World Academy of Science, Engineering and Technology
International Journal of Civil and Environmental Engineering
Vol:15, No:12, 2021

Soil-Structure Interaction Models for the Reinforced Foundation System: A State-of-the-Art Review

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Open Science Index, Civil and Environmental Engineering Vol:15, No:12, 2021 publications.waset.org/10012375.pdf

Abstract**—**Challenges of weak soil subgrade are often resolved either by stabilization or reinforcing it. However, it is also practiced to reinforce the granular fill to improve the load-settlement behavior of it over weak soil strata. The inclusion of reinforcement in the engineered granular fill provided a new impetus for the development of enhanced Soil-Structure Interaction (SSI) models, also known as mechanical foundation models or lumped parameter models. Several researchers have been working in this direction to understand the mechanism of granular fill-reinforcement interaction and the response of weak soil under the application of load. These models have been developed by extending available SSI models such as the Winkler Model, Pasternak Model, Hetenyi Model, Kerr Model etc., and are helpful to visualize the load-settlement behavior of a physical system through 1-D and 2-D analysis considering beam and plate resting on the foundation, respectively. Based on the literature survey, these models are categorized as 'Reinforced Pasternak Model,' 'Double Beam Model,' 'Reinforced Timoshenko Beam Model,' and 'Reinforced Kerr Model'. The present work reviews the past 30+ years of research in the field of SSI models for reinforced foundation systems, presenting the conceptual development of these models systematically and discussing their limitations. A flow-chart showing procedure for compution of deformation and mobilized tension is also incorporated in the paper. Special efforts are taken to tabulate the parameters and their significance in the load-settlement analysis, which may be helpful in future studies for the comparison and enhancement of results and findings of physical models.

Keywords**—**Geosynthetics, mathematical modeling, reinforced foundation, soil-structure interaction, ground improvement, soft soil.

I. INTRODUCTION

THE problems related to soil-structure interaction (SSI) are often resolved using the rheological concept of modeling by both geotechnical and structural engineers. Many idealized SSI models have been developed in the past to analyze the complex behavior of the soil. One of the earliest SSI models was the Winkler Model, which consists of closely spaced, independent linear springs. The deficiency of the Winkler model in predicting settlement outside the loaded region has led to many two-parameter and three-parameter foundation models. These models eliminate the discontinuity in the Winkler model by considering mechanical interaction between the springs through various elements like thin stretched elastic membrane [1], flexible beam [2], incompressible shear layer [3] etc. Kerr [4], Selvadurai [5] and Horvath [6] presented a very informative review of these models.

Geosynthetic reinforcement is one of the established techniques of subgrade improvement and base reinforcement [7]. With the inclusion of reinforcement in the foundation system, proven to be an efficient and cost-effective solution for many engineering problems, it created a necessity to upgrade existing SSI models with consideration of soil-reinforcement-structure interaction. Many Researchers [8]-[13] have developed a new mathematical formulation considering the soil-reinforcement-structure interaction mechanism and the response of weak soil subgrade under the applied load. For differentiating them from existing SSI models, these models are named and referred to as 'Reinforced Soil-structure Interaction (RSSI) models' in this paper. Based on the literature survey, these mathematical models can be categorized into four types viz. 'Reinforced Pasternak Model', 'Double Beam Model', 'Reinforced Timoshenko Beam Model' and 'Reinforced Kerr Model'. To consider structural forces and reactions, these RSSI models have been analyzed with finite or infinite beam (1-D analysis) or plate elements (2-D analysis), resting on the foundation, and thus are applicable in the design of combined footings, railway tracks, pavements and rafts, storage tanks or silos, etc.

The paper reviews the past 30+ years of research, in the field of RSSI models, presenting the theoretical development of these models systematically, noting down their limitations. Role of various parameters in the load-settlement analysis is also incorporated in the tabular form for easy reference.

The present study is focused on the rheological concept of modeling to solve soil-structure interaction problem of reinforced foundation, other studies such as empirical or semi-empirical methods based on bearing capacity approach and or finite element analysis based on continuum mechanics approach are not cited in this paper.

II. GRANULAR FILL -REINFORCEMENT INTERACTION MECHANISM

To start the analysis with the RSSI model, one needs to idealize each component of a foundation system with a rheological element, which represents its behavior in a mathematical form. A reinforced foundation system generally consists of three major components viz. a granular fill/base material supporting the structure, subgrade soil and a reinforcement (placed either at the interface of base material and subgrade or interposed between two layers of base material). Fig.1 shows the chart of rheological elements which represents various components of RSSI model.

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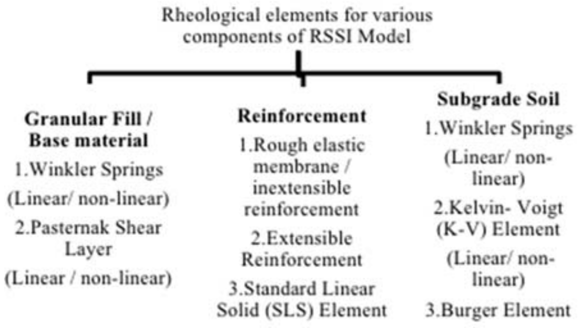

Fig. 1 Chart showing rheological elements for various components of RSSI model

The primary purpose of reinforcement in the foundation is to prevent tensile strains in the soil due to the applied load and thus, support the tensile stresses which soil cannot withstand. The response of interface behavior of reinforcement and granular fill, under the applied pressure, depends on the characteristics of the chosen reinforcing material. Shukla et al. [13] presented a critical review of the basic soil-reinforcement mechanism. The interaction mechanism between the reinforcement and the granular fill is generally idealized using two approaches viz. a rough interface (with inextensible reinforcement) and a smooth interface (with extensible reinforcement).

A. Rough Interaction Mechanism

The interaction between granular fill and the reinforcement is considered to be rough (no-slip condition) when reinforcement is idealized as a rough elastic membrane or inextensible reinforcement such as steel or metal strips, fiber-reinforced plastic sheet etc. The tension in the membrane is believed to be mobilized by the transfer of shearing resistance acted at the top and bottom of the reinforcement layer. The term ‘stretched rough elastic membrane’ [14] addresses prestressing effect on the reinforcement. The prestressed membrane distributes the load over a wider area reducing excessive settlement due to the *membrane effect* of the reinforcement layer. Bourdeau [15] explained the ‘membrane effect’ of the geosynthetic layer very well, considering the stresses at the interface of soil and reinforcement at the top and bottom surfaces.

Further to incorporate the compaction induced stresses at the interface of reinforcement and compacted granular fill, Shukla et al. [16] considered the coefficient of lateral stress at rest for granular fill, with horizontal as well as a vertical shear transfer mechanism introducing a rigid-perfectly plastic friction model with vertical and horizontal shear stresses at the interfaces.

B. Smooth Interaction Mechanism

To consider a smooth interaction between the reinforcement and granular fill, a reinforcement is assumed to be an extensible layer, which suggests that the relative modulus of elasticity of soil and reinforcement is very small. Extensible reinforcement such as geogrid, geotextile, etc. deforms as much as surrounding soil deforms under the loading. This condition eliminates consideration of interface coefficient due to shear stresses and incorporates stiffness in the reinforcement layer. Yin [10] incorporated extensible reinforcement in their model, by considering no slip and the compatibility condition at the interface between the reinforcing layer and fill. It was observed that consideration of compatibility condition shows slightly higher settlement values than the settlement obtained by considering interfacial friction coefficients.

III. REINFORCED PASTERNAK MODEL

To enhance the response of the Winkler model, Pasternak [3] assumed a thick shear layer attached to Winkler Springs, which deforms in the transverse shear. To incorporate the inclusion of reinforcement Madhav and Poorooshasb [8] introduced a three-parameter model, as an extension of the Pasternak Model, with a new element, a rough elastic membrane for the reinforcement layer, interposed between two layers of granular fill. They [17], [18] verified and quantified the confinement effect of reinforcement in the granular fill due to the mobilized tensile forces at the interface of reinforcement and granular fill. This type of idealization is considered to be appropriate if the granular fill is in loose or medium dense condition with relative density <65% [19]. Fig. 2 shows a typical sketch of the Reinforced Pasternak Model, where reinforced granular fill is placed on the top of the soft soil to support the embankment and to distribute the surcharge load, including self-weight over a large area.

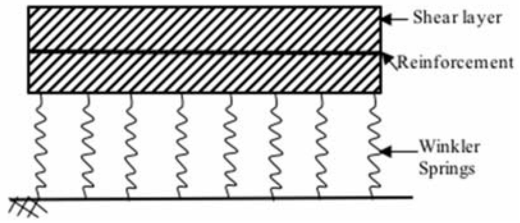

Fig. 2 Typical Sketch of Reinforced Pasternak Model (adapted from [8])

Further development in the model is summarized in Table I to show the influence of various parameters on the response of the Reinforced Pasternak Foundation. The table is subdivided based on the application area.

TABLE I
INFLUENCE OF VARIOUS PARAMETERS ON THE RESPONSE OF REINFORCED PASTERNAK MODEL

Parameters	Deformation in foundation	Mobilized tension in reinforcement	Limiting criteria	References
a) Unpaved foundation system				
Prestressing in the reinforcement	↑↓	↑↑	1. Not effective for higher shear modulus and higher interface friction coefficient between reinforcement and fill material. 2. Significant till the length of reinforcement is equal to twice the width of loading area from either side of the center of footing.	[14], [9], [10]
Compaction of granular fill	↑↓	↑↓	Not effective with pre-tensioned reinforcement.	[16], [9]
Compressibility of granular fill	↑↑	↑↑	Not significant for 1. Lightly loaded structures and higher shear modulus of granular fill material. 2. For higher relative stiffness of granular fill than soft soil.	[20], [9]
Degree of Consolidation	↑↓	↑↑		[20],[21]
Stiffness of granular fill	↑↓	↑↑	Effective when the stiffness of soil is comparatively very less.	[22]
Stiffness of reinforcement	↑↓	↑↑	-	[10], [23]
Extensible Reinforcement	↑↑	↑↓	-	[24]
b) Beam/ Plate resting on the foundation subjected to moving load				
Magnitude and velocity of moving load	↑↑	↑↑	The increase in deflection and mobilized tension increases by the ratio in which the intensity of load increases.	[12], [13], [24]-[28]
Stiffness of reinforcement	negligible	↑↑	An increase in stiffness shows a reduction in deflection when damping in the foundation is considered.	[12], [24]-[27] [29], [30]
Compressibility of granular fill	↑↓	↑↓	-	[25]-[27]
Shear modulus of granular fill	negligible	↑↑	-	[25]-[27]
Interfacial friction coefficient	negligible	↑↑	-	[25]-[27]
Viscous damping of foundation	↑↓	↑↓	Significant for the higher velocity of moving load.	[31], [32]
Non-linear behavior of soil	↑↑	↑↑	-	[32]
The magnitude of Elastic coefficients of Burger Element	↑↑	-	Mobilized tension is not applicable as geocell beam reinforcement is considered.	[28]
The magnitude of viscous coefficients of Burger Element	↑↓	-	Mobilized tension is not applicable as geocell beam reinforcement is considered.	[28]
Separation of the beam from ground	↑↑	↑↑		[25], [33]
Degree of consolidation	↑↑	↑↑		[29], [30]

		c) Foundation system with stone-column inclusion in the soft subgrade soil		
Modular Ratio (E_c/E_s)	↑↓	↑↑	The differential settlement was found to increase with the increase of modular ratio. Stress concentration ratio increases with an increase in modular ratio. Influence is negligible when the ratio is equal to or more than 25.	[12] [34] [35], [36]
Stiffness of Stone column	↑↓	↑↑	-	[12], [35]
Intensity of load	↑↑	↑↑	Reinforcement is more effective for higher values of applied load.	[12], [37]
Ultimate bearing capacity of soft soil (q_u)	↑↓	↑↑	After a certain value of q_u, soil behaves linearly and no further change in settlement occurs.	[12], [38]
Ultimate bearing capacity of stone columns	↑↓	-	Higher values showed a significant effect on settlement and bending moment of foundation.	[35], [38]
Degree of consolidation	↑↑	↑↑	Marginal increase in deflection after 90% of consolidation.	[12],[34], [36]-[38]
Flexural rigidity of the beam	↑↓	-	Effect on bending moment found negligible.	[35]
Spacing to diameter ratio of stone column (s/d)	↑↑	-	Beyond certain value of diameter of stone column, settlement shows reduction hence the optimum value of s/d ratio needs to be considered.	[35], [37]-[40]
Intensity and velocity of moving load	↑↑	↑↑	Negligible influence for a lower velocity of load.	[38]
Damping in foundation system	↑↓	-	Negligible influence for the lower velocity of load.	[38]
Multilayer Reinforcement	↑↓	↑↑	1. More effective for settlement reduction in case of lower flexural rigidity of the beam, lower shear modulus of granular fill and lower ultimate bearing capacity of the soft soil with higher loading intensity. 2. Not significant for stone columns induced soil. 3. Effective when the top of the reinforcement layers are placed closer to the interface of granular fill and subgrade soil. 4. As the stiffness of reinforcement increases, mobilized tension increases.	[12], [41]-[46]

Note: Both arrows going up indicates that the parameter is directly proportional to the response of the model, whereas one arrow going up with the second arrow going down indicates that the parameter is inversely proportional to the response of the model

A. Limitation of Reinforced Pasternak Model

The proposed model considers only vertical shear deformation at the interface of granular fill and reinforcement, neglecting the bending behavior in the reinforcement.

IV. DOUBLE BEAM MODEL

This mathematical model is based on Hetenyi's [2] concept of a beam on an elastic foundation. As shown in Fig. 3, the model consists of an upper and lower beam to represent the structure and the reinforcement respectively hence named as: Double Beam Model.

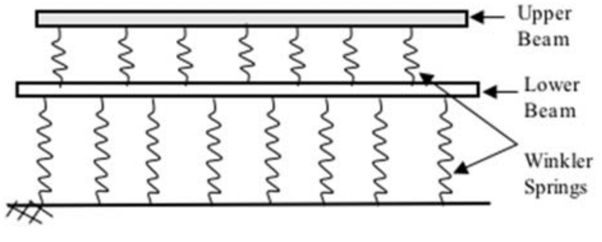

Fig. 3 Typical Sketch of Double Beam Model (adapted from [47])

Compacted dense granular soil above the reinforcement as well as subgrade soil below the reinforcement is idealized as Winkler Springs. Reinforcements such as geogrid, geocells, geomats, etc. offers a bending stiffness under the application of load and are hence, considered as a lower beam in the analysis. Maheshwari et al. [47] developed the Double Beam

Model to perform load settlement analysis for a strip footing subjected to concentrated load, with surcharge load [47] and without surcharged load [48] on the reinforcement. Further analysis has been performed considering rail ties or combined footing with edge concentrated loading [49], with consideration of separation effect between upper beam and soil below. Day and Basudhar [50] studied the varying distribution of modulus of subgrade reaction along the length of the upper beam. They found that the non-uniform distribution of subgrade modulus shows lesser deflection of footing as compared to deflection due to uniform subgrade reaction. Further development in the model related to the influence of various parameters is mentioned in Table II.

A. Limitations of Double Beam Model

1. Hetenyi's concept of modeling is used while developing a solution for the proposed Double Beam Model, However, in Hetenyi's model, the lower beam is imaginary whereas, in the present model, it is a physical element [51] because of which the model still possess the limitations of the Winkler Model.
2. The proposed model does not consider the tension membrane effect in the reinforcement and hence is not suitable for the reinforcing material such as geotextile, geomembrane etc.

V. REINFORCED TIMOSHENKO BEAM MODEL

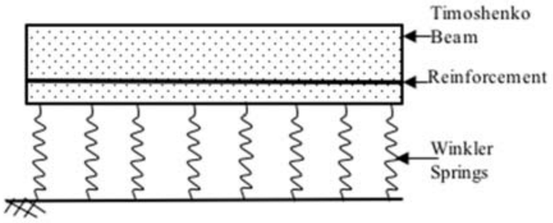

Fig. 1 Typical Sketch of Reinforced Timoshenko Beam Model (adapted from [52])

This model idealizes reinforced granular fill as a Reinforced Timoshenko Beam (RTB) resting on an elastic medium. Fig. 4 shows a definition sketch of the RTB model (adapted from Yin [52]). The model is based on the concept, that compacted dense reinforced granular fill (relative density $\geq 65\%$) act as a beam, deforming in shear as well as in bending under the application of load [19].

Yin [53] derived the analytical solution for Infinite Timoshenko beam on elastic medium, subjected to the concentrated load and compared the analysis with Winkler beam model and Finite Element Model. He concluded that RTB Model shows a better response than Winkler Model and the Finite Element Model. Ghosh et al. [54] applied RTB Model to design the thickness of load transfer platform, with the controlled column supported embankment. They indicated that bending stiffness of granular fill is a more influencing parameter than shear stiffness of granular fill in settlement of loading platform. They also found that the use of geosynthetic reinforcement in the granular platform reduced the settlement significantly when the thickness of the platform was between 0.2 m to 0.6 m. However, for a higher thickness than 0.6 m, geosynthetic is not helpful for the load transfer mechanism. Table III shows the influence of various parameters on the response of the RTB Model.

A. Limitations of RTB model

1. The RTB model possess similar limitations as that of the Winkler model i.e., discontinuity in the response of model, beyond the loaded region.
2. The model idealizes a granular fill layer as a beam element therefore, the reinforcement may develop negative tensile forces for particular reinforcing material such as geogrid or geocomposites.

VI. REINFORCED KERR MODEL

Kerr [4] developed a three-parameter model consisting of two elastic spring layers interconnected by an elastic shear layer. Shukla and Chandra [55] proposed a reinforced foundation model as an extension of the Kerr Model. As shown in Fig. 5, they considered the reinforced Pasternak Shear layer sandwiched between two layers of Winkler Springs. The reinforcement is idealized as a rough elastic membrane for materials like thin rubber sheets wire or rope, geosynthetics, metallic sheets etc. interposed between two compressible soil layers. Through elastic springs and shear layer interaction, this model simulates the punching shear failure that occurs in highly compressible soils.

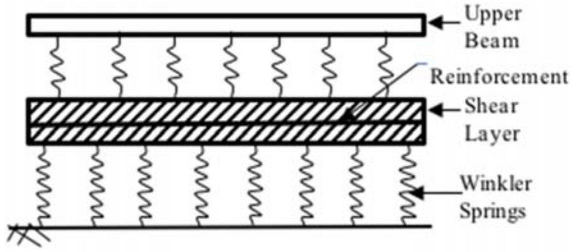

Fig. 2 Typical Sketch of Kerr Model (adapted from [55])

The proposed model is a four-parameter model. The list of parameters involved and their significance on the settlement of the foundation is noted in Table IV.

TABLE I
Influence of Various Parameters on the Response of Double Beam Model

Parameters	Deformation of upper structural beam	Deformation of lower reinforcement Beam	Limiting Criteria	References
The ratio of flexural rigidity of upper to the lower beam	↑↑	↑↓	For the upper beam maximum deflection is observed at the center of the beam. Showing discontinuity of deformation at the edge.	[48], [56], [11]
The relative stiffness of the upper soil layer to the lower soil layer	↑↑	↑↑	For upper beam, heave is observed at the edge of the beam. Maximum negative bending moment shifts towards the edge as relative stiffness increases for both upper and lower beams.	[48], [56], [11]
Surcharge load on the lower beam	↑↑	↑↑	For the lower beam deflection remains constant towards the edge of the beam.	[11], [51]
Depth of placement of the lower beam	↑↑	↑↑	For the lower beam deflection remains constant towards the edge of the beam.	[11], [51]
Interface resistance between lower beam and soil	↑↓	↑↓		[56], [57]

Note: Both arrows going up indicates that the parameter is directly proportional to the response of the model, whereas one arrow going up with the second arrow going down indicates that the parameter is inversely proportional to the response of the model.

TABLE III
Influence of Various Parameters on the Response of Reinforced Timoshenko Beam Model

Parameter	Deformation in foundation	Bending Moment of RTB	Mobilized tension in the reinforcement	References
Tension modulus of reinforcement	↑↓	↑↑	↑↑	[52], [58]
Stiffness of granular fill (Shear and bending)	↑↓	↑↓	↑↓	[59]
Modulus of subgrade reaction of soft soil	↑↓	-	-	[59]–[61]
Depth of placement of reinforcement from the center of Timoshenko beam	↑↓	-	-	[61]
Degree of consolidation	↑↑	↑↑	↑↑	[19]

Note: Both arrows going up indicates that the parameter is directly proportional to the response of the model, whereas one arrow going up with the second arrow going down indicates that the parameter is inversely proportional to the response of the model.

TABLE II
Influence of Various Parameters on the Response of Reinforced Kerr Model

Parameters	Effect on the deflection in the foundation	Limiting Criteria	References
The ratio of Spring constants (for upper and lower soil layer)	↑↓	The sudden drop in the deflection at the edge of the loaded region for small spring constant	[55]
Intensity of loading	↑↑	The discontinuity at the edge of the loading region for high load intensity	
Pre-tensioned reinforcement	↑↓	More significant at the center of the loading region than at the edge	
Shear parameters of granular Fill	↑↓	More significant at the center of the loading region than at the edge	
Interface frictional coefficients at the top and bottom of the reinforcement	↑↓	The deflection decreases beyond the loading region	

Note: Both arrows going up indicates that the parameter is directly proportional to the response of the model, whereas one arrow going up with the second arrow going down indicates that the parameter is inversely proportional to the response of the model

A. Limitations of the Reinforced Kerr Model

1. The model suffers from similar limitations as that of the Winkler Model, showing discontinuity in the response of model outside the loaded region.
2. The model is very sensitive to the parameters such as the ratio of spring constants and the modular ratio of shear layer, hence users need to provide special attention while calculating these parameters.
3. The model does not consider bending stiffness in the reinforcement; hence it is not suitable for reinforcing material such as geocell, geogrid etc.

VII. FLOW-CHART

A flow-chart showing procedure for modelling RSSI model and compution of deformation in the foundation system and mobilized tension in the reinforcement is presented in Fig. 6.

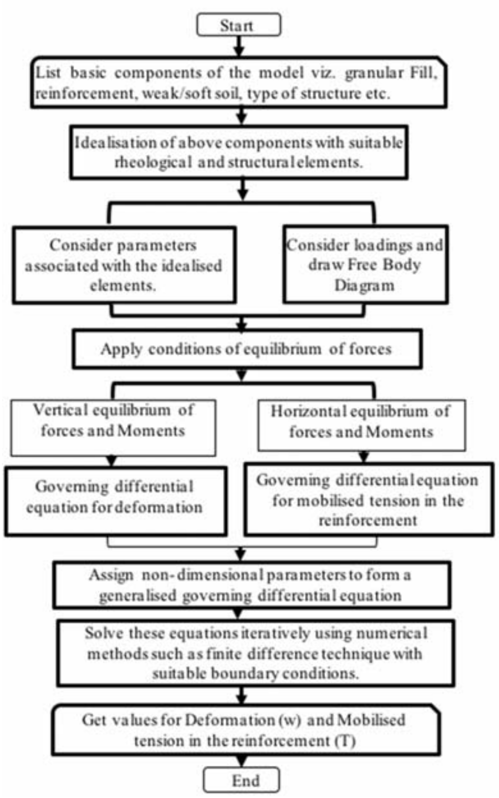

Fig. 6 Flow-chart for computing deformation of foundation and mobilised tension in the reinforcement using RSSI Model

VIII. DISCUSSION

1. It is observed from the Table I that the Reinforced Pasternak Model is the most generalized model and shows wide applicability in the field of geotechnical engineering.
2. RSSI models are evolving to provide a simple and reliable tool considering wide practical aspects of field conditions, however, very few models [23],[57] have been validated with field and laboratory experimentation.
3. A major advantage of RSSI models is that they enable parametric studies, as mentioned in Tables I-IV which can be used to enhance the results and findings of physical modeling or field studies.
4. The advancement in the formulation of reinforced foundation models, using rheological concepts provide a more realistic behavior of foundation system, however, this leads to the difficulty in evaluation of parameters either by experimentation or by correlation with soil properties. Thus, higher-order elements show limited use in a practical application.
5. Literature review shows that, while analyzing pavement or railway track foundation using RSSI model, loadings such as uniformly distributed load [29], [56], [57], [60] and moving load with constant velocity [26]- [28], [31]- [33], [36] have been considered, however, the design of such systems generally required strain accumulation due to repetitive loading conditions, which cannot be directly evaluated through these models.

IX. CONCLUDING REMARKS

It is observed that the use of conventional rheological elements for solving complex behavior of soil-reinforcement-structure interaction problems is quite simplified and successful. However, to gain confidence in a practical application, more research on validation with field or laboratory experimentation and application-based case studies is needed.

ACKNOWLEDGMENT

The authors would like to acknowledge the Director, College of Engineering Pune and Technical Education Quality Improvement Programme [TEQIP]-Phase III (The World Bank Assisted Project) for providing research facilities, funding for the research paper.

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