Applying the Crystal Model Approach on Light Nuclei for Calculating Radii and Density Distribution

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Abstract-A new model namely, the crystal model, has been modified to calculate radius and density distribution of light nuclei up to 8Be. The crystal model has been modified according to solid state physics which uses the analogy between nucleon distribution and atoms distribution in the crystal. The model has analytical analysis to calculate the radius where the density distribution of light nuclei has been obtained from the analogy of crystal lattice. The distribution of nucleons over crystal has been discussed in general form. The equation used to calculate binding energy was taken from the solid-state model of repulsive and attractive force. The numbers of the protons were taken to control repulsive force where the atomic number was responsible for the attractive force. The parameter has been calculated from the crystal model was found to be proportional to the radius of the nucleus. The density distribution of light nuclei was taken as a summation of two clusters distribution as in ⁶Li=alpha+deuteron configuration. A test has been done on the data obtained for radius and density distribution using double folding for d+6.7Li with M3Y nucleon-nucleon interaction. Good agreement has been obtained for both radius and density distribution of light nuclei. The model failed to calculate the radius of ⁹Be, so modifications should be done to overcome discrepancy.

Keywords—Nuclear lattice, crystal model, light nuclei, nuclear density distributions.

I. INTRODUCTION

CTUDYING nuclear properties for light nuclei has been Dattracted the attention of scientists and still under considerations. Light nuclei tend to form clusters as in case of ^{6,7}Li and ^{7,8,9}Be nuclei [1], [2]. In an article review [3], the author has discussed the anthropic considerations and the role of Hoyle-state and its closeness of the triple-alpha process to the energy of ⁴He+⁸Be. Many subjects have been discussed in a previous review which discussed the nuclear forces and chiral symmetry. Monte Carlo and ab-initio solutions have been used to study nuclear systems up to four nucleons. The quark mass dependence on nuclear forces is fulfilled to about 95%. For a long time, it was a goal of nuclear physics and astrophysics to find the mechanism of nuclear reactions and the structure of the nucleus for light, intermediate and heavy nuclei [4]. A lot of attempts have been done on nuclear systems to obtain and discuss the ambiguities of nuclear parameters like coupledcluster approach [5], [6] ab-initio calculations [7], [8] and Coupled channel scattering [9]-[12] which have been studied by using lattice calculations at crystal lattice range in the Fermi scale. Variation of the Monte Carlo method also has been used in the same track [13], [14]. Guatam and Dean Lee paved the

way for a general method for all nuclear reaction calculations on the crystal lattice. They studied the neutron radiative reaction (n,γ) on the lattice, and also included halo nuclei like ${}^{14}C(n, \gamma){}^{15}C$ as an example of the applicability of their theory by calculating the photo-nuclear reaction rates lattice [4]. Michelle Pine et al. [15] demonstrated and tested the adiabatic projection method on neutron-deuteron scattering. They calculated the s-wave phase shift and inelastic scattering reactions on the lattice for neutron-deuteron scattering.

The cluster structure was a helpful phenomenon that scientists used to simplify their studies. Ulf Meißner has discussed clustering using ab-initio method on the lattice, he focused on alpha clustering because its properties as high binding energy and spin (iso-spin) saturated. Nuclei like ¹²C, ¹⁶O and ²⁴Mg have been included in the study [16]. The authors asked; how large a nucleus can be calculated from first principles using the framework of chiral nuclear EFT and what are the remaining challenges [17]; they answered their question by calculating ground states of α -type nuclei from ⁴He to ²⁸Si. The Nuclear Lattice (EFT) or Nuclear Lattice Simulations method has been used by authors of the previous article, who succeeded in an explanation of clustering in lattice without any constrains which can be used as a benchmark for ab-initio calculations of larger nuclei using chiral nuclear EFT. Matter nuclear distribution for 6Li, 6He and 9Be has been discussed theoretically using the three body models in [18].

Binding energy and nuclear density distribution have been calculated for light nuclei depending on alpha as a core of a cluster. It was chosen ⁶Li, ⁶He as $(\alpha+n+n)$ and ⁹Be as $\alpha+\alpha+n$, where in case of ⁶Li it was an approximation to take the neutron instead of the proton in the configuration. The authors have summarized the form of nuclear density distribution of light nuclei in the form of summation of two Gaussian in [19]. The authors in [1] have concluded the possibility of using dynamic cluster in the future to study ⁸Be and ¹²C. Research [20] has been performed to study nucleus as crystal. Our work is devoted to study the form of expected nuclear density distribution for light nuclei.

II. CRYSTAL MODEL APPROACH

The potential between nucleons inside the nucleus will be calculated using the equations, which have been used to calculate a parameter proportional to the radius of the nucleus [21]:

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$$U = (4\varepsilon) \left(\frac{a}{r^{12}} - \frac{b}{r^7}\right) \tag{1}$$

where a is taken to be the number of the protons and b is the mass number of the nucleus and ε is taken an arbitrary constant. The first term of (1) has been used as the repulsion force where the second term is taken to be the attractive force. The crystal model has been developed to calculate a parameter that is proportional to the radii of light nuclei and to expect the form of matter density distribution of light nuclei. The cluster model and crystal model are close to each other where the distribution of nucleons inside nuclei in our point of view looks like the distribution of atoms on crystal. ⁶Li as an example contains alpha as a base and two nucleons are located on the upper face of the crystal, and alpha is on the lower face, as shown in Figs. 2 and 3. The density distribution of alpha and deuteron of the general form:

$$\rho(\mathbf{r}) = \mathbf{A}\{\operatorname{Bexp}\left(-\mathbf{C}\right) + \operatorname{Dexp}(-\mathbf{E}) - \mathbf{F}\}$$
(2)

where A, B, D and F are constants where C and E are functions of r. Let $A = \rho_0$, B=D=F=1 and hence, (2) could be modified to take the form:

$$\rho(\mathbf{r}) = \mathbf{A}\{[1 + \exp{(-B)}] + [1 + \exp{(-B)}] - 1\} \quad (3)$$

where $A = \rho_0$, and B is a function of r. For single parameterization, the nucleon density distribution in the nucleus is approximated by:

$$\rho(\mathbf{r}) = \left(\frac{3}{2\pi R_{\rm m}^2}\right)^{3/2} \exp\left(\frac{-3r^2}{2R_{\rm m}^2}\right) \tag{4}$$

where R_m is the rms of matter radius of the nucleus. The standard Fermi of the nuclear matter distribution is written in the form:

$$\rho(\mathbf{r}) = \rho_0 (1 + e^{\frac{(\mathbf{r} - \mathbf{R})}{a}})^{-1}$$
(5)

which is suitable for description of intermediate and heavy nuclei. For light nuclei, it is suitable to modify (3) to be symmetrized in Fermi-density [17]:

$$\rho(\mathbf{r}) = \rho_0 \{ (1 + e^{\frac{(\mathbf{r} - \mathbf{R})}{a}})^{-1} + (1 + e^{\frac{(\mathbf{r} - \mathbf{R})}{a}})^{-1} - 1 \}$$
 (6)

$$\rho(\mathbf{r}) = \rho_0 \frac{\sinh\left(\frac{R}{a}\right)}{\cosh\left(\frac{R}{a}\right) + \cosh\left(\frac{r}{a}\right)} \tag{7}$$

With normalization,

$$\rho_0 = \frac{3}{4\pi R^3} \frac{1}{1 + (\frac{\pi a}{R})^2} \tag{8}$$

where R is the half-density radius.

If we compare (6) and (3) from the crystal model approach, we find that the first term of (9) $\left[\rho_0(1 + e^{\frac{(r-R)}{a}})^{-1}\right]$ represents the density of alpha in our model as simple Gaussian

distribution where the second and third term, $\{\rho_0[(1 + e^{\frac{(r-R)}{a}})^{-1} - 1]\}$ is represented in the density distribution of deuteron. Where B is taken to be $\frac{(r-R)}{a}$ from the analogy with our expectation and those parameters from [17]. R_m (rms of nuclear matter radius) is related to R and a by the relation:

$$\rho_0 = \left(\frac{3}{2}\right)^{1/2} R \left[1 + \left(\frac{7}{3}\right) \left(\frac{\pi a}{R}\right)^2\right]^{\frac{1}{2}}$$
(9)

In another form, from the principles of nuclear structure which states that overlapping between two constituents of nucleus is forbidden [18], the density distribution of many bodies systems may be written as:

$$\rho(\mathbf{R}) = \sum_{\text{cluster}=i,j,k} \rho_{\text{cluster}}(\mathbf{R})$$
(10)

Applying (10) on ⁷Li for example, it will be written as:

$$\rho(\mathbf{R}) = \rho_{\alpha}(\mathbf{R}) + \rho_{t}(\mathbf{R}) \tag{11}$$

Also, the calculations of alpha nucleus have been done here using the relation:

$$p(\mathbf{r}) = 0.17\{(1 + e^{\frac{(\mathbf{r}-1.97)}{0.65}})^{-1}\}$$
(12)

where R=1.97fm, a=0.65fm, f and ρ_0 =0.17(1/fm3).

III. RESULTS AND DISCUSSION

A. Calculating Binding Energy for Light Nuclei

By imagining the nucleus as a crystal, where the nucleons are arranged on the corners of the crystal with analogy of atoms in the solid-state physics. The model has been developed by us to study light nuclei [20], and succeeded to explain up to alpha particles. For nuclei heavier than alpha, the model should be modified to explain the distribution of the nucleons inside the nucleus. The distribution of the nucleons in case 8Be as hexagonal crystal is discussed. The two alphas will be taken on the up and down faces of a hexagonal crystal. The situation of ⁸Be is very interesting as the life-time is about 10⁻¹⁶ sec which reflects the instability of such nuclei. Dynamic cluster model should be applied in case of instability from mechanical force. The distribution of nucleons inside the nucleus starts with cubic (on the corners of the crystal as one proton beside one neutron). There is a change in color from white tin to gray by changing the temperature which introduces another crystal structure. The same situation takes place in the case of the 8Be nucleus where the structure in the moment of formation is cubic and during about 10⁻¹⁶ sec, the structure changes into a hexagonal close packed in a process known as a dynamic cluster of the nucleus. The change in structure inside the nucleus of ⁸Be increases the volume of it, and decreases the nuclear density of ⁸Be nucleus. ⁸Be will disintegrate into two alphas. The shape of the nucleus in the case of 8Be will change from cubic into hexagonal closed packed (HCP) produces stretching as shown in Fig. 1 where two alphas will occupy the two faces of HCP. The radius of 8Be

have been calculated from (1) to be 0.8697992638 where U was taken to be Q-value as shown in Table I compared with calculated radii from the literature [22]. A new parameter α (R₁/R₂) has been calculated by dividing the radius of the nucleus taken from the literature and those calculated from the crystal model in the present work. Monte Carlo transfer matrix algorithm calculations have been achieved to distinguish between atomic number A≤8 and higher ones [23]. The value of transformation parameter α (R₁/R₂) has been calculated from the crystal model of nucleus (CMN) is ~2.2-3.5 which has low value at lower binding energy as in the case of deuteron 2.59. As seen in Table I, the calculated value of transformation parameter for ⁷Li is smaller than that value calculated for ⁶Li, which agrees with the values of them in the literature [2], [24], [25] (see Table I).

The aim now is to calculate the parameters of the hexagonal close packed (HCP).



Fig. 1 Determination the parameters of hexagonal close packed

The volume of unit cell could be obtained by the product of the base area times the cell height, c. The lattice parameter (a) has a relation with the nucleon radius r as:

$$a=2r$$
 (13)

$$V_{\rm c} = 6r^2 c \sqrt{3} \tag{14}$$

The volume of ⁸Be nucleus has been calculated before and after separation into two alphas. r is taken to be 1 to 1.25fm and c is about two alphas radius to be 2×1.6755 fm= 3.351fm. So, the minimum volume of ⁸Be is:

$$V_{c} = 6r^{2}c\sqrt{3} = 6 * 1.25 * 1.25 * 3.351 * \sqrt{3} = 54.41 \text{ fm}^{3}$$
(15)

Dynamical torque is required to break down ⁸Be into two alphas; the dynamical torque is responsible for separation of two alphas from each other. The secrete on such configuration arises from the distance c between two alphas arranged on the faces of hexagonal close packed. The large distance between the two alphas gives an opportunity for two alphas to separate from each other. The changing on the allotropic transformation on ⁸Be produces stretching and dynamic torque leads to separate ⁸Be into two alphas. Another approach could be applied if the transformation parameter (α) is used for ⁸Be shown in Table I used to calculate the radius of ⁸Be on the separation moment; which was found in the range 3.05 fm. The radius of ⁸Be on the moment of separation into two alphas is nearly equal the radius of ²⁰Ne which contains more than the twice number of nucleons of ⁸Be nucleus and equal ²⁴Mg which three times of nucleon number of ⁸Be. The solution of (1) succeeded to give us the transformation parameter (α) which proportional to the radius of nuclei from deuteron up to ⁸Be as shown in Table I. The binding energy of ⁸Be has been calculated from (1). There is a condition which should be applied on ⁸Be, which is r≤3.5fm. The binding energy of ⁸Be nucleus that is calculated from the crystal model approach: U = (4 × 10³) $\left(\frac{4}{(R*\alpha)^{12}} - \frac{8}{(R*\alpha)^7}\right)$ which is applied on the nuclei under consideration with R and α taken from Table II. The binding energy of ⁸Be has been calculated:

$$U = (4 \times 10^3) \left(\frac{4}{(3.35)^{12}} - \frac{8}{(3.35)^7}\right) = 52 \text{MeV}$$
(16)

where U is binding energy of ⁸Be and ε is extracted from experimental data from literature and has the value 1000 MeV.

B. Matter Density Distribution for Light Nuclei

An approximation has been applied on the nuclei using crystal model approach that the density distribution ⁶Li, ⁷Li, and ⁷Be nuclei are taken as a summation of two Gaussian distributions as one for the first cluster is alpha and the other part of nucleus is the residual cluster of nuclei. For example, ⁶Li is taken as d+alpha or t+³He (two configurations with the same probability) and the density distribution of ⁶Li now is a summation of two Gaussian, one for deuteron and the other for alpha. That is could be described for the ⁷Li≡ α +t configuration as in cluster structure and hence, alpha will be the core of the nucleus and triton as valence (See Figs. 2 and 3).



Fig. 2 Cluster model distribution for ⁷Li and ⁶Li



Fig. 3 Distribution of the nucleons on crystal model for ^{6,7}Li where the distribution of the nucleons of ⁶Li as alpha on the base and deuteron on the upper face (left panel) and ⁷Li case with alpha on the base and triton on the upper face (right panel)

The imagination of ${}^{6}Li$ in the present work agrees with the phenomenological charge and matter distributions and the shell-model distribution has the form [26]:

$$A\rho(r) = a\alpha^{3}e^{-\alpha^{2}r^{2}} + (b + c\beta^{2}r^{2})\beta^{3}e^{-\beta^{2}r^{2}}$$
(17)

The constants of ⁶Li were taken from Table III.

Also, density distribution of ⁷Li nucleus was taken in the form which is two summations of Gaussian [27]:

$$\rho(R) = \rho_0 \left(1 + \alpha \left(\frac{R}{a}\right)^2 \right) \exp\left(- \left(\frac{R}{a}\right)^2 \right), \qquad (18)$$

where for harmonic oscillator a=1.77 fm and $\alpha=0.327$ fm. ⁶Li is taken in the crystal model approach to be alpha on the base of the lattice and deuteron on the upper face of the crystal which is shown in Fig. 4.



Fig. 4 Matter density distribution ⁶Li nucleus from crystal model approach

The calculations have been done for ${}^{6}\text{Li}$ (${}^{6}\text{Li}\equiv\alpha+d$):

$$\rho(r) = 0.17\{(1 + e^{\frac{(r-2.5)}{1.2}})^{-1} + (1 + e^{\frac{(-r-2.5)}{1.2}})^{-1} - 1\}$$
(19)

where R=2.5fm, a=1.2fm (as very smooth nucleus) and $\rho_0=0.17(1/\text{fm}^3)$. Also, the calculations of alpha nucleus have been done here using the relation:

$$\rho(\mathbf{r}) = 0.17\{(1 + e^{\frac{(\mathbf{r}-1.97)}{0.65}})^{-1}\}$$
(20)

where R=1.97fm, a=0.65fm, and ρ_0 =0.17(1/fm³).

Double folding calculations have been applied on deuteron elastically scattered by ^{6,7}Li using density distribution from the present analysis as shown in Figs. 5 and 6.

Double folding model has been used to calculate microscopic potential in the present work. The real part of the calculated potential is given in the following form [28]:

$$V_F(r) = \int dr_P \int dr_1 \rho_p(r_p) \rho_1(r_1) v(r_{pt})$$
(21)

where ρp is the matter density distribution of the projectile, ρt is the matter density distribution of the target and rpt = r + rt - rp. M3Y form has been used for nucleon-nucleon interaction at the present calculation:

$$v(r) = 7999 \frac{\exp(-4r)}{4r} - 2134 \frac{\exp(-2.5r)}{2.5r} - 276(1 - \frac{0.005E}{A_{p}})\delta(R)$$
(22)

The form of nuclear central optical potential during double folding model calculations was taken in the shape: U(r)=VF(r)+iW(r), where $V_F(r)$ is calculated from (21) and W(r) is phenomenological Woods-Saxon potential. The imaginary part was included into the Fresco program [29] to reproduce a differential cross section.

Double folding calculations for ⁶Li+d with alpha transfer has been done using density distribution of ⁶Li from crystal model approach at deuterons energy 25 MeV and 14.7 MeV, and was found to be in agreement with experimental data as shown in Figs. 5 and 6. The importance of using double folding potential appears at backward angles and the effect of spectroscopic amplitudes is very obvious for ⁷Li elastically scattered by deuteron.



Fig. 5 Represents double folding calculations for ⁶Li+d for elastic scattering with alpha transfer using density distribution of ⁶Li from crystal model approach at deuterons energy 14.7 MeV where solid line represents CM, the dash line is the optical model and dash-dot line represents the coupled reaction channel. Experimental data were taken from [30]



Fig. 6 Represents double folding calculations for ⁶Li+d for elastic scattering with alpha transfer using density distribution of ⁶Li from crystal model approach at deuterons energy 25 MeV where solid line represents CM, dash line is the optical model and dash-dot line

represents the coupled reaction channel. Experimental data were taken from [31]

The density distribution of ⁷Li nucleus is simpler than ⁶Li as triton density distribution is not complicated as deuteron and will be taken in the form:

$$\rho(\mathbf{r}) = \rho_0 \{\operatorname{Aexp} (-B) + \operatorname{Aexp} (-C) - D\}$$
(23)

where ρ_0 , A, and D are constants but B and C are functions of r. The same situation may be applied on ⁷Li where the base is alpha and triton on the upper face of crystal as shown in Figs. 2 and 3. The density distribution of ⁷Li is taken:

$$\rho(\mathbf{r}) = 0.17\{(1 + e^{\frac{(\mathbf{r}-2.4)}{1.1}})^{-1} + (1 + e^{\frac{(\mathbf{r}-2.4)}{1.1}})^{-1} - 1\}$$
(24)

where R=2.4fm, a=1.1fm, and ρ^0 =0.17(1/fm³).

⁸Be was assumed as alpha+alpha configuration that is because it decays into alpha+alpha during 10⁻¹⁶s. The distribution of the nucleons on ⁸Be is shown in Figs. 9 and 10. Dynamic cluster is needed to explain ⁸Be decay into two alphas where crystal model could achieve such mission with analogy of solid state physics. The density distribution of ⁸Be could be written in the form:

$$\rho(r) = \rho_0 A\{Bexp(-C) + Bexp(-C) - 1\}$$
(25)

The analysis of (8) gives an impression that the density distribution of ⁸Be is two identical Gaussians with the form Bexp(-C).



Fig. 7 Nuclear density distribution of ⁷Li from the crystal model approach



Fig. 8 Double folding calculations for deuteron elastically scattered by ⁷Li with alpha transfer using density distribution of ⁷Li from crystal model approach at deuterons energy 25 MeV. The solid line represents CM and dash line is the coupled reaction channel where experimental data were taken from [32]



Fig. 9 The distribution of nucleons for ⁸Be nucleus



Fig. 10 Nuclear matter distribution of ⁸Be nucleus using parameters from crystal model approach

$$\mathbf{p}(\mathbf{r}) = 0.17\{(1 + e^{\frac{(\mathbf{r}-1.97)}{0.65}})^{-1} + (1 + e^{\frac{(\mathbf{r}-1.97)}{0.65}})^{-1} - 1\}$$

General form for light nuclei:

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$$\rho(\mathbf{r}) = 0.17\{(1 + e^{\frac{(\mathbf{r}-1.97)}{0.65}})^{-1} + (1 + e^{\frac{(\mathbf{r}-1.97)}{0.65}})^{-1}\}$$
(26)

⁸Be density distribution is just the summation of two alphas. Where R=1.97fm, a=0.65fm, and ρ_0 =0.17(1/fm³). So, from the matter density distribution for ⁸Be we can expect the continuum state of ⁸Be. ⁷Be is a stable nucleus as it contains four protons and three neutrons with the configuration ⁷Be=⁴He+³He. The density distribution of alpha is just a simple Gaussian distribution {Aexp(-B)} where A, B and C are functions on r in the form:

$$p(r) = A\{exp(-B) + exp(-C) - 1\}$$
(27)



Fig. 11 7Be density matter distribution from crystal model approach

The same calculations will be applied on ⁷Be and the results are presented as:

$$\rho(\mathbf{r}) = 0.17\{(1 + e^{\frac{(\mathbf{r}-1.97)}{0.65}})^{-1} + (1 + e^{\frac{(\mathbf{r}-1.97)}{0.65}})^{-1} - 1\}$$
(28)

Hence, the density distribution of light nuclei could be studied using crystal model approach. In the near future further study will be done for ¹²C, ¹⁶O and ²⁰Ne. As discussed in [20], the matter density is a summation of two clusters forming the light nucleus.

IV. CONCLUSION

The crystal model approach produces an imagination for nuclear structure and arrangement of nucleons inside light nuclei. Transformation parameters which are proportional to the radii of nuclei have been calculated using the crystal model approach. The crystal lattice space has been calculated for different nuclei. The binding energies for light nuclei have been calculated. The distribution of nucleons on the space of crystal has been discussed in a satisfied manner which reflects the success of the model to interpret phenomena in nuclear systems such as ⁸Be decay. Density distribution also has been predicted according to our model, crystal model, for such nuclei under investigation. The model failed to calculate the radius of ⁹Be, so modifications should be done on (1) to overcome discrepancy. Double folding calculations have been done for deuterons elastically scattered by ^{6,7}Li using matter densities distribution from the crystal model approach.

TABLE I CALCULATED PARAMETERS FROM CRYSTAL MODEL OF NUCLEUS APPROACH

(CMN)										
Ζ	А	R1 from	R2	(a)	U or					
		Literature	parameter	lattice	(Q-vale)					
		[22]	calculated	spacing	MeV					
			from CMN)		[24], [25]					
1	2 (deuteron)	2.1421 fm	0.8270	2.52fm	2.21					
2	4 (Alpha)	1.6755 fm	0.7515	1.970fm	28.4					
3	6 (Li)	2.589 fm	0.7690	2.1fm	32.1					
3	7 (Li)	2.444 fm	0.7530	2.24fm	39.2					
4	7 (Be)	2.646 fm	0.7718	2.24fm	46.55					
4	8 (Be)		0.8690		53.2					
4	9 (Be)	2.519 fm	0.7501	2.141fm	58.2					

TABLE II EXTRACTED BINDING ENERGY FROM CM Extracted value Binding energy From of a from CMN Ζ A Present work in (Present work) (MeV) 1 2 (deuteron) 3.35 2.193 2 4 (Alpha) 2.5 27.79 3 6 (Li) 2.630.33 3 7 (Li) 2.6 35.02 4 7 (Be) 2.4 51.01 4 8 (Be) 2.5 52

TABLE III Parameters Defining Ground State Distributions of ⁶ Li								
Distribution	а	b	с	α (fm-1)	β (fm-1)			
Apph	1.07	-0.19	0.130	0.5388	0.384			
Aρm	1.07	-0.20	0.138	0.575	0.398			
ΑρSΜ	0.71	0	0.239	0.613	0.505			

ACKNOWLEDGMENT

We acknowledge helpful discussions with Prof. Dr. Osama Hemada, Prof. Dr. Reda Elshater. We acknowledge Dr. Ahmed Elbanna and Prof. Dr. Mohammed Shehab as Computing process has been provided by them.

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