Fighter Aircraft Selection Using Neutrosophic Multiple Criteria Decision Making Analysis

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Abstract—Fuzzy set and intuitionistic fuzzy set are dealing with the imprecision and uncertainty inherent in a complex decision problem. However, sometimes these theories are not sufficient to model indeterminate and inconsistent information encountered in real-life problems. To overcome this insufficiency, the neutrosophic set, which is useful in practical applications, is proposed, triangular neutrosophic numbers and trapezoidal neutrosophic numbers are examined, their definitions and applications are discussed. In this study, a decision making algorithm is developed using neutrosophic set processes and an application is given in fighter aircraft selection as an example of a decision making problem. The estimation of the fighter aircraft selection with the neutrosophic multiple criteria decision analysis method is examined.

Keywords—Neutrosophic set, multiple criteria decision making analysis, fighter aircraft selection, MCDMA, neutrosophic numbers.

I. INTRODUCTION

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function that assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established [1]. Decision-making in a fuzzy environment was introduced to address the uncertainty in decision making analysis [1, 2]. A neutrosophic set [3, 4] is a generalization of the concepts of classical set and fuzzy set [1], intuitionistic fuzzy set [5], and interval-valued intuitionistic fuzzy set [6]. Unlike interval-valued intuitionistic fuzzy sets, uncertainty characterized in a neutrosophic set. A neutrosophic set has three principal components, truth membership \( T \), indeterminacy membership \( I \), and falsity membership \( F \), which are defined independently of each other. In practice, a neutrosophic set is an instance of the single-valued neutrosophic set (SVNS) and interval neutrosophic set (INS) concepts are proposed and set-theoretical operators and various properties of SVNSs and INSs are provided [7-17]. Neutrosophic set provides uncertainty, inconsistent and incomplete information, and it is more appropriate to deal with indeterminate and inconsistent information.

Due to the inherent uncertainty and complexity of the problems, decision-making methods are often needed. With multiple criteria decision making analysis (MCDMA) methods, the best alternative can be determined among multiple alternatives according to some decision criteria. Recently, many quantitative MCDMA techniques have been developed for the selection problem in decision making research. Some of these techniques are additive weighted model (AWM) [18-22], multiplicative weighted model (MWM) [23], analytical hierarchy process (AHP) [24-26], composite programming [27-28], compromise programming [29-31], entropic programming [32], preference analysis for reference ideal solution (PARIS) [33-38], elimination et choix traduisant la réalité (ELECTRE) [39-40], preference ranking organization method for enrichment of evaluations (PROMETHEE) [41-45], the technique for order preference by similarity to ideal solution (TOPSIS) [46-51], všekriterijumska optimizacija i kompromisno rešenje (VIKOR) [51-54].

However, most of these methods were developed based on crisp data and therefore lack a few influence factors such as uncertainty preferences, additional qualitative criteria, and incomplete information. Therefore, fuzzy set, intuitionistic fuzzy set, and neutrosophic set theory are more suitable for tackling problems in the decision making process.

Single-valued neutrosophic information is a generalization of intuitionistic fuzzy information, while intuitionistic fuzzy information generalizes fuzzy information. A single-valued neutrosophic set is an instance of a neutrosophic set, which gives an additional possibility to represent uncertain, imprecise, incomplete, and inconsistent information. It can identify and process indeterminate information and inconsistent information.

However, there has not been much work on the single-valued neutrosophic sets integrated with the multiple criteria decision making methods that take into account the criteria values of the alternatives.

An extended single-valued neutrosophic AHP and MULTIMOORA method to evaluate the optimal training aircraft for flight training organizations was proposed to deal with inconsistent environments [55]. In addition, the study has extended a single-valued neutrosophic analytic hierarchy process (AHP) based on multi-objective optimization based on ratio analysis plus a full multiplicative form (MULTIMOORA) to rank the training aircraft as the alternatives.

A sensitivity analysis was performed to demonstrate the stability of the developed single-valued neutrosophic AHP and MULTIMOORA method. Also, a comparison between the results of the developed approach and existing approaches to validate the model was discussed.

Fuzzy triangular and trapezoidal numbers were applied to aircraft selection problems [56-57]. A stealth fighter aircraft selection model was proposed for the aircraft evaluation problem by the neutrosophic multiple criteria decision analysis method [58]. The study deals with the problem of

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selecting a stealth fighter aircraft using the single-valued neutrosophic set.

These studies reveal that the fighter aircraft selection problem can be modeled with a single-valued neutrosophic set approach. The main objectives of this study are (1) to define a formulation to calculate the performance importance weights of decision makers expressed with single-valued neutrosophic numbers, (2) to create a multiple criteria decision-making analysis method for the fighter aircraft selection problem under single-valued neutrosophic numbers, and (3) to illustrate the application and effectiveness of the proposed method with a decision making case.

The remainder of the paper is organized as follows: In Section 2 neutrosophic sets are briefly summarized. The steps of neutrosophic MCDMA are briefly summarized. In Section 3, a fighter aircraft selection application is carried out and, the results are analyzed. In Section 4, the paper concludes with a recommendation for further work.

II. METHODOLOGY

This section presents some basic definitions of neutrosophic sets, single-valued neutrosophic numbers, and theoretical concepts of multiple criteria decision analysis.

Definition 1. [1] Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a fixed set. A fuzzy set \( J \) in \( X \) is an object having the form

\[
J = \{x, \mu_j(x) \mid x \in X\}
\]

that is characterized by a function: membership function \( \mu_j: X \to [0,1] \) with the condition for all \( x \in X \).

Definition 2. [5] Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a fixed set. An intuitionistic fuzzy set \( K \) in \( X \) is an object having the form

\[
K = \{(x, \mu_j(x), \gamma_j(x)) \mid x \in X\}
\]

that is characterized by two functions: membership function \( \mu_j: X \to [0,1] \) and non-membership function \( \gamma_j: X \to [0,1] \) with the condition \( 0 \leq \mu_j(x) + \gamma_j(x) \leq 1 \), for all \( x \in X \).

Definition 3. [12] Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a fixed set. A single-valued neutrosophic set (SVN set) \( L \) in \( X \) is an object having the form

\[
L = \{(x, T_j(x), I_j(x), F_j(x)) \mid x \in X\}
\]

that is characterized by three functions: truth-membership function \( T_j: X \to [0,1] \), indeterminacy-membership function \( I_j: X \to [0,1] \), and falsity-membership function \( F_j: X \to [0,1] \) with the condition \( 0 \leq T_j(x) + I_j(x) + F_j(x) \leq 3 \), for all \( x \in X \).

The operations of SVN-sets [59, 60] are given as: Assume that \( A \) and \( B \) are two SVN-sets. Then

\[
A \oplus B = \left\{ \left(x, T_j(x) + T_j(x) - T_j(x)T_j(x), I_j(x) + I_j(x) - I_j(x)I_j(x), F_j(x) + F_j(x) - F_j(x)F_j(x) \right) \mid x \in X \right\}
\]

\[
A \ominus B = \left\{ \left(x, T_j(x)T_j(x), I_j(x) + I_j(x) - I_j(x)I_j(x), F_j(x) + F_j(x) - F_j(x)F_j(x) \right) \mid x \in X \right\}
\]

\[
\xi A = \left\{ \left(x, (1 - T_j(x))^{\xi}, I_j(x)^{\xi}, F_j(x)^{\xi} \right) \mid x \in X \right\}
\]

\[
A^\xi = \left\{ \left(x, T_j(x)^{\xi}, 1 - (1 - I_j(x))^{\xi}, 1 - (1 - F_j(x))^{\xi} \right) \mid x \in X \right\}
\]

where \( \xi \in R \). For convenience, the notation \( \langle T, I, F \rangle \) is adopted, instead of \( \langle x, (T_j(x), I_j(x), F_j(x)) \rangle \) for a single-valued neutrosophic element.

Definition 4. [59] Let \( X = \{x_1, x_2, \ldots, x_n\} \) be a set of alternatives, \( G = \{g_1, g_2, \ldots, g_m\} \) be the set of attributes. The ratings of alternatives \( x_i \in X \) are given as \( g_j \in G \) are expressed with SVN-number \( A_j = \{T_j(x), I_j(x), F_j(x)\} \). Then,

\[
g_j \left[ \{T_{11}, I_{11}, F_{11}\}, \{T_{12}, I_{12}, F_{12}\}, \ldots, \{T_{1n}, I_{1n}, F_{1n}\} \right]
\]

\[
[A_j]_{\max} = \left[ \{T_{21}, I_{21}, F_{21}\}, \{T_{22}, I_{22}, F_{22}\}, \ldots, \{T_{2m}, I_{2m}, F_{2m}\} \right]
\]

\[
[A_j]_{\min} = \left[ \{T_{31}, I_{31}, F_{31}\}, \{T_{32}, I_{32}, F_{32}\}, \ldots, \{T_{3n}, I_{3n}, F_{3n}\} \right]
\]

\[
[A_j]_{\max} \text{ is called a multiple criteria decision making matrix.}
\]

The importance weight vector of attribute set \( G \) is given as

\[
\omega = (\omega_1, \omega_2, \ldots, \omega_n) = (\{a_1, b_1, c_1\}, \{a_2, b_2, c_2\}, \ldots, \{a_n, b_n, c_n\})
\]

Then, the weighted multiple criteria decision making matrix \( \{A_j\}_{\max} = a[A_j]_{\max} \) is presented as

\[
g_j \left[ \{T_{11}, I_{11}, F_{11}\}, \{T_{12}, I_{12}, F_{12}\}, \ldots, \{T_{1n}, I_{1n}, F_{1n}\} \right]
\]

\[
[A_j]_{\min} = \left[ \{T_{21}, I_{21}, F_{21}\}, \{T_{22}, I_{22}, F_{22}\}, \ldots, \{T_{2n}, I_{2n}, F_{2n}\} \right]
\]

\[
\left[ \{T_{31}, I_{31}, F_{31}\}, \{T_{32}, I_{32}, F_{32}\}, \ldots, \{T_{3n}, I_{3n}, F_{3n}\} \right]
\]

where

\[
\langle T_j(x), I_j(x), F_j(x) \rangle = \omega_j A_j = (a_1, b_1, c_1) \langle T_j(x), I_j(x), F_j(x) \rangle = \langle a_1 T_j(x), b_1 - I_j(x), c_1 + F_j(x) \rangle
\]
Using arithmetic average operator [60], comprehensive evaluation of each alternative \( x_i \in \mathcal{X}(i = 1,2,\ldots,m) \), \( V_i \), is given by

\[
V_i = \sum_{j=1}^{n} \left( \frac{T_{ij} - F_j}{T_{ij}} \right) = \{T_i, I_i, F_i\}
\]

The score, accuracy and certainty function [61,62] to compare two alternatives are defined as

- Score function of \( V_i(i = 1,2,\ldots,m) \), denoted as \( s(V_i) \), defined as
  \[
s(V_i) = \frac{2 + T_i - F_i - I_i}{3}
\]

- Accuracy function of \( V_i(i = 1,2,\ldots,m) \), denoted as \( a(V_i) \), defined as
  \[
a(V_i) = T_i - F_i
\]

and then for \( s,t \in \{i = 1,2,\ldots,m\} \),

(a) If \( s(V_i) < s(V_t) \), then \( V_i \) is smaller than \( V_t \), denoted by \( V_i < V_t \).

(b) If \( s(V_i) = s(V_t) \);

(i) If \( a(V_i) < a(V_t) \), then \( V_i \) is smaller than \( V_t \), denoted by \( V_i < V_t \).

(ii) If \( a(V_i) < a(V_t) \), then \( V_i \) and \( V_t \) are the same, denoted by \( V_i = V_t \).

- Certainty function of \( V_i(i = 1,2,\ldots,m) \), denoted as \( c(V_i) \), defined as
  \[
c(V_i) = T_i
\]

III. APPLICATION

In this section, the fighter aircraft selection problem was presented as an illustrative example to show its applicability and effectiveness in decision making problems.

Assume that \( \mathcal{X} = \{x_1,x_2,\ldots,x_3\} \) be a set of fighter aircraft alternatives and \( \mathcal{G} = \{g_1, g_2, g_3\} \) be a set of attributes (\( g_1 \): survivability, \( g_2 \): maneuverability, \( g_3 \): reliability). The evaluation attributes of fighter aircraft alternatives are briefly defined as follows:

- Aircraft combat survivability is defined as the capability of an aircraft to avoid or withstand a man-made hostile environment. Aircraft combat maneuverability is defined as the ability to change the aircraft flight path by application of forces from the rotors or other control devices. Also, agility is defined as how quickly the aircraft flight path can be changed. The purpose of an aircraft reliability program is to ensure that the aircraft maintenance program tasks are effective, and their intervals are acceptable.

In this decision making problem, a decision maker wants to select the best alternative fighter aircraft considering the three evaluation attributes. All evaluation attributes (\( g_j \)) are considered as benefit criteria for the direction of optimization. Using information of criteria weights, such as the vector of the importance weights of the attributes, allows the decision maker to set priorities in the decision problem. Therefore, the algorithmic decision making process is given as

Step 1. Decision making matrix \( [A_{\text{ij}}]_{m \times 3} \) is established as

\[
[A_{\text{ij}}]_{m \times 3} = \begin{bmatrix}
(0.7,0.2,0.7) & (0.7,0.6,0.9) & (0.1,0.5,0.7) \\
(0.8,0.3,0.8) & (0.3,0.1,0.4) & (0.3,0.1,0.9) \\
(0.9,0.1,0.7) & (0.8,0.6,0.3) & (0.7,0.3,0.8)
\end{bmatrix}
\]

Step 2. The importance weight vector of attributes is determined as

\[
\omega = \{0.6,0.9,0.7\}, \{0.9,0.5,0.9\}, \{0.8,0.6,0.4\}
\]

Step 3. The weighted decision making matrix \( [\bar{A}_{\text{ij}}]_{m \times 3} \) is found as

\[
[\bar{A}_{\text{ij}}]_{m \times 3} = \begin{bmatrix}
(0.42,0.92,0.91) & (0.63,0.8,0.99) & (0.08,0.8,0.82) \\
(0.48,0.93,0.94) & (0.27,0.55,0.94) & (0.24,0.64,0.94) \\
(0.54,0.91,0.91) & (0.72,0.8,0.93) & (0.56,0.72,0.88)
\end{bmatrix}
\]

Step 4. The score function \( (V_i) \) of the alternatives \( x_i \in \mathcal{X}(i = 1,2,\ldots,m) \) is calculated as

\[
V_1 = \{1-(0.42)(1-0.48)(1-0.54), 0.92x0.93x0.91, 0.91x0.94x0.91\}
V_2 = 0.861264, 0.778596, 0.778414
V_3 = \{1-(0.63)(1-0.27)(1-0.72), 0.8x0.55x0.8, 0.99x0.94x0.93\}
V_4 = 0.924372, 0.352, 0.865458
V_5 = \{1-(0.08)(1-0.24)(1-0.56), 0.8x0.64x0.72, 0.82x0.94x0.88\}
V_6 = 0.692352, 0.36864, 0.678304
\]

respectively.

Step 5. The scores of \( V_i(i = 1,2,\ldots,m) \) are calculated as

\[
s(V_1) = 0.434751
s(V_2) = 0.568971
s(V_3) = 0.548469
\]
respectively. Then, the ranking of alternatives is found as

\[ x_2 > x_3 > x_1 \]

According to the decision making analysis results, the alternative \((x_2)\) is found to be the best fighter aircraft in the decision making problem.

Step 6. The accuracies of \(V_i(i = 1, 2, \ldots, m)\) are calculated as

\[ a(V_1) = 0.082850 \]
\[ a(V_2) = 0.058914 \]
\[ a(V_3) = 0.014048 \]

respectively.

Step 7. Certainties of \(V_i(i = 1, 2, \ldots, m)\) are found as

\[ c(V_1) = 0.861264 \]
\[ c(V_2) = 0.924372 \]
\[ c(V_3) = 0.692352 \]

respectively.

Neutrosophic sets (NSs) have widely been recognized as successful mathematical tools for solving ambiguous and imprecise problems. In this study, a single-valued neutrosophic set approach was used to determine the final ranking scores of alternatives. Using the proposed approach to address initial neutrosophic information, a fighter aircraft selection problem with three criteria and three candidate alternatives was applied to demonstrate its applicability and effectiveness.

Consequently, neutrosophic decision analysis procedure successfully ranked fighter aircraft candidates using the importance weight vector and the three evaluation attributes of the alternatives.

IV. CONCLUSION

Recently, multiple criteria decision making analysis problems have gained extensive awareness in single-valued neutrosophic sets. A multiple criteria decision making analysis problem with a single valued neutrosophic sets (SVN-sets) was explored with both criteria weights and attribute ratings expressed by single-valued neutrosophic information. Firstly, some basic concepts concerning SVN-sets were reviewed for the subsequent decision making analysis.

Secondly, a multiple criteria decision making method of SVN-sets was developed to describe the ranking order of the alternatives. Finally, fighter aircraft selection problem was presented as an illustrative example to show its applicability and effectiveness in decision making problems. The proposed neutrosophic MCDMA approach can be applied to various fields of study in solving real-life problems.

REFERENCES


