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Applying the Crystal Model Approach on Light Nuclei for Calculating Radii and Density Distribution

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***Abstract*—**A new model namely, the crystal model, has been modified to calculate radius and density distribution of light nuclei up to ^{8}Be. The crystal model has been modified according to solid state physics which uses the analogy between nucleon distribution and atoms distribution in the crystal. The model has analytical analysis to calculate the radius where the density distribution of light nuclei has been obtained from the analogy of crystal lattice. The distribution of nucleons over crystal has been discussed in general form. The equation used to calculate binding energy was taken from the solid-state model of repulsive and attractive force. The numbers of the protons were taken to control repulsive force where the atomic number was responsible for the attractive force. The parameter has been calculated from the crystal model was found to be proportional to the radius of the nucleus. The density distribution of light nuclei was taken as a summation of two clusters distribution as in ^{6}Li=alpha+deuteron configuration. A test has been done on the data obtained for radius and density distribution using double folding for d+^{6,7}Li with M3Y nucleon-nucleon interaction. Good agreement has been obtained for both radius and density distribution of light nuclei. The model failed to calculate the radius of ^{9}Be, so modifications should be done to overcome discrepancy.

***Keywords*—**Nuclear lattice, crystal model, light nuclei, nuclear density distributions.

I. Introduction

STUDYING nuclear properties for light nuclei has been attracted the attention of scientists and still under considerations. Light nuclei tend to form clusters as in case of ^{6,7}Li and ^{7,8,9}Be nuclei [1], [2]. In an article review [3], the author has discussed the anthropic considerations and the role of Hoyle-state and its closeness of the triple-alpha process to the energy of ^{4}He+^{8}Be. Many subjects have been discussed in a previous review which discussed the nuclear forces and chiral symmetry. Monte Carlo and ab-initio solutions have been used to study nuclear systems up to four nucleons. The quark mass dependence on nuclear forces is fulfilled to about 95%. For a long time, it was a goal of nuclear physics and astrophysics to find the mechanism of nuclear reactions and the structure of the nucleus for light, intermediate and heavy nuclei [4]. A lot of attempts have been done on nuclear systems to obtain and discuss the ambiguities of nuclear parameters like coupled-cluster approach [5], [6] ab-initio calculations [7], [8] and Coupled channel scattering [9]-[12] which have been studied by using lattice calculations at crystal lattice range in the Fermi scale. Variation of the Monte Carlo method also has been used in the same track [13], [14]. Guatam and Dean Lee paved the way for a general method for all nuclear reaction calculations on the crystal lattice. They studied the neutron radiative reaction (n,γ) on the lattice, and also included halo nuclei like ^{14}C(n, γ)^{15}C as an example of the applicability of their theory by calculating the photo-nuclear reaction rates lattice [4]. Michelle Pine et al. [15] demonstrated and tested the adiabatic projection method on neutron-deuteron scattering. They calculated the s-wave phase shift and inelastic scattering reactions on the lattice for neutron-deuteron scattering.

The cluster structure was a helpful phenomenon that scientists used to simplify their studies. Ulf Meißner has discussed clustering using ab-initio method on the lattice, he focused on alpha clustering because its properties as high binding energy and spin (iso-spin) saturated. Nuclei like ^{12}C, ^{16}O and ^{24}Mg have been included in the study [16]. The authors asked; how large a nucleus can be calculated from first principles using the framework of chiral nuclear EFT and what are the remaining challenges [17]; they answered their question by calculating ground states of α-type nuclei from ^{4}He to ^{28}Si. The Nuclear Lattice (EFT) or Nuclear Lattice Simulations method has been used by authors of the previous article, who succeeded in an explanation of clustering in lattice without any constrains which can be used as a benchmark for ab-initio calculations of larger nuclei using chiral nuclear EFT. Matter nuclear distribution for ^{6}Li, ^{6}He and ^{9}Be has been discussed theoretically using the three body models in [18].

Binding energy and nuclear density distribution have been calculated for light nuclei depending on alpha as a core of a cluster. It was chosen ^{6}Li, ^{6}He as (α+n+n) and ^{9}Be as α+α+n, where in case of ^{6}Li it was an approximation to take the neutron instead of the proton in the configuration. The authors have summarized the form of nuclear density distribution of light nuclei in the form of summation of two Gaussian in [19]. The authors in [1] have concluded the possibility of using dynamic cluster in the future to study ^{8}Be and ^{12}C. Research [20] has been performed to study nucleus as crystal. Our work is devoted to study the form of expected nuclear density distribution for light nuclei.

II. Crystal Model Approach

The potential between nucleons inside the nucleus will be calculated using the equations, which have been used to calculate a parameter proportional to the radius of the nucleus [21]:

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$$U = (4\varepsilon)\left(\frac{a}{r^{12}} - \frac{b}{r^{7}}\right) \quad (1)$$

where a is taken to be the number of the protons and b is the mass number of the nucleus and ε is taken an arbitrary constant. The first term of (1) has been used as the repulsion force where the second term is taken to be the attractive force. The crystal model has been developed to calculate a parameter that is proportional to the radii of light nuclei and to expect the form of matter density distribution of light nuclei. The cluster model and crystal model are close to each other where the distribution of nucleons inside nuclei in our point of view looks like the distribution of atoms on crystal. ^{6}Li as an example contains alpha as a base and two nucleons are located on the upper face of the crystal, and alpha is on the lower face, as shown in Figs. 2 and 3. The density distribution of ^{6}Li could be taken as a summation of two densities of alpha and deuteron of the general form:

$$\rho(r) = A\{B\exp(-C) + D\exp(-E) - F\} \quad (2)$$

where A, B, D and F are constants where C and E are functions of r. Let $A = \rho_0$, B=D=F=1 and hence, (2) could be modified to take the form:

$$\rho(r) = A\{[1 + \exp(-B)] + [1 + \exp(-B)] - 1\} \quad (3)$$

where $A = \rho_0$, and B is a function of r. For single parameterization, the nucleon density distribution in the nucleus is approximated by:

$$\rho(r) = \left(\frac{3}{2\pi R_m^2}\right)^{3/2}\exp\left(\frac{-3r^2}{2R_m^2}\right) \quad (4)$$

where R_m is the rms of matter radius of the nucleus. The standard Fermi of the nuclear matter distribution is written in the form:

$$\rho(r) = \rho_0\left(1 + e^{\frac{(r-R)}{a}}\right)^{-1} \quad (5)$$

which is suitable for description of intermediate and heavy nuclei. For light nuclei, it is suitable to modify (3) to be symmetrized in Fermi-density [17]:

$$\rho(r) = \rho_0\{(1 + e^{\frac{(r-R)}{a}})^{-1} + (1 + e^{\frac{(r-R)}{a}})^{-1} - 1\} \quad (6)$$

$$\rho(r) = \rho_0 \frac{\mathrm{Sinh}\left(\frac{R}{a}\right)}{\mathrm{Cosh}\left(\frac{R}{a}\right) + \mathrm{Cosh}\left(\frac{r}{a}\right)} \quad (7)$$

With normalization,

$$\rho_0 = \frac{3}{4\pi R^3}\frac{1}{1 + \left(\frac{\pi a}{R}\right)^2} \quad (8)$$

where R is the half-density radius.

If we compare (6) and (3) from the crystal model approach, we find that the first term of (9) $[\rho_0(1 + e^{\frac{(r-R)}{a}})^{-1}]$ represents the density of alpha in our model as simple Gaussian distribution where the second and third term, $\{\rho_0[\left(1 + e^{\frac{(r-R)}{a}}\right)^{-1} - 1]\}$ is represented in the density distribution of deuteron. Where B is taken to be $\frac{(r-R)}{a}$ from the analogy with our expectation and those parameters from [17]. R_m (rms of nuclear matter radius) is related to R and a by the relation:

$$\rho_0 = \left(\frac{3}{2}\right)^{1/2} R\left[1 + \left(\frac{7}{3}\right)\left(\frac{\pi a}{R}\right)^2\right]^{\frac{1}{2}} \quad (9)$$

In another form, from the principles of nuclear structure which states that overlapping between two constituents of nucleus is forbidden [18], the density distribution of many bodies systems may be written as:

$$\rho(R) = \sum_{\mathrm{cluster=i,j,k}} \rho_{\mathrm{cluster}}(R) \quad (10)$$

Applying (10) on ^{7}Li for example, it will be written as:

$$\rho(R) = \rho_\alpha(R) + \rho_t(R) \quad (11)$$

Also, the calculations of alpha nucleus have been done here using the relation:

$$\rho(r) = 0.17\{(1 + e^{\frac{(r-1.97)}{0.65}})^{-1}\} \quad (12)$$

where R=1.97fm, a=0.65fm, f and ρ_0=0.17(1/fm3).

III. Results and Discussion

A. Calculating Binding Energy for Light Nuclei

By imagining the nucleus as a crystal, where the nucleons are arranged on the corners of the crystal with analogy of atoms in the solid-state physics. The model has been developed by us to study light nuclei [20], and succeeded to explain up to alpha particles. For nuclei heavier than alpha, the model should be modified to explain the distribution of the nucleons inside the nucleus. The distribution of the nucleons in case ^{8}Be as hexagonal crystal is discussed. The two alphas will be taken on the up and down faces of a hexagonal crystal. The situation of ^{8}Be is very interesting as the life-time is about 10^{-16} sec which reflects the instability of such nuclei. Dynamic cluster model should be applied in case of instability from mechanical force. The distribution of nucleons inside the nucleus starts with cubic (on the corners of the crystal as one proton beside one neutron). There is a change in color from white tin to gray by changing the temperature which introduces another crystal structure. The same situation takes place in the case of the ^{8}Be nucleus where the structure in the moment of formation is cubic and during about 10^{-16} sec, the structure changes into a hexagonal close packed in a process known as a dynamic cluster of the nucleus. The change in structure inside the nucleus of ^{8}Be increases the volume of it, and decreases the nuclear density of ^{8}Be nucleus. ^{8}Be will disintegrate into two alphas. The shape of the nucleus in the case of ^{8}Be will change from cubic into hexagonal closed packed (HCP) produces stretching as shown in Fig. 1 where two alphas will occupy the two faces of HCP. The radius of ^{8}Be

have been calculated from (1) to be 0.8697992638 where U was taken to be Q-value as shown in Table I compared with calculated radii from the literature [22]. A new parameter α (R_1/R_2) has been calculated by dividing the radius of the nucleus taken from the literature and those calculated from the crystal model in the present work. Monte Carlo transfer matrix algorithm calculations have been achieved to distinguish between atomic number A≤8 and higher ones [23]. The value of transformation parameter α (R_1/R_2) has been calculated from the crystal model of nucleus (CMN) is ~2.2-3.5 which has low value at lower binding energy as in the case of deuteron 2.59. As seen in Table I, the calculated value of transformation parameter for ^{7}Li is smaller than that value calculated for ^{6}Li, which agrees with the values of them in the literature [2], [24], [25] (see Table I).

The aim now is to calculate the parameters of the hexagonal close packed (HCP).

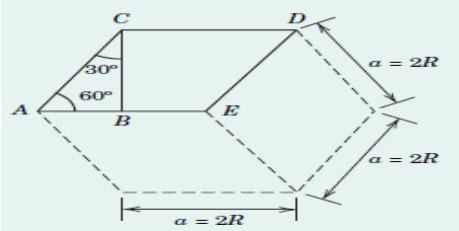

Fig. 1 Determination the parameters of hexagonal close packed

The volume of unit cell could be obtained by the product of the base area times the cell height, c. The lattice parameter (a) has a relation with the nucleon radius r as:

$$a=2r \tag{13}$$

$$V_c = 6r^2c\sqrt{3} \tag{14}$$

The volume of ^{8}Be nucleus has been calculated before and after separation into two alphas. r is taken to be 1 to 1.25fm and c is about two alphas radius to be 2×1.6755fm= 3.351fm. So, the minimum volume of ^{8}Be is:

$$V_c = 6r^2c\sqrt{3} = 6 * 1.25 * 1.25 * 3.351 * \sqrt{3} = 54.41\ \text{fm}^3 \tag{15}$$

Dynamical torque is required to break down ^{8}Be into two alphas; the dynamical torque is responsible for separation of two alphas from each other. The secrete on such configuration arises from the distance c between two alphas arranged on the faces of hexagonal close packed. The large distance between the two alphas gives an opportunity for two alphas to separate from each other. The changing on the allotropic transformation on ^{8}Be produces stretching and dynamic torque leads to separate ^{8}Be into two alphas. Another approach could be applied if the transformation parameter (α) is used for ^{8}Be shown in Table I used to calculate the radius of ^{8}Be on the separation moment; which was found in the range 3.05 fm. The radius of ^{8}Be on the moment of separation into two alphas is nearly equal the radius of ^{20}Ne which contains more than the twice number of nucleons of ^{8}Be nucleus and equal ^{24}Mg which three times of nucleon number of ^{8}Be. The solution of (1) succeeded to give us the transformation parameter (α) which proportional to the radius of nuclei from deuteron up to ^{8}Be as shown in Table I. The binding energy of ^{8}Be has been calculated from (1). There is a condition which should be applied on ^{8}Be, which is r≤3.5fm. The binding energy of ^{8}Be nucleus that is calculated from the crystal model approach: $U = (4 \times 10^3)\left(\frac{4}{(R*\alpha)^{12}} - \frac{8}{(R*\alpha)^{7}}\right)$ which is applied on the nuclei under consideration with R and α taken from Table II. The binding energy of ^{8}Be has been calculated:

$$U = (4 \times 10^3)\left(\frac{4}{(3.35)^{12}} - \frac{8}{(3.35)^{7}}\right) = 52\text{MeV} \tag{16}$$

where U is binding energy of ^{8}Be and ε is extracted from experimental data from literature and has the value 1000 MeV.

B. Matter Density Distribution for Light Nuclei

An approximation has been applied on the nuclei using crystal model approach that the density distribution ^{6}Li, ^{7}Li, and ^{7}Be nuclei are taken as a summation of two Gaussian distributions as one for the first cluster is alpha and the other part of nucleus is the residual cluster of nuclei. For example, ^{6}Li is taken as d+alpha or t+^{3}He (two configurations with the same probability) and the density distribution of ^{6}Li now is a summation of two Gaussian, one for deuteron and the other for alpha. That is could be described for the ^{7}Li≡α+t configuration as in cluster structure and hence, alpha will be the core of the nucleus and triton as valence (See Figs. 2 and 3).

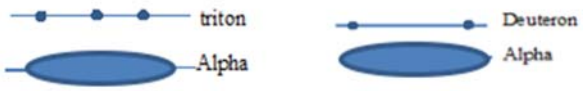

Fig. 2 Cluster model distribution for ^{7}Li and ^{6}Li

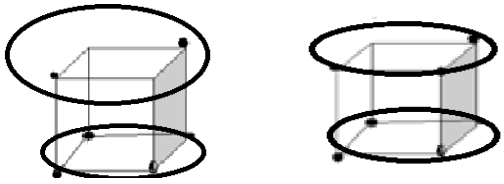

Fig. 3 Distribution of the nucleons on crystal model for ^{6,7}Li where the distribution of the nucleons of ^{6}Li as alpha on the base and deuteron on the upper face (left panel) and ^{7}Li case with alpha on the base and triton on the upper face (right panel)

The imagination of ^{6}Li in the present work agrees with the phenomenological charge and matter distributions and the shell-model distribution has the form [26]:

$$A\rho(r) = a\alpha^3 e^{-\alpha^2 r^2} + (b + c\beta^2 r^2)\beta^3 e^{-\beta^2 r^2} \tag{17}$$

The constants of ^{6}Li were taken from Table III.

Also, density distribution of ^{7}Li nucleus was taken in the form which is two summations of Gaussian [27]:

$$\rho(R) = \rho_0\left(1+\alpha(\frac{R}{a})^2\right)\exp\left(-(\frac{R}{a})^2\right), \quad (18)$$

where for harmonic oscillator a=1.77fm and α=0.327fm. ^{6}Li is taken in the crystal model approach to be alpha on the base of the lattice and deuteron on the upper face of the crystal which is shown in Fig. 4.

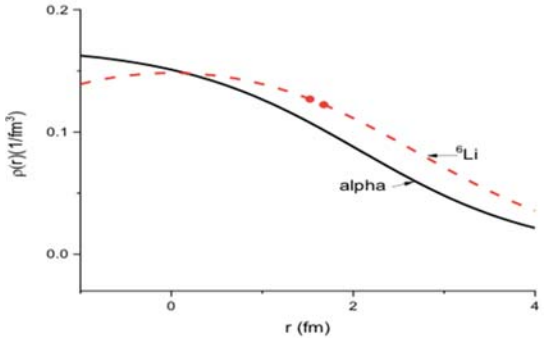

Fig. 4 Matter density distribution ^{6}Li nucleus from crystal model approach

The calculations have been done for ^{6}Li (^{6}Li≡α+d):

$$\rho(r) = 0.17\{(1+e^{\frac{(r-2.5)}{1.2}})^{-1} + (1+e^{\frac{(-r-2.5)}{1.2}})^{-1} - 1\} \quad (19)$$

where R=2.5fm, a=1.2fm (as very smooth nucleus) and ρ_0=0.17(1/fm^3). Also, the calculations of alpha nucleus have been done here using the relation:

$$\rho(r) = 0.17\{(1+e^{\frac{(r-1.97)}{0.65}})^{-1}\} \quad (20)$$

where R=1.97fm, a=0.65fm, and ρ_0=0.17(1/fm^3).

Double folding calculations have been applied on deuteron elastically scattered by ^{6,7}Li using density distribution from the present analysis as shown in Figs. 5 and 6.

Double folding model has been used to calculate microscopic potential in the present work. The real part of the calculated potential is given in the following form [28]:

$$V_F(r) = \int dr_P \int dr_1\, \rho_p(r_p)\rho_1(r_1)v(r_{pt}) \quad (21)$$

where ρp is the matter density distribution of the projectile, ρt is the matter density distribution of the target and rpt = r + rt - rp. M3Y form has been used for nucleon-nucleon interaction at the present calculation:

$$v(r) = 7999\,\frac{\exp(-4r)}{4r} - 2134\,\frac{\exp(-2.5r)}{2.5r} - 276\left(1-\frac{0.005\,E}{A_p}\right)\delta(R) \quad (22)$$

The form of nuclear central optical potential during double folding model calculations was taken in the shape: U(r)=VF(r)+iW(r), where V_F(r) is calculated from (21) and W(r) is phenomenological Woods-Saxon potential. The imaginary part was included into the Fresco program [29] to reproduce a differential cross section.

Double folding calculations for ^{6}Li+d with alpha transfer has been done using density distribution of ^{6}Li from crystal model approach at deuterons energy 25 MeV and 14.7 MeV, and was found to be in agreement with experimental data as shown in Figs. 5 and 6. The importance of using double folding potential appears at backward angles and the effect of spectroscopic amplitudes is very obvious for ^{7}Li elastically scattered by deuteron.

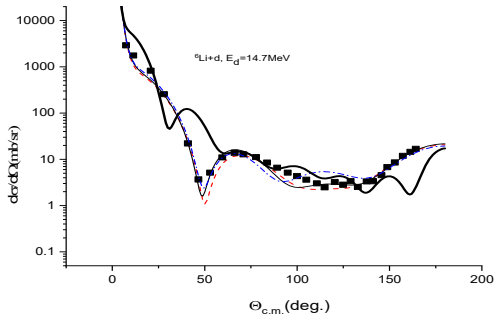

Fig. 5 Represents double folding calculations for ^{6}Li+d for elastic scattering with alpha transfer using density distribution of ^{6}Li from crystal model approach at deuterons energy 14.7 MeV where solid line represents CM, the dash line is the optical model and dash-dot line represents the coupled reaction channel. Experimental data were taken from [30]

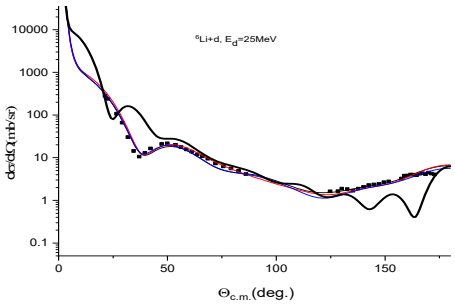

Fig. 6 Represents double folding calculations for ^{6}Li+d for elastic scattering with alpha transfer using density distribution of ^{6}Li from crystal model approach at deuterons energy 25 MeV where solid line represents CM, dash line is the optical model and dash-dot line represents the coupled reaction channel. Experimental data were taken from [31]

The density distribution of ^{7}Li nucleus is simpler than ^{6}Li as triton density distribution is not complicated as deuteron and will be taken in the form:

$$\rho(r) = \rho_0\{A\exp(-B) + A\exp(-C) - D\} \quad (23)$$

where ρ_0, A, and D are constants but B and C are functions of r. The same situation may be applied on ^{7}Li where the base is alpha and triton on the upper face of crystal as shown in Figs. 2 and 3. The density distribution of ^{7}Li is taken:

$$\rho(r) = 0.17\{(1+e^{\frac{(r-2.4)}{1.1}})^{-1} + (1+e^{\frac{(r-2.4)}{1.1}})^{-1} - 1\} \quad (24)$$

where R=2.4fm, a=1.1fm, and ρ^0=0.17(1/fm^3).

^{8}Be was assumed as alpha+alpha configuration that is because it decays into alpha+alpha during 10^{-16}s. The distribution of the nucleons on ^{8}Be is shown in Figs. 9 and 10. Dynamic cluster is needed to explain ^{8}Be decay into two alphas where crystal model could achieve such mission with analogy of solid state physics. The density distribution of ^{8}Be could be written in the form:

$$\rho(r) = \rho_0 A\{Bexp(-C) + Bexp(-C) - 1\} \quad (25)$$

The analysis of (8) gives an impression that the density distribution of ^{8}Be is two identical Gaussians with the form Bexp(-C).

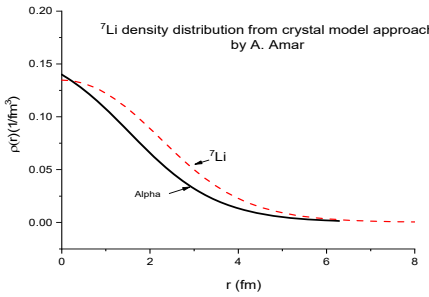

Fig. 7 Nuclear density distribution of ^{7}Li from the crystal model approach

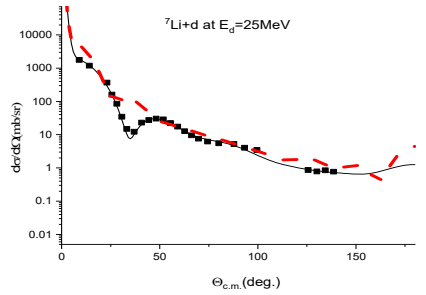

Fig. 8 Double folding calculations for deuteron elastically scattered by ^{7}Li with alpha transfer using density distribution of ^{7}Li from crystal model approach at deuterons energy 25 MeV. The solid line represents CM and dash line is the coupled reaction channel where experimental data were taken from [32]

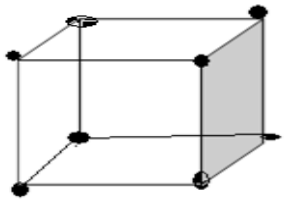

Fig. 9 The distribution of nucleons for ^{8}Be nucleus

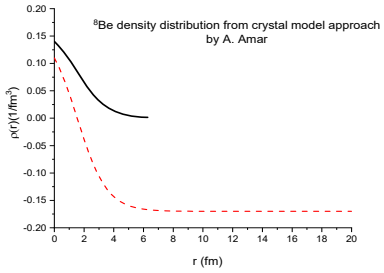

Fig. 10 Nuclear matter distribution of ^{8}Be nucleus using parameters from crystal model approach

$$\rho(r) = 0.17\{(1+e^{\frac{(r-1.97)}{0.65}})^{-1} + (1+e^{\frac{(r-1.97)}{0.65}})^{-1} - 1\}$$

General form for light nuclei:

$$\rho(r) = 0.17\{(1+e^{\frac{(r-1.97)}{0.65}})^{-1} + (1+e^{\frac{(r-1.97)}{0.65}})^{-1}\} \quad (26)$$

^{8}Be density distribution is just the summation of two alphas. Where R=1.97fm, a=0.65fm, and ρ_0=0.17(1/fm^3). So, from the matter density distribution for ^{8}Be we can expect the continuum state of ^{8}Be. ^{7}Be is a stable nucleus as it contains four protons and three neutrons with the configuration ^{7}Be≡^{4}He+^{3}He. The density distribution of alpha is just a simple Gaussian distribution {Aexp(-B)} where A, B and C are functions on r in the form:

$$\rho(r) = A\{exp(-B) + exp(-C) - 1\} \quad (27)$$

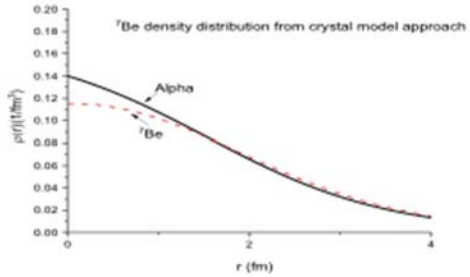

Fig. 11 ^{7}Be density matter distribution from crystal model approach

The same calculations will be applied on ^{7}Be and the results are presented as:

$$\rho(r) = 0.17\{(1+e^{\frac{(r-1.97)}{0.65}})^{-1} + (1+e^{\frac{(r-1.97)}{0.65}})^{-1} - 1\} \quad (28)$$

Hence, the density distribution of light nuclei could be studied using crystal model approach. In the near future further study will be done for ^{12}C, ^{16}O and ^{20}Ne. As discussed in [20], the matter density is a summation of two clusters forming the light nucleus.

IV. Conclusion

The crystal model approach produces an imagination for nuclear structure and arrangement of nucleons inside light

nuclei. Transformation parameters which are proportional to the radii of nuclei have been calculated using the crystal model approach. The crystal lattice space has been calculated for different nuclei. The binding energies for light nuclei have been calculated. The distribution of nucleons on the space of crystal has been discussed in a satisfied manner which reflects the success of the model to interpret phenomena in nuclear systems such as ^{8}Be decay. Density distribution also has been predicted according to our model, crystal model, for such nuclei under investigation. The model failed to calculate the radius of ^{9}Be, so modifications should be done on (1) to overcome discrepancy. Double folding calculations have been done for deuterons elastically scattered by ^{6,7}Li using matter densities distribution from the crystal model approach.

TABLE I
CALCULATED PARAMETERS FROM CRYSTAL MODEL OF NUCLEUS APPROACH (CMN)

Z	A	R1 from Literature [22]	R2 parameter calculated from CMN)	(a) lattice spacing	U or (Q-vale) MeV [24], [25]
1	2 (deuteron)	2.1421 fm	0.8270	2.52fm	2.21
2	4 (Alpha)	1.6755 fm	0.7515	1.970fm	28.4
3	6 (Li)	2.589 fm	0.7690	2.1fm	32.1
3	7 (Li)	2.444 fm	0.7530	2.24fm	39.2
4	7 (Be)	2.646 fm	0.7718	2.24fm	46.55
4	8 (Be)	---	0.8690		53.2
4	9 (Be)	2.519 fm	0.7501	2.141fm	58.2

TABLE II
EXTRACTED BINDING ENERGY FROM CM

Z	A	Extracted value of α from CMN (Present work)	Binding energy From Present work in (MeV)
1	2 (deuteron)	3.35	*2.193*
2	4 (Alpha)	2.5	*27.79*
3	6 (Li)	2.6	*30.33*
3	7 (Li)	2.6	*35.02*
4	7 (Be)	2.4	*51.01*
4	8 (Be)	2.5	*52*

TABLE III
PARAMETERS DEFINING GROUND STATE DISTRIBUTIONS OF ^{6}LI

Distribution	a	b	c	α (fm-1)	β (fm-1)
Aρph	1.07	-0.19	0.130	0.5388	0.384
Aρm	1.07	-0.20	0.138	0.575	0.398
AρSM	0.71	0	0.239	0.613	0.505

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