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Strongly Coupled Finite Element Formulation of Electromechanical Systems with Integrated Mesh Morphing using Radial Basis Functions

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Abstract**—**The paper introduces a method to efficiently simulate nonlinear changing electrostatic fields occurring in micro-electromechanical systems (MEMS). Large deflections of the capacitor electrodes usually introduce nonlinear electromechanical forces on the mechanical system. Traditional finite element methods require a time-consuming remeshing process to capture exact results for this physical domain interaction. In order to accelerate the simulation process and eliminate the remeshing process, a formulation of a strongly coupled electromechanical transducer element will be introduced which uses a combination of finite-element with an advanced mesh morphing technique using radial basis functions (RBF). The RBF allows large geometrical changes of the electric field domain while retain high element quality of the deformed mesh. Coupling effects between mechanical and electrical domains are directly included within the element formulation. Fringing field effects are described accurate by using traditional arbitrary shape functions.

Keywords**—**Electromechanical, electric field, transducer, simulation, modeling, finite-element, mesh morphing, radial basis function.

I. Introduction

THE continually improving manufacturing technology of micro-electromechanical systems (MEMS) leads to increasingly smaller and more geometrical complex structures. With the decreasing size of functional elements, the physical behavior can show an increase of nonlinear effects. In electrostatic driven actuators or sensors, the large deflection of the mechanical structures results in nonlinear changing electric fields. In addition, fringing field effects can become more dominant. At this point electromechanical lumped elements are not valid anymore. To efficiently describe the behavior of such complex systems a strongly coupled electromechanical finite element is introduced. This element is combined with an advanced mesh morphing technique to avoid time consuming remeshing of the electric domain. By using traditional shape function in the element formulation, the new element is compatible with the interfacing mechanical structure. The strong coupling between mechanical and electrical domain of the element gives the possibility to apply small signal analysis as modal or harmonic analysis.

In the following first the state of the art of simulating electromechanical coupled domains is investigated. After this the theory behind the new element formulation and the mesh morphing algorithm using radial basis functions is depicted. The resulting transducer element is validated on well-known electrode setups used in MEMS. The advantages of the element using an integrated and automated mesh morphing algorithm is shown on some more complex examples.

II. State of the Art

The investigation of electromechanical coupled field elements has been done with different methods in the literature. There are mainly weakly coupled and strongly coupled approaches. The weakly coupled approaches need sequentially computation of the mechanical and electrical domain [1]. This leads to slower convergence rates and the use of small signal analysis procedures is not possible.

Using boundary element method (BEM), a meshless electromechanical domain can be described [2]. The problem here is the element formulation itself which requires special solvers and the BEM domain must interfacing with FE domain in non-traditional ways.

Strongly coupled electromechanical elements which can convert electrostatic energy into mechanical energy and vice versa were formulated with different approaches. A variational approach was applied to a 2D domain in [3]. Another energy based concept was proven in [4] on a 2D with only triangular shape function which limits the complexity. In all approaches the deformation of the mesh is one of the key problems. The quality of the mesh correlates with the quality of the results.

III. Element Formulation

The coupling between electrical and mechanical domains can be described as an energy transfer between these two domains. The capacitance changes with a mechanical motion of the electrodes. Thereby electrical energy is converted to mechanical and produces a nonlinear mechanical force on the domain interface. For this reason, we are using an energy approach. The electrostatic energy W_e is given in (1) where ε is the electric permittivity, E is the electric field and Ω the domain volume.

$$W_\mathrm{e} = \frac{1}{2}\int_\Omega E^T \, \varepsilon E \, \mathrm{d}\Omega \tag{1}$$

The electric field E is a function of the electric potential ϕ and the shape function $N(P)$ depending on the global node coordinates P of the element [1].

$$E = -\nabla N(P)\,\phi = -J^{-1}\nabla N(\xi)\,\phi \tag{2}$$

Applying the virtual work principle, the tangent element stiffness matrix which couples mechanical degree of freedom (DOFs) x and electrical DOFs ϕ can be modeled as following [4]:

$$\begin{bmatrix} K_{xx} & K_{x\phi} \\ K_{\phi x} & K_{\phi\phi} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta\phi \end{bmatrix} = \begin{bmatrix} \Delta F_{\mathrm{el}} \\ \Delta Q \end{bmatrix} \tag{3}$$

The components of the right-hand side are the electrostatic forces F_{el} and charges Q. Using the fact that the shape functions is a function of the isoparametric coordinates ξ and the Jacobi matrix J, the electric energy can be differentiated against mechanical and electric DOFs, since the Jacobi matrix can be expressed as a function of mechanical DOFs. The domain volume Ω is also a function of the Jacobi matrix which makes it differentiable.

$$F_{\mathrm{el}} = \frac{\partial W_{\mathrm{e}}}{\partial x} = \int \frac{\partial E}{\partial x}^{T} \varepsilon E \,\mathrm{d}\Omega + \int \frac{1}{2} E^{T} \varepsilon E \frac{\partial \Omega}{\partial x} \tag{4}$$

$$Q = \frac{\partial W_{\mathrm{e}}}{\partial \phi} = \int \frac{\partial E}{\partial \phi}^{T} \varepsilon E \,\mathrm{d}\Omega \tag{5}$$

The components of the stiffness matrix consist of 3 different terms since the matrix is symmetrical ($K_{\phi x} = K_{x\phi}^{T}$). All terms are generated in the same way as the right-hand side by using the product rule. We drop high order derivatives, but hold mixed partial derivatives.

$$K_{\phi\phi} = \frac{\partial^2 W_{\mathrm{e}}}{\partial \phi^2} = \int \frac{\partial E}{\partial \phi}^{T} \varepsilon \frac{\partial E}{\partial \phi} \,\mathrm{d}\Omega \tag{6}$$

$$K_{xx} = \frac{\partial^2 W_{\mathrm{e}}}{\partial x^2} = \int \frac{\partial E}{\partial x}^{T} \varepsilon \frac{\partial E}{\partial x} \,\mathrm{d}\Omega + 2\int \frac{\partial E}{\partial x}^{T} \varepsilon E \frac{\partial \Omega}{\partial x} \tag{7}$$

$$K_{\phi x} = \frac{\partial^2 W_{\mathrm{e}}}{\partial x \partial \phi} = \int \frac{\partial^2 E}{\partial x \partial \varphi}^{T} \varepsilon E \,\mathrm{d}\Omega + \int \frac{\partial E}{\partial x}^{T} \varepsilon \frac{\partial E}{\partial \varphi} \,\mathrm{d}\Omega + \int \frac{\partial E}{\partial \varphi}^{T} \varepsilon E \frac{\partial \Omega}{\partial x} \tag{8}$$

$K_{\phi\phi}$ describes the electric field, K_{xx} is the electrostatic softening effect on the mechanical structure and $K_{\phi x}$ is the strong coupling between mechanical and electrical domain. These definitions make nonlinear coupled field simulation possible.

It is important to mention that only on the domain interface nodes the mechanical DOFs are considered in the element definition. These mechanical DOFs are coincident with the ones on the mechanical structure and don't create additional DOFs. In the electrical domain every node has an electrical DOF. The coordinates of non-interface nodes of all nodes are updated using an advanced mesh morphing algorithm.

IV. Mesh Morphing

The main problem on quite every electromechanical FE technology are large geometrical changes. This is caused by the missing mechanical force equilibrium. The electrical domain usually consists of air, which has no mechanical properties in a traditional way. This makes it hard to apply force balancing algorithms. Remeshing can be done, but it's the one of the most time-consuming operations.

The preferred way to compute the node displacements of the inner nodes is to use mesh smoothing or mesh morphing algorithms. Mesh smoothing can be done by using a Laplace smoother [5]. This method works quite well as long as the mesh deformation doesn't exceed a critical level. If a critical level is reached, elements could overlap each other or even invert. Both cases cause false results. These critical levels are reached fast especially around singularities, like sharp corners. With application to typical MEMS structures like comb drives, perforated membranes or micro mirrors, large displacements and deformations typically occur. To overcome these problems a mesh morphing algorithm using radial basis functions (RBF) is used [6]-[8].

The resulting meshes of an example structure with large displacement is compared in the following figure (Figure 1). The mechanical structure (grey filled) is moved and the resulting mesh is shown after applying the Laplace smoother and the RBF mesh morphing algorithm. An element overlapping occur with the Laplace equation because every direction in space is independently solved. A perfect alignment of the deformed mesh is achieved by using RBFs.

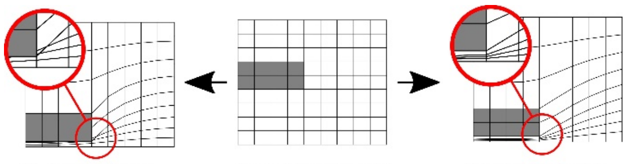

Fig. 1 Mesh deformation on a sample structure. (a) Laplace smoother result, (b) Undeformed mesh, (c) Mesh morphing with RBF

The theory behind this mesh morphing technique is presented in the following. The displacements x on the mechanical interface/boundary nodes and n_{m} are known. The remaining node displacements are approximated using a sum of basis functions and a linear polynomial.

$$x(P) = \sum_{j=1}^{n_{\mathrm{b}}} \alpha_j \Phi(\|P - P_{\mathrm{m},j}\|_2) + p(P) \tag{9}$$

There are two different coefficient vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ which are determined through the known displacements at the interface and boundary nodes on the mechanical domain. The first coefficient $\boldsymbol{\alpha}$ is related to the distance or Euclidian norm wrt. the mechanical nodes. The second coefficient $\boldsymbol{\beta}$ is related to a linear polynomial term $p(P)$ which takes the global node coordinates P into account. This term generates an additional dependence between all directions in space (10), where P_{m} is a $n_{\mathrm{m}} \times 4$ matrix consists of 3D interface and boundary node coordinates with its components $[1 \quad P_{\mathrm{m},x} \quad P_{\mathrm{m},y} \quad P_{\mathrm{m},z}]$.

$$p(P) = [1 \quad P_x \quad P_y \quad P_z] \cdot \boldsymbol{\beta} \tag{10}$$

TABLE I
COMMON RADIAL BASIS FUNCTIONS

Radial basis function	$\Phi(r)$
Polyharmonic spline	r^k , odd k
Multiquadratic	$\sqrt{1+r^2}$
Inverse multiquadratic	$\frac{1}{\sqrt{1+r^2}}$
Inverse quadratic	$\frac{1}{1+r^2}$
Gaussian	e^{-r^2}

The function Φ represents a meshless interpolation function. It computes the distance from any point to every mechanical interface node and weight them with a radial basis function. There are many different radial symmetric functions available. In the following table the most common and useable RBFs are given [9].

It turns out that polyharmonic splines and normal/inverse multiquadratic basis function are the best choice in terms of mesh morphing. The polyharmonic splines have the advantage of less computation effort.

To compute the coefficient vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ the following system of equations in matrix form is solved in (11). The matrix $M_{\mathrm{m,m}}$ is a $n_{\mathrm{m}} \times n_{\mathrm{m}}$ contains the evaluated radial basis functions $\Phi_{\mathrm{m}_i,\mathrm{m}_j} = \Phi(\|P_{\mathrm{m}_i} - P_{\mathrm{m},j}\|_2)$. Thereby only nodes with known displacements x are considered. The coefficient vectors are solved for every direction in space and can be done fast since the system matrix is symmetrical.

$$\begin{bmatrix} x_{\mathrm{m}} \\ 0 \end{bmatrix} = \begin{bmatrix} M_{\mathrm{m,m}} & P_{\mathrm{m}} \\ P_{\mathrm{m}}^T & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} \tag{11}$$

The back transformation from computed coefficients to unknown displacements of the mesh can be done using (9). In that way a mesh morphing algorithm can be realized. The computation of the coefficients is either precomputed in respect to the undeformed mesh or iteratively computed with the use of the updated node coordinates. Both ways are acceptable to achieve good results in a large deformed mesh. The electrical properties are nearly independent to the degree of deformation. That means the elements can have large distortion and the resulting effects are still in an acceptable range.

V. VALIDATION

To validate the strongly coupled electromechanical element with integrated mesh morphing we investigate in typical MEMS capacitive actuators. We compare the results with an ANSYS model containing exactly the same mesh. The electrical domain in the ANSYS model is remeshed.

A good validation sample is a comb drive cell. It consists of one moving and one fixed comb drive finger structure. In addition, there a top and bottom electrode modelled which are connected to ground potential. The moving finger can move in every space direction. We cover gap-varying and area-varying capacitances. The capacitance C and electrostatic force F_{el} are compared to the ANSYS results. We can prove that there is a negligible small error between the remeshed ANSYS model and our model using mesh morphing.

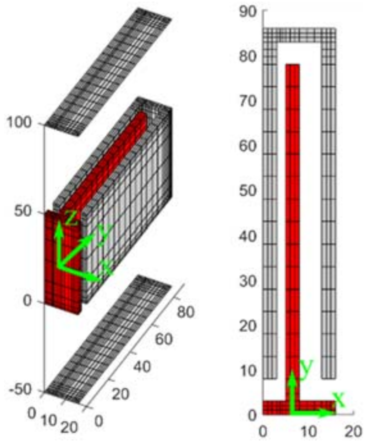

Fig. 2 Comb cell used for validation

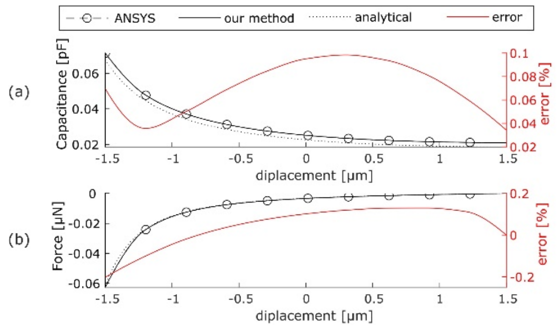

Fig. 3 (a) Capacitance vs. x-displacement, (b) electrostatic force vs. x-displacement

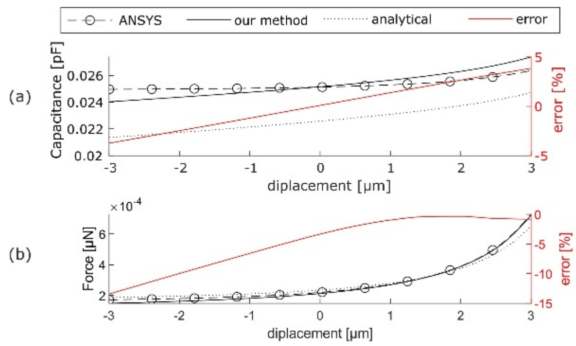

Fig. 4 (a) Capacitance vs. y-displacement, (b) electrostatic force vs. y-displacement

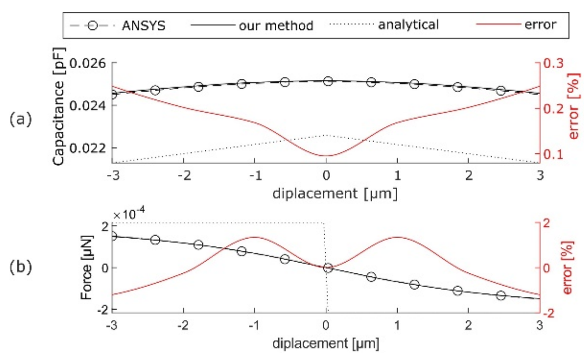

Fig. 5 (a) Capacitance vs. z-displacement, (b) electrostatic force vs. z-displacement

VI. Further Examples

In order to show the spectrum of possible usages of this FE formulation, more complex models were created. Only a sectional view is shown to make the complex meshes depictable. The mechanical structure (solid gray) is shown undeflected on the top and large deflected on bottom. An electrical field was also computed to demonstrate the independence of element distortion. The deformed meshes containing no invalid/inverted elements.

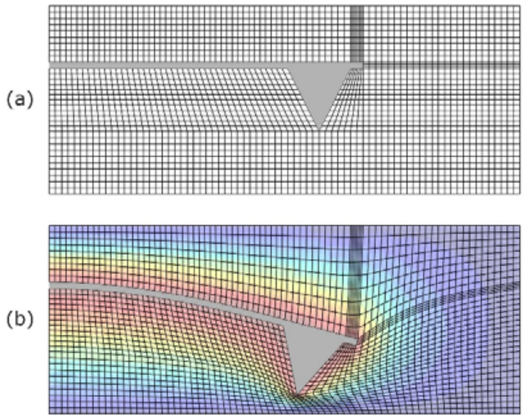

Fig. 6 Atomic force microscopy-tip - (a) undeflected, (b) deflected

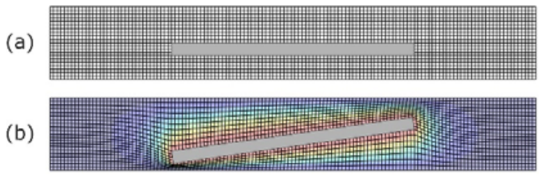

Fig. 7 Micro mirror - (a) undeflected, (b) deflected

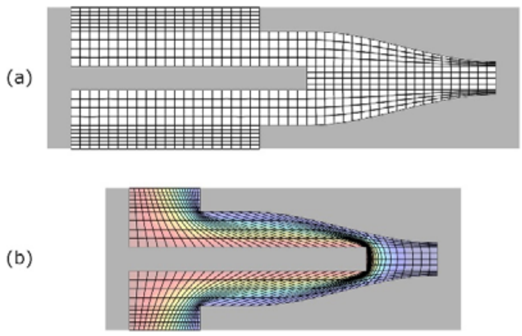

Fig. 8 Complex comb drive - (a) undeflected, (b) deflected

VII. Conclusion

A new formulation for electromechanical transducer elements was introduced. By differentiating the electrical energy an accurate description of the energy transfer phenomena in electrostatic field problem was found. This formulation uses an integrated mesh morphing algorithm which makes a time consuming remeshing of the model while simulating unnecessary. Moreover, the radial basis function approach in the mesh morphing algorithm allows also highly deformed meshes while overlapping or inverting elements are avoided. Usage of traditional shape functions and solvers makes this method fully integrable into existing FE simulation tools. The spectrum of application is widespread in the field of MEMS. Some more complex models which demonstrates that the FE formulation is independent of the complexity. Structures like micro mirrors, atomic force microscopy (AFM) tips, micro switches or even more complex is aimed to solve. The effects of nonlinear changing capacitances, electrostatic softening and fringing field effects are included in the method. The strongly coupled matrix formulation opens up the possibility to use any kind of analysis, as modal or harmonic analysis. Furthermore, additional reduced order model techniques can be to create an even more faster simulation model.

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