Fighter Aircraft Evaluation and Selection Process Based on Triangular Fuzzy Numbers in Multiple Criteria Decision Making Analysis Using the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

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***Abstract*—**This article presents a multiple criteria evaluation approach to uncertainty, vagueness, and imprecision analysis for ranking alternatives with fuzzy data for decision making using the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). The fighter aircraft evaluation and selection decision making problem is modeled in a fuzzy environment with triangular fuzzy numbers.

The fuzzy decision information related to the fighter aircraft selection problem is taken into account in ordering the alternatives and selecting the best candidate. The basic fuzzy TOPSIS procedure steps transform fuzzy decision matrices into matrices of alternatives evaluated according to all decision criteria. A practical numerical example illustrates the proposed approach to the fighter aircraft selection problem.

***Keywords*—**Triangular fuzzy number (TFN), multiple criteria decision making analysis, decision making, aircraft selection, MCDMA, fuzzy TOPSIS.

I. Introduction

FIGHTER aircraft evaluation and selection process is considered as a multiple criteria decision making analysis problem. Multiple criteria decision making analysis (MCDMA) theory ranks alternatives and selects the optimal alternative with respect to a set of conflicting decision criteria.

A multiple criteria decision making problem is characterized by the ratings of alternatives with respect to evaluation criteria and the importance weights criteria. The MCDMA model belongs to the class of vector optimization problems, where decision criteria can be divided into two groups: the criteria for which the maximum value is optimal and the criteria for which the minimum value is optimal.

Also, the MCDMA problems can be solved with the accuracy of multiple nondominant alternatives. Achieving a single solution can only be implemented based on some compromise scheme that reflects the decision maker's preferences.

The MCDMA methods for solving the decision-making problem can be divided into two large categories: compensatory methods that use the aggregation of all alternatives by all criteria and the solution of the resulting single-criteria problem, outranking methods are associated with the pairwise comparison procedure and stepwise aggregation [1-8].

The first MCDMA category includes methods composite programming [3-4], compromise programming [3-4], preference analysis for reference ideal solution (PARIS) [5-8], analytical hierarchical process (AHP) [9-11], VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [12-14], technique for order of preference by similarity to ideal solution (TOPSIS) [15-18], the second MCDMA category includes preference ranking organization method for enrichment evaluation (PROMETHEE) [19-22], and ÉLimination et Choix Traduisant la REalité (ELECTRE) [23-24].

Classical MCDMA methods assume that the ratings of alternatives and the importance weights of criteria are crisp numbers, but this assumption is impossible in real-life situations. Under conditions of uncertainty, ambiguity, and imprecise information, classical MCDMA models that require precise information may not be permanently applicable in real-life problems. Fuzzy set theory was proposed to manage uncertainty, vagueness, and imprecision in decision making [25].

Modeling using fuzzy sets is an effective way to formulate decision problems when available information is subjective and imprecise. Fuzzy numbers represent a specific range for a given value. Because of this range, it is easier for the evaluator to state a preference. In many practical situations, the preference of the expert is uncertain, which makes it difficult to make a numerical comparison. Also, various fuzzy [25-31], intuitionistic [32], neutrosophic [33], and plithogenic [34] decision making methods are widely used in the evaluation of uncertainty, vagueness, and imprecision information problems.

In fuzzy modeling, a single linguistic rating is translated into a fuzzy number consisting of multiple numbers. In this way, linguistic rating is reflected as a range. Both triangular and trapezoidal fuzzy numbers can be used for fuzzy set theory. Because of their computational ease, triangular fuzzy numbers (TFNs) are generally suitable for use in decision making problems [35].

In current practice, TFNs are generally convenient to work with because of their computational simplicity and are useful

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in promoting representation and information processing in a fuzzy environment.

This paper discusses the TOPSIS method with triangular fuzzy numbers for fighter aircraft selection problem. This mathematical method is very popular for solving multiple criteria analysis problems under certain conditions. This TOPSIS method regards the principle that the chosen alternative should have the shortest distance from the ideal solution and the longest distance from the negative ideal solution [15].

The MCDMA research provides enough information on the applicability of various methods of multiple criteria decision making [36-40].

The reminder of paper is organized as follows: Triangular fuzzy numbers with TOPSIS method are presented in Section 2. In Section 3, the proposed model was utilized for a case study of fighter aircraft selection. Finally, conclusions and future directions are presented in Section 4.

II. Methodology

A. Triangular Fuzzy Numbers

Fuzzy set theory assigns membership degrees to linguistic variables and treats them as probability distributions. Fuzzy set theory uses fuzzy numbers to achieve this. Although fuzzy numbers have various shapes such as trapezoidal, triangular or Gaussian, triangular fuzzy numbers (TFN) is the most preferred in the literature [35]. The outlines of fuzzy sets and TFNs are briefly given below.

Definition 1: A fuzzy number is a special fuzzy set $Z=\{(x,\mu_Z(x),x\in\Re)\}$, where *x* takes its values on the real line, $\Re:-\infty\leq x\leq\infty$ and $\mu_Z(x)$ is a membership function in the closed interval [0,1].

Definition 2: A triangular fuzzy number T_i can be defined by a triplet (l_i,m_i,u_i). A TFN expresses the relative strength of each pair of elements in the same hierarchy and can be denoted as $T_i=(l_i,m_i,u_i)$ where $0\leq l_i\leq m_i\leq u_i\leq 1$. The triplet parameters (l_i,m_i,u_i) indicate the lower bound value (l_i), the center (m_i), and the upper bound value (u_i) in a fuzzy event, respectively. Triangular type membership function $\mu_T(x)$ of *T* fuzzy number is

$$\mu_T(x)=\begin{cases}0 & ,x<l\\ (x-l)/(m-l) & ,l\leq x\leq m\\ (u-x)/(u-m) & ,m\leq x\leq u\\ 0 & ,x>u\end{cases} \tag{1}$$

Consider two TFNs $T_1=(l_1,m_1,u_1)$, $T_2=(l_2,m_2,u_2)$, and γ a positive scalar number. The basic operational laws related to triangular fuzzy numbers (TFNs), T_1 and T_2, are shown as respectively

$$(l_1,m_1,u_1)\oplus(l_2,m_2,u_2)=(l_1+l_2,m_1+m_2,u_1+u_2) \tag{2}$$

$$(l_1,m_1,u_1)\otimes(l_2,m_2,u_2)=(l_1l_2,m_1m_2,u_1u_2) \tag{3}$$

$$\gamma\otimes(l_i,m_i,u_i)=(\gamma l_i,\gamma m_i,\gamma u_i)$$

$$(l_1,m_1,u_1)/(l_2,m_2,u_2)\cong(l_1/u_2,m_1/m_2,u_1l_2) \tag{4}$$

$$(l_i,m_i,u_i)^{-1}\approx\left(\frac{1}{u_i},\frac{1}{m_i},\frac{1}{l_i}\right) \tag{5}$$

where $0\leq l_i\leq m_i\leq u_i\leq 1$, l_i and u_i stand for the lower and upper values of the support of T_i, and m_i stands for the modal values.

Definition 3: The graded mean integration representation (GMIR)

Defuzzifed values are obtained by applying GMIR. Let $a_j=(l_j,m_j,u_j)$ be TFN and GMIR $R(a_j)$ of a_i can be calculated as

$$R(a_j)=\frac{l_j+4m_j+u_j}{6} \tag{6}$$

B. Classical TOPSIS Programming

The technique for order of preference by similarity to ideal solution (TOPSIS) method is a mathematical MCDMA method that has been used in numerous real-life problems and extended in different uncertain environments. In the TOPSIS method, the evaluation process of alternatives is conducted with respect to the distances from the ideal and anti-ideal solutions.

Suppose that, given a set of alternatives I, $a_i=(a_1,...,a_i)$, $i\in\{i=1,...,I\}$, a set of criteria J, $g_j=(g_1,...,g_j)$, $j\in\{j=1,...,J\}$, and the importance weight of each criterion (ω_j, $j\in\{j=1,...,J\}$) is known. The procedural steps of TOPSIS method are presented as follows [15]:

Step 1. The construction of a decision matrix

A multiple criteria decision making problem can be concisely expressed in matrix format as

$$X=\begin{pmatrix}a_1\\ \vdots\\ a_i\end{pmatrix}\begin{pmatrix} \overset{g_1}{x_{11}} & \cdots & \overset{g_j}{x_{1j}}\\ \vdots & \ddots & \vdots\\ x_{i1} & \cdots & x_{ij}\end{pmatrix}_{ixj} \tag{7}$$

where $X=(x_{ij})_{ixj}$ represents the decision matrix and x_{ij} is the value of *i*th alternative with respect to *j*th indicator g_j

Step 2. Determination of the normalized values of the decision matrix

The linear scale transformation is used to transform the various criteria scales into a comparable scale. Therefore, it is possible to obtain the normalized decision matrix denoted by $R = (r_{ij})_{ixj}$

$$r_{ij} = \begin{cases} \dfrac{x_{ij}}{\max_i x_{ij}} & if \quad j \in B \\ \dfrac{\max_i x_{ij}}{x_{ij}} & if \quad j \in C \end{cases} \tag{8}$$

where $i = 1,...,m,...,I$ (set of alternatives), and $j = 1,...,n,...,J$ (set of criteria), B and C are the sets of benefit and cost criteria, respectively. The normalization method preserves the property that the ranges of normalized matrix elements (r_{ij}) belong to [0,1].

Step 3. Calculation of the weighted normalized values

Considering the different importance weights of the criteria, the weighted normalized decision matrix $V = (v_{ij})_{ixj}$ is created as follows

$$v_{ij} = \omega_j r_{ij} \tag{9}$$

where (ω_j) is the importance weights of criteria, and (r_{ij}) is the normalized matrix element.

Step 4. Determination of the ideal and anti-ideal solutions based on the weighted normalized values

The positive ideal solution (a_i^*) and negative ideal solution (a_i^-) are defined as

$$a_i^* = \left\{v_1^*,...,v_j^*\right\} = \left\{(max_i\, v_{ij} \mid j \in B), (\min_i v_{ij} \mid j \in C\right\} \tag{10}$$

$$a_i^- = \left\{v_1^-,...,v_j^-\right\} = \left\{(max_i\, v_{ij} \mid j \in B), (\min_i v_{ij} \mid j \in C\right\} \tag{11}$$

where B and C are the sets of benefit and cost criteria, respectively.

Step 5. Calculation of the Euclidean distance of alternatives from the ideal (d_i^*) and anti-ideal (d_i^-) solutions

The distance of each alternative (a_i) from (a_i^*) and (a_i^-) are calculated as

$$d_i^+ = \sqrt{\sum_{j=1}^{J} (v_{ij} - v_j^*)^2} \tag{12}$$

$$d_i^- = \sqrt{\sum_{j=1}^{J} (v_{ij} - v_j^-)^2} \tag{13}$$

Step 6. Calculation of the closeness coefficient (CC_i) of each alternative

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \tag{14}$$

Step 7. Rank the alternatives in decreasing order of the closeness coefficient values (CC_i)

C. Fuzzy TOPSIS Programming

In this section, the problem in which the decision maker makes decisions in linguistic form is addressed. The procedural stages of the fuzzy TOPSIS method are considered.

Suppose that, given a set of alternatives I, $a_i = \left(a_1,...,a_i\right)$, $i \in \{ i = 1,...,I \}$, a set of criteria J, $g_j = \left(g_1,...,g_j\right)$, $j \in \{ j = 1,...,J \}$, and the importance weight of each criterion (ω_j, $j \in \{ j = 1,...,J \}$) is known. The procedural steps of fuzzy TOPSIS method are presented as follows [14]:

Step 1. Determine the linguistic variables for the decision making problem

First, the criteria weight importance and linguistic variables for decisions with triangular fuzzy numbers are defined.

Table 1. Linguistic variables for the decision problem

Linguistic variables for the weight of criteria	Triangular fuzzy numbers	Linguistic variables for the performance ratings	Triangular fuzzy numbers
Absolutely Low (AL)	(0.0,0.0,0.1)	Absolutely Poor (AP)	(0.0,0.0,0.1)
Very Low (VL)	(0.0,0.1,0.2)	Very Poor (VP)	(0.0,0.1,0.2)
Low (L)	(0.1,0.2,0.3)	Poor (P)	(0.1,0.2,0.3)
Medium Low (ML)	(0.3,0.4,0.5)	Medium Poor (MP)	(0.3,0.4,0.5)
Medium (M)	(0.4,0.5,0.6)	Medium (M)	(0.4,0.5,0.6)
Medium High (MH)	(0.5,0.6,0.7)	Medium Good (MG)	(0.5,0.6,0.7)
High (H)	(0.7,0.8,0.9)	Good (G)	(0.7,0.8,0.9)
Very High (VH)	(0.8,0.9,1.0)	Very Good (VG)	(0.8,0.9,1.0)
Absolutely High(AH)	(0.9,1.0,1.0)	Absolutely Good (AG)	(0.9,1.0,1.0)

Step 2. Establish the linguistic decisions as the decision matrix with m - number of alternatives and n - number of criteria. The MCDMA problem representation is given by

A fuzzy multiple criteria decision making problem can be concisely expressed in matrix format as

$$X = \begin{pmatrix} a_1 \\ \vdots \\ a_i \end{pmatrix} \begin{pmatrix} \overset{g_1}{x_{11}} & \cdots & \overset{g_j}{x_{1j}} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} \end{pmatrix}_{ixj} \tag{15}$$

where $X = (x_{ij})_{ixj}$ represents the decision matrix and x_{ij} is the value of ith alternative with respect to jth indicator g_j. $x_{ij} = (a_{ij}, b_{ij}, c_{ij})$ is representation of triangular fuzzy numbers of linguistic terms.

Step 3. Calculate normalized fuzzy decision matrix $R = (r_{ij})_{ixj}$, $i = 1,...,I$, $j = 1,...,J$

The linear scale transformation is used to transform the various criteria scales into a comparable scale. Therefore, it is possible to obtain the normalized fuzzy decision matrix denoted by $R = (r_{ij})_{ixj}$

$$r_{ij} = (\frac{a_{ij}}{d_l^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}), j \in B \quad (16)$$

$$r_{ij} = (\frac{a_j^*}{d_{ij}}, \frac{a_j^*}{c_{ij}}, \frac{a_j^*}{b_{ij}}), j \in C \quad (17)$$

where

$$\begin{aligned} d_j^* &= \max_i d_{ij}, j \in B \\ a_j^* &= \min_i a_{ij}, j \in C \end{aligned} \quad (18)$$

where B and C represent the maximization criteria set, and minimization criteria set respectively. The normalization method preserves the property that the ranges of normalized triangular fuzzy numbers (r_{ij}) belong to [0,1].

Step 4. Calculate weighted normalized fuzzy decision matrix

Considering the different importance weights of the criteria, the weighted normalized fuzzy decision matrix $V = (v_{ij})_{ixj}$ is created as follows

$$V = (v_{ij}), i = 1,...m,...I, j = 1,...n,...J \quad (19)$$

where $v_{ij} = v_{ij} \otimes \omega_j, i = 1,...m,...I, j = 1,...n,...J$

where (ω_j) is the importance weights of criteria, and (r_{ij}) is the normalized element.

Step 5. Determine positive and negative ideal solutions

The fuzzy positive ideal solution (a^+) and fuzzy negative ideal solution (a^-) are defined as

$$\begin{aligned} a^+ &= (v_1^+, ..., v_n^+) \\ a^- &= (v_1^-, ..., v_n^-) \end{aligned} \quad (20)$$

where

$$\begin{aligned} v_1^+ &= (1,1,1) \\ v_1^- &= (0,0,0) \end{aligned} \quad (21)$$

Step 6. Calculate distances between decisions and positive and negative ideal solutions

$$d_i^+ = \sum_{j=1}^{J} d(v_{ij}^+, v_j^+), j = 1,...,n,...J \quad (22)$$

$$d_i^- = \sum_{j=1}^{J} d(v_{ij}^-, v_j^-), j = 1,...,n,...J \quad (23)$$

The vertex method is defined to calculate the distance between fuzzy numbers as

$$d(A,B) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]} \quad (24)$$

Step 7. Calculate closeness coefficient (CC_i) for all alternatives

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-}, i = 1,...,m,...I \quad (25)$$

Step 8. Determine acceptance level of decisions

Table 2. Acceptance criteria

Closeness Coefficient (CC_i)	Evaluation
$CC_i \in [0, 0.2)$	Not recommended
$CC_i \in [0.2, 0.4)$	Recommended with high risk
$CC_i \in [0.4, 0.6)$	Recommended with low risk
$CC_i \in [0.6, 0.8)$	Acceptable
$CC_i \in [0.8, 1.0)$	Accepted and preferred

Step 9. Ranking the alternatives

The ranking of the alternatives is based on the final values of the utility functions. It is desirable that an alternative has the highest possible value of the utility function.

Step 10. Select optimal decision with maximum of closeness coefficient (CC_i)

D.Implementing Procedure

The methodological calculations for multiple criteria decision making fuzzy TOPSIS procedure are implemented as follows

Step 1. Identify the evaluation criteria and alternatives.

Step 2.Select the appropriate linguistic variables for the importance weight of criteria and the linguistic ratings for alternatives with respect to criteria.

Step 3. Construct the fuzzy decision matrix

Step 4. Construct the normalized fuzzy decision matrix

Step 5. Construct the weighted normalized fuzzy decision matrix

Step 6. Determine the fuzzy positive ideal solution and fuzzy negative ideal solution

Step 7.Calculate the distance of each alternative from positive and negative ideal solutions, respectively

Step 8. Calculate the closeness coefficient of each alternative

Step 9. Determine acceptance level of decisions

Step 10. According to the closeness coefficient, determine the ranking order of all alternatives

III. Application

In this section, selection of fighter aircraft using triangular fuzzy numbers for multiple criteria decision making analysis problem is considered as a practical numerical example.

Consider a multiple attribute decision making problem with *m* alternatives and *n* attributes. Let $a_i = \{a_1, a_2, ..., a_m\}$, $g_j = \{g_1, g_2, ..., g_n\}$,and $\omega_j = \{\omega_1, \omega_2, ..., \omega_n\}$ denote the alternatives, attributes, and criteria importance respectively.

Alternatives and attributes for the decision making problem were determined from the analysis of the research data on aircraft evaluation and selection problems. The decision dataset can be established using characteristics of the alternatives with respect to the decision attributes. In this decision problem, fuzzy dataset was used to generate decision solutions

Alternatives are indicated by α_1, α_2, and α_3, a set of fighter aircraft candidates. Attributes, characteristics of fighter aircraft are defined as follows:

g_1 : Maximum takeoff weight (kg)
g_2 : Payload (kg)
g_3 : Avionics
g_4 : Maximum speed (km/h)
g_5 : Range (km)
g_6 : Service ceiling (km)
g_7 : Combat radius (km)
g_8 : Maneuverability
g_9 : Reliability

In the fuzzy dataset, optimal decision is maximum for all decision criteria. Application of fuzzy TOPSIS method is considered for this decision analysis problem.

Table 3. Presentation of decisions in linguistic decision matrix

g_j	ω_j	α_1	α_2	α_3
g_1	ω_1	G	G	G
g_2	ω_2	G	MG	G
g_3	ω_3	VG	MG	MG
g_4	ω_4	G	G	MG
g_5	ω_5	G	G	G
g_6	ω_6	VG	MG	MG
g_7	ω_7	G	G	G
g_8	ω_8	G	MG	G
g_9	ω_9	G	G	G

The importance weight of criteria is directly assigned. The vector of criteria importance weights (ω_j) is presented as

ω_j ={H,VH,H,MH,MH,M,H,VH,VH]

Table 4. Convert linguistic presentation in triangular fuzzy numbers

g_j	ω_j	α_1	α_2	α_3
g_1	ω_1	(0.7,0.8,0.9)	(0.7,0.8,0.9)	(0.7,0.8,0.9)
g_2	ω_2	(0.7,0.8,0.9)	(0.5,0.6,0.7)	(0.7,0.8,0.9)
g_3	ω_3	(0.8,0.9,1.0)	(0.5,0.6,0.7)	(0.5,0.6,0.7)
g_4	ω_4	(0.7,0.8,0.9)	(0.7,0.8,0.9)	(0.5,0.6,0.7)
g_5	ω_5	(0.7,0.8,0.9)	(0.7,0.8,0.9)	(0.7,0.8,0.9)
g_6	ω_6	(0.8,0.9,1.0)	(0.5,0.6,0.7)	(0.5,0.6,0.7)
g_7	ω_7	(0.7,0.8,0.9)	(0.7,0.8,0.9)	(0.7,0.8,0.9)
g_8	ω_8	(0.7,0.8,0.9)	(0.5,0.6,0.7)	(0.7,0.8,0.9)
g_9	ω_9	(0.7,0.8,0.9)	(0.7,0.8,0.9)	(0.7,0.8,0.9)

The vector of criteria importance weights (ω_j) is determined as

ω_j ={ (0.7,0.8,0.9), (0.8,0.9,1.0), (0.7,0.8,0.9), (0.5,0.6,0.7), (0.5,0.6,0.7), (0.4,0.5,0.6), (0.7,0.8,0.9), (0.8,0.9,1.0), (0.8,0.9,1.0) }.

After the linguistic values were converted into corresponding triangular fuzzy numbers, the fuzzy decision matrix is normalized as shown in Table 5.

Table 5. Calculated normalized fuzzy decision matrix

g_j	ω_j	α_1	α_2	α_3
g_1	ω_1	(0.7,0.8, 0.9)	(0.8,0.9,1)	(0.8,0.9,1)
g_2	ω_2	(0.8,0.9,1.0)	(0.6,0.7,0.8)	(0.8,0.9,1)
g_3	ω_3	(0.7,0.8,0.9)	(0.6,0.7,0.8)	(0.6,0.7,0.8)
g_4	ω_4	(0.5,0.6,0.7)	(0.8,0.9,1)	(0.6,0.7,0.8)
g_5	ω_5	(0.5,0.6,0.7)	(0.8,0.9,1)	(0.8,0.9,1)
g_6	ω_6	(0.4,0.5,0.6)	(0.6,0.7,0.8)	(0.6,0.7,0.8)
g_7	ω_7	(0.7,0.8,0.9)	(0.8,0.9,1)	(0.8,0.9,1)
g_8	ω_8	(0.8,0.9,1.0)	(0.6,0.7,0.8)	(0.8,0.9,1)
g_9	ω_9	(0.8,0.9,1.0)	(0.8,0.9,1)	(0.8,0.9,1)

After normalization procedure, a weighted normalized fuzzy decision matrix is formed by multiplying the corresponding vector of criteria weights, and the weighted normalized fuzzy decision matrix is shown in Table 6.

Table 6. Calculated weighted normalized fuzzy decision matrix

g_j	ω_j	α_1	α_2	α_3
g_1	ω_1	(0.49,0.64,0.81)	(0.54,0.71,0.90)	(0.54,0.71,0.90)
g_2	ω_2	(0.56,0.72,0.90)	(0.44,0.60,0.78)	(0.62,0.80,1.00)
g_3	ω_3	(0.56,0.72,0.90)	(0.39,0.53,0.70)	(0.39,0.53,0.70)
g_4	ω_4	(0.35,0.48,0.63)	(0.39,0.53,0.70)	(0.28,0.40,0.54)
g_5	ω_5	(0.35,0.48,0.63)	(0.39,0.53,0.70)	(0.39,0.53,0.70)
g_6	ω_6	(0.32,0.45,0.60)	(0.22,0.33,0.47)	(0.22,0.33,0.47)
g_7	ω_7	(0.49,0.64,0.81)	(0.54,0.71,0.90)	(0.54,0.71,0.90)
g_8	ω_8	(0.56,0.72,0.90)	(0.44,0.60,0.78)	(0.62,0.80,1.00)
g_9	ω_9	(0.56,0.72,0.90)	(0.62,0.80,1.00)	(0.62,0.80,1.00)

After determining positive ideal solutions and negative ideal solutions, the distances of each alternative from positive ideal solutions and negative ideal solutions with respect to each criterion are calculated using vertex method.

Table 7. Calculated distance between decisions and positive ideal solutions

g_j	ω_j	α_1	α_2	α_3
g_1	ω_1	0,070967	0,050165	0,050165
g_2	ω_2	0,054750	0,084300	0,056070
g_3	ω_3	0,054750	0,100700	0,100700
g_4	ω_4	0,097148	0,092015	0,135307
g_5	ω_5	0,099457	0,086585	0,091048
g_6	ω_6	0,108657	0,154300	0,154300
g_7	ω_7	0,053339	0,049692	0,043475
g_8	ω_8	0,040154	0,069836	0,040928
g_9	ω_9	0,038970	0,032035	0,032035

The calculated distance values are shown in Table 7 to Table 8.

Table 8. Calculated distances between decisions and negative ideal solutions

g_j	ω_j	α_1	α_2	α_3
g_1	ω_1	0,217633	0,268683	0,268683
g_2	ω_2	0,273667	0,193745	0,337860
g_3	ω_3	0,273667	0,154280	0,154280
g_4	ω_4	0,124967	0,154280	0,088930
g_5	ω_5	0,124967	0,154280	0,154280
g_6	ω_6	0,110817	0,063045	0,063045
g_7	ω_7	0,217633	0,268683	0,268683
g_8	ω_8	0,273667	0,193745	0,337860
g_9	ω_9	0,273667	0,337860	0,337860

Finally, after calculating the distances between the alternatives and the positive and negative ideal solutions, the closeness coefficients for all alternatives are calculated and the ranking results are given in Table 9.

Table 9. Calculated closeness coefficients (CC_i) and ranking of alternatives

a_i	d_i^+	d_i^-	CC_i	Ranking
α_1	0,618192	1,890683	0,753598	1
α_2	0,719628	1,788601	0,713093	3
α_3	0,704028	2,011481	0,740738	2

The ranking order of the alternatives was determined according to the closeness coefficients given in Table 9, and the priorities of the fighter aircraft alternatives are $a_1 \succ a_3 \succ a_2$.

The first alternative is determined as the most suitable fighter aircraft for the Air Force, as it is closer to positive ideal solution and further from negative ideal solution. Similarly, the second alternative with the lowest closeness coefficient was determined as the least preferred because it was farther from positive ideal solution and closer to negative ideal solution.

According to the acceptance criteria of the alternatives, all military combat aircraft alternatives are designated as "Acceptable". Alternative α_1 is optimal selection since the closeness coefficients are ordered from largest to smallest as $CC_1 \succ CC_3 \succ CC_2$. Therefore, alternative (α_1) is selected as the best military combat aircraft candidate for the Air Force.

IV. Conclusion

Fuzzy set theory can be used to overcome problems involving uncertainty, vagueness, and imprecision. More reliable results can be obtained if the importance weights of criteria are integrated with the fuzzy set theory that best expresses the human thought and reasoning structure.

In this study, therefore, fuzzy sets are combined with fuzzy TOPSIS method. In addition, the vector of importance weights of criteria and performance ratings were taken into account by using linguistic variables instead of exact values in the decision making process.

The fuzzy TOPSIS programming method contributes to the decision making analysis problem for fighter aircraft selection. The fuzzy TOPSIS programming is widely used as a mathematical method to solve decision analysis problems.

Also, practical application stages of fuzzy TOPSIS method with triangular fuzzy numbers were discussed using the decision making process. Fighter aircraft selection problem with nine decision criteria and three alternatives was considered as a practical decision making problem.

The results of the solution at all stages for the decision making problem were presented. From the fuzzy TOPSIS decision analysis results, it was concluded that alternative (a_1) fighter aircraft was selected as the best aircraft for the Air Force.

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