a_1$ | 0,06 | 0,21 | 0,19 | 0,15 |
| ATAK T629 | $a_2$ | 0,06 | 0,32 | 0,42 | 0,15 |
| AH-1Z VIPER | $a_3$ | 0,07 | 0,21 | 0,32 | 0,15 |
| Mi-28NE | $a_4$ | 0,06 | 0,32 | 0,46 | 0,14 |
| Mi-35M | $a_5$ | 0,06 | 0,25 | 0,43 | 0,14 |
| Ka-52 Alligator | $a_6$ | 0,06 | 0,32 | 0,41 | 0,13 |
| Denel Rooivalk | $a_7$ | 0,06 | 0,21 | 0,33 | 0,15 |
| TIGER HAD | $a_8$ | 0,05 | 0,32 | 0,25 | 0,10 |
| AH-64E APACHE | $a_9$ | 0,06 | 0,32 | 0,39 | 0,15 |

The ranking of alternatives based on the global scores of the unweighted normalized decision matrix (UW) and the variance weighted normalized decision matrix (VW) is given in Table 12.

Table 12. Ranking of alternatives based on the unweighted normalized decision matrix (UW), and the variance weighted normalized decision matrix (VW)

| Alternatives | | $\zeta_i^{UW}$ | Rank | $\zeta_i^{VW}$ | Rank |
|---|---|---|---|---|---|
| ATAK T129 | $a_1$ | 0,0527 | 9 | 0,6091 | 9 |
| ATAK T629 | $a_2$ | 0,0699 | 1 | 0,9481 | 2 |
| AH-1Z VIPER | $a_3$ | 0,0623 | 6 | 0,7528 | 7 |
| Mi-28NE | $a_4$ | 0,0684 | 2 | 0,9723 | 1 |
| Mi-35M | $a_5$ | 0,0642 | 5 | 0,8786 | 5 |
| Ka-52 Alligator | $a_6$ | 0,0668 | 4 | 0,9224 | 3 |
| Denel Rooivalk | $a_7$ | 0,0598 | 7 | 0,7541 | 6 |
| TIGER HAD | $a_8$ | 0,0544 | 8 | 0,7219 | 8 |
| AH-64E APACHE | $a_9$ | 0,0678 | 3 | 0,9216 | 4 |

Step 7. Calculate the $L_n$ norm distance function measures of each alternative from the ideal solutions using equations from (11) to (17) as shown in Table 13, Table 14, and Table 15.

Table 13. Ranking of alternatives based on the variance weighted normalized decision matrix (VW) using the $L_1$ norm distance function measure

| Alternatives | | $\xi_{i,L_1}^{+}$ | $\xi_{i,L_1}^{-}$ | $\zeta_{i,L_1}$ | Rank |
|---|---|---|---|---|---|
| ATAK T129 | $a_1$ | 3,3909 | 0,6091 | 0,5136 | 9 |
| ATAK T629 | $a_2$ | 3,0519 | 0,9481 | 0,0343 | 2 |
| AH-1Z VIPER | $a_3$ | 3,2472 | 0,7528 | 0,3104 | 7 |
| Mi-28NE | $a_4$ | 3,0277 | 0,9723 | 0,0000 | 1 |
| Mi-35M | $a_5$ | 3,1214 | 0,8786 | 0,1325 | 5 |
| Ka-52 Alligator | $a_6$ | 3,0776 | 0,9224 | 0,0705 | 3 |
| Denel Rooivalk | $a_7$ | 3,2459 | 0,7541 | 0,3086 | 6 |
| TIGER HAD | $a_8$ | 3,2781 | 0,7219 | 0,3541 | 8 |
| AH-64E APACHE | $a_9$ | 3,0784 | 0,9216 | 0,0716 | 4 |

Table 14. Ranking of alternatives based on the variance weighted normalized decision matrix (VW) using the $L_2$ norm distance function measure

| Alternatives | | $\xi_{i,L_2}^{+}$ | $\xi_{i,L_2}^{-}$ | $\zeta_{i,L_2}$ | Rank |
|---|---|---|---|---|---|
| ATAK T129 | $a_1$ | 3,3909 | 0,6091 | 0,5136 | 9 |
| ATAK T629 | $a_2$ | 3,0519 | 0,9481 | 0,0343 | 2 |
| AH-1Z VIPER | $a_3$ | 3,2472 | 0,7528 | 0,3104 | 7 |
| Mi-28NE | $a_4$ | 3,0277 | 0,9723 | 0,0000 | 1 |
| Mi-35M | $a_5$ | 3,1214 | 0,8786 | 0,1325 | 5 |
| Ka-52 Alligator | $a_6$ | 3,0776 | 0,9224 | 0,0705 | 3 |
| Denel Rooivalk | $a_7$ | 3,2459 | 0,7541 | 0,3086 | 6 |
| TIGER HAD | $a_8$ | 3,2781 | 0,7219 | 0,3541 | 8 |
| AH-64E APACHE | $a_9$ | 3,0784 | 0,9216 | 0,0716 | 4 |

Table 15. Ranking of alternatives based on the variance weighted normalized decision matrix (VW) using the $L_\infty$ norm distance function measure

| Alternatives | | $\xi_{i,L_\infty}^{+}$ | $\xi_{i,L_\infty}^{-}$ | $\zeta_{i,L_\infty}$ | Rank |
|---|---|---|---|---|---|
| ATAK T129 | $a_1$ | 0,9437 | 2,1E-01 | 0,2426 | 9 |
| ATAK T629 | $a_2$ | 0,9363 | 4,2E-01 | 0,0428 | 2 |
| AH-1Z VIPER | $a_3$ | 0,9258 | 3,2E-01 | 0,1399 | 8 |
| Mi-28NE | $a_4$ | 0,9439 | 4,6E-01 | 0,0180 | 1 |
| Mi-35M | $a_5$ | 0,9399 | 4,3E-01 | 0,0266 | 3 |
| Ka-52 Alligator | $a_6$ | 0,9399 | 4,1E-01 | 0,0510 | 4 |
| Denel Rooivalk | $a_7$ | 0,9381 | 3,3E-01 | 0,1270 | 6 |
| TIGER HAD | $a_8$ | 0,9457 | 3,2E-01 | 0,1362 | 7 |
| AH-64E APACHE | $a_9$ | 0,9413 | 3,9E-01 | 0,0648 | 5 |

**Sensitivity Analysis:** The MCDMA results for the military attack helicopter selection problem can be analyzed as follows:

a) In ranking of the alternatives according to the global sum of normalized decision matrices by the unweighted method, the ATAK T629 helicopter took first place, the Mi-28NE helicopter took second place, and the AH-64E APACHE helicopter took third place as shown in Table 6.

b) In ranking of the alternatives according to the global sum of the normalized decision matrices with the mean weight method, the ATAK T629 helicopter took first place, the Mi-28NE helicopter took second place, and the AH-64E APACHE helicopter took third place as shown in Table 6.

c) In ranking of the alternatives according to $L_1$ norm distance function measures with the mean weight method, the ATAK T629 helicopter took first place, the Mi-28NE helicopter took second place, and the AH-64E APACHE helicopter took third place as shown in Table 8.

d) In ranking of the alternatives according to $L_2$ norm distance function measures with the mean weight method, the ATAK T629 helicopter took first place, the Mi-28NE helicopter took second place, and the AH-64E APACHE helicopter took third place as shown in Table 9.

e) In ranking of the alternatives according to $L_\infty$ norm distance function measures with the mean weight method, the ATAK T629 helicopter took first place, the Ka-52 Alligator helicopter took second place, and the AH-64E APACHE helicopter took third place as shown in Table 10.

f) In ranking of the alternatives according to the global sum of the normalized decision matrices with the variance weight method, the Mi-28NE helicopter took first place, the ATAK T629 helicopter took second place, and the Ka-52 Alligator helicopter took third place as shown in Table 12.

g) In ranking of the alternatives according to $L_1$ norm distance function measures with the variance weight method, the Mi-28NE helicopter took first place, the ATAK T629 helicopter took second place, and the Ka-52 Alligator helicopter took third place as shown in Table 13.

h) In ranking of the alternatives according to $L_2$ norm distance function measures with the variance weight method, the Mi-28NE helicopter took first place, the ATAK T629 helicopter took second place, and the Ka-52 Alligator helicopter took third place as shown in Table 14.

i) In ranking of the alternatives according to $L_\infty$ norm distance function measures with the variance weight method, the Mi-28NE helicopter took first place, the ATAK T629 helicopter took second place, and the Mi-35M helicopter took third place as shown in Table 15.

Consequently, the variance-weighted method significantly influenced the assigned importance weights of the attributes; main weapon ($g_2$) and maximum take-off weight (MTOW) ($g_3$), and the ranking results of the military attack helicopters.

## IV. Application

In this study, a quantitative MCDMA evaluation based on distance function measures was made to select the best military attack helicopter for the air forces. While applying the sensitivity analysis with the variance weight method, the ranking results were significantly affected.

ATAK T629, Mi-28NE, Ka-52 Alligator and AH-64E APACHE are military attack helicopters that stand out in unweighted, mean-weighted, and variance-weighted evaluations in MCDMA application.

This study, which uses distance function measures, is thought to contribute to research studies on the evaluation of military attack helicopters. In the future, it is suggested that the study should be handled with the neutrosophic sets analysis approach.

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