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Function of Fractals: Application of Non-linear Geometry in Continental Architecture

Mohammadsadegh Zanganehfar

***Abstract*—**Since the introduction of fractal geometry in 1970, numerous efforts have been made by architects and researchers to transfer this area of mathematical knowledge in the discipline of architecture and postmodernist discourse. The discourse of complexity and architecture is one of the most significant ongoing discourses in the discipline of architecture from the 70's until today and has generated significant styles such as deconstructivism and parametricism in architecture. During these years, several projects were designed and presented by designers and architects using fractal geometry, but due to the lack of sufficient knowledge and appropriate comprehension of the features and characteristics of this nonlinear geometry, none of the fractal-based designs have been successful and satisfying. Fractal geometry as a geometric technology has a long presence in the history of architecture. The current research attempts to identify and discover the characteristics, features, potentials and functionality of fractals despite their aesthetic aspect by examining case studies of pre-modern architecture in Asia and investigating the function of fractals.

***Keywords*—**Asian architecture, fractal geometry, fractal technique, geometric properties.

I. Introduction

THE contradictory discourse between architecture and complexity theory had been addressed by various thinkers and architects since the late 20th century. One of the most significant themes of this discourse driven from scientific theories has been fractal architecture and the application of fractal geometry in the architecture discipline [1].

When Carl Bovill published his research book “Fractal Geometry in Architecture and Design” in 1996, he argued that fractal geometry, which is interpreted as an extension of the classical geometry, could be considered a geometric technology and a powerful design tool which has to be used wisely and elegantly [2]. Since the advent of chaos theory and complexity discourse, numerous attempts had been made to translate this term of knowledge into architecture territory as a new complexity paradigm [3]. Probably the most famous character is Peter Eisenman who designed various projects and drawings in this direction.

The discourse of complexity and architecture addressed the confrontation of Euclidean geometry as the governing geometric tool of modern architecture and fractals as the scientific integral connection between geometry of nature and architecture [2]-[4]. Although, through the recent history of this attempt, architects were occupied with the aesthetic aspect of fractal geometry and its advantages on creating ornaments, attractive façades and skin design of buildings while, the characteristics of fractal geometry and its properties could play a fundamental role in redefining architectural structure and spatial behavior [4]. The evidence of fractals could be traced to ancient Maya settlements, Middle Eastern civilizations and European cathedrals during different eras and geographies [5], [6].

We could argue that the misuse of fractal geometry and chaos science in contemporary architecture and failing attempts of translating this term of knowledge into architecture discipline is mostly due to the lack of proper research on the characteristics and properties of this geometric technology [1]. Post-modern architects merely adopt a literal interpretation of fractals and attempt to create a fractal-based architecture, which mostly result in unsatisfactory design outcomes. As mentioned, the important issue in the discipline of architecture is the spatial properties, characteristics and function of fractals as an architecture tool which bring us insight into the proper use of fractals. This paper attempts to address this issue by investigating three case studies in the Asia-Pacific continent with specific use of fractals in architecture adopting a visual analysis to identify the fractal patterns and self-similar elements [7].

The aim of this research is to address the issue of function of fractals and the role of fractal geometry regardless of aesthetics aspects in order to adopt a deeper comprehension from this geometric technology in architecture discipline. We attempt to recognize fractals as a non-linear geometric technology with specific characteristics and properties based on the evidence of pre-modern architecture. This research is focused on the argument that how classic architects adopt fractal geometry to respond and solve the existing challenges and issues of their time and geography, and also how they use fractal properties for the benefit of their architecture.

II. Methodology

A qualitative analysis research based on case study through library and academic resources, with a critical approach had been conducted for this research. By using the visual analysis method, architectural drawings analysis, and study of three cases from Asian continent architecture, an attempt has been made to conduct the research. The main area of the research includes functionality of fractal geometry, application of fractals in Asian architecture and the properties of this geometry in spatial structure.

Mohammadsadegh Zanganehfar is with School of Housing, Building and Planning, University Sains Malaysia,11800 USM, Pulau Pinang Malaysia (e-mail: mohamadsz.arc@gmail.com)

III. Results

Due to the importance of having a clear understanding of fractal geometry, we will address a brief summary of fractals as a prerequisite of the research discussion. In the following, this research contains a case study of "Sakyamuni Pagoda of Fogong Temple" of China, "Borujerdi house" of Iran and "Ali Qapu palace music hall" in Iran.

Fractal Geometry

Fractals were introduced and discussed in the 1970's by mathematician Benoit Mandelbrot, who had dedicated his research career to what he labeled as "the art of roughness" and "geometry of life" [8]. His famous quotation and description of the geometry of nature is probably the statement "clouds are not spherical, mountains are not conical, coastlines are not circle and barks are not smooth" which emphasizes the notion that Euclidian geometry is incapable of describing the natural geometry and phenomenon of the universe [4], [6], [9], [10]. Mandelbrot asserted that a new geometry is necessary to describe shapes, irregularities and properties of natural phenomenon. He chooses the name "fractal" for his theory which was adopted from the Latin word "fractus", meaning a stone crushed in an irregular way [1], [9]. The advent of fractal theory and development of fractal geometry leads to the emergence of a new scientific model for describing the complexities and geometric value of nature [11], [12].

Although the characteristics and properties of fractal geometry are ample and detailed, but to describe in general, we could address its major properties relevant to our research briefly. Probably the most important property of fractals is 'self-similarity' or 'self-affinity' over scale. Mandelbrot mathematically defines fractals as a rough and fragmented geometric shape which could be subdivided in parts and each of which is a scaled copy of the whole. In other words, fractal is a self-similar pattern which repeats itself [13], [14]. This property explains that in a mathematical fractal even an infinitely small part is representing the exact or almost approximately the same geometry of the whole [15]. The common property which fractal objects have in common is their geometric complexity, whereas the underlying and generative laws behind their formation are very simple [15]. We could generalize that fractals are formed by repetition of a mathematical equation or shape based on a pattern [6]. Due to the fundamental and mathematical discipline of fractal geometry, in the fractal structure irregularities would emerge which are not justifiable by classic geometry. Rough edges, drastic curves and unpredictable fluctuations in the shape of fractal structures are almost impossible to be calculated or described by the principles of Euclidian geometry. Alongside the repeating formation and self-similarity, a non-integer dimension is one of the characteristics of a fractal [6].

Sakyamuni Pagoda of Fogong Temple

Sakyamuni Pagoda of Fogong Temple is a five-story wooden structure built in 1056 CE, which is known as the oldest wooden multi-story structure in the world [16], located in Shaxi, China, with height of 67.58 m. This wooden structure is consisting of beams, columns and Dougong brackets which are the primary load bearing elements of this high-rise structure [17]. Due to this unique and complex structural system consisting of 22,743 pieces, Yingxian pagoda has survived in its unstable geography with numerous earthquakes for nearly 1000 years [18], [19].

Fig. 1 Sakyamuni Pagoda of Fogong Temple [20]

Fig. 2 Dougong bucket structures on the exterior façade of Sakyamuni Pagoda of Fogong Temple [22]

Dougong (bracket set) is an ancient unique technique of Chinese architecture, and one of the complex, supporting systems of ancient high-rise buildings. Terminologically, the word is consisting of two words "dou" interpreted as invert supporting cap and timber rectangular element with a cross grove, and "gong" interpreted as the timber bow-like block supporting the load, placed on the cross grove section of the dou element [19], [21].

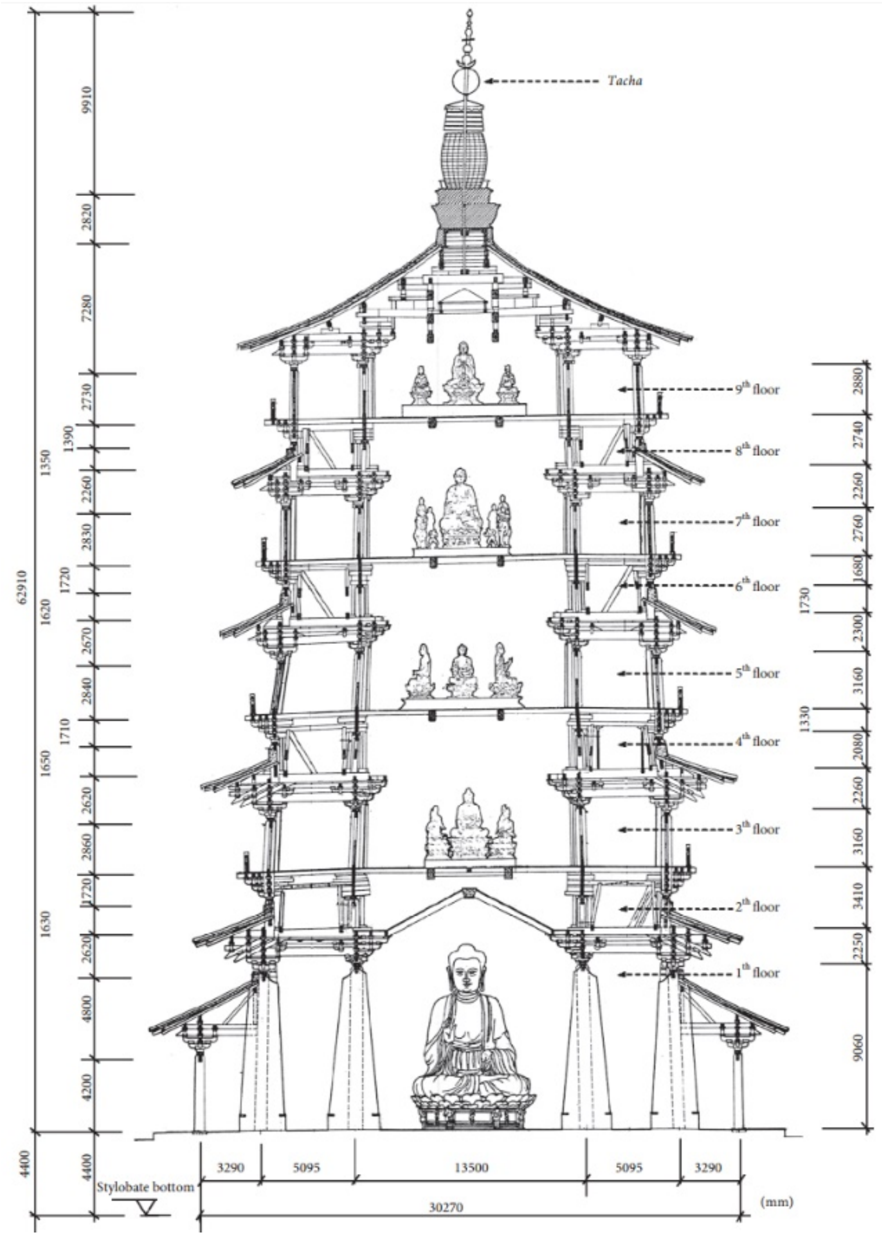


Fig. 3 Sakyamuni Pagoda of Fogong Temple section, elaborating load baring structure and duo-gong components [18]

Due to the obstacles and constrictions of using huge timber construction and the fact that large size timber brackets do not have the required stiffness and durability confronting weather conditions, Chinese constructors have replaced the large brackets with several small brackets [23]. By repeating this strategy and minimizing the size of the bracket over scale, they accomplish enough stiffness in the overall structure of this system. This geometric operation and technique could be explained by the Iteration Function System (IFS) method of fractal geometry generation [23].

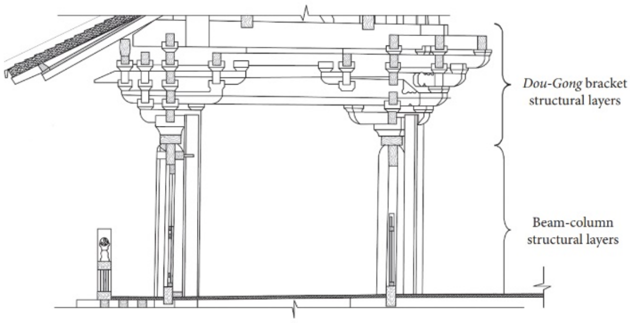


Fig. 4 Application of the Duogong system [18]

The bracket sets models, due to their organic construction potentials, used to be the most important content of the East Asian tradition and design method [24]. It has been discussed by researchers that the dougong structure performs as the most important element to resisting seismic action by transmitting roof and beam loads to the columns and foundation of the buildings [21], [23], [25]. Numerous static and dynamic experiments and analysis by researchers proved the non-linear stiffness of dougong [21].

Chinese ancient architecture followed a certain pattern of similar appearance followed by construction techniques. Dougong correspondence with the column and roof determines the location of the housing body and also the roof construction system due to the application of balancing the weight of the roof structure [27], [25]. Dougong brackets, through the application of uniform distribution of horizontal load and transformation of weight on horizontal beams over a larger area to the vertical columns, caused the opportunity of increasing the roof support area and could be recognized as the main contributor of emerging a new module of construction and building in Chinese architecture [28].

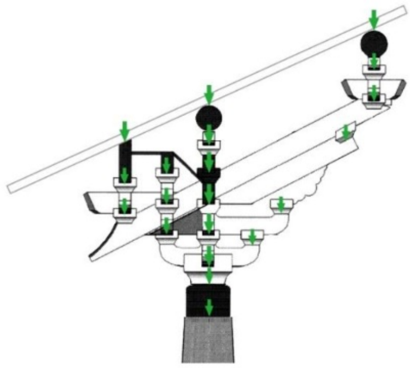


Fig. 5 Load transfer in the dougong system [26]

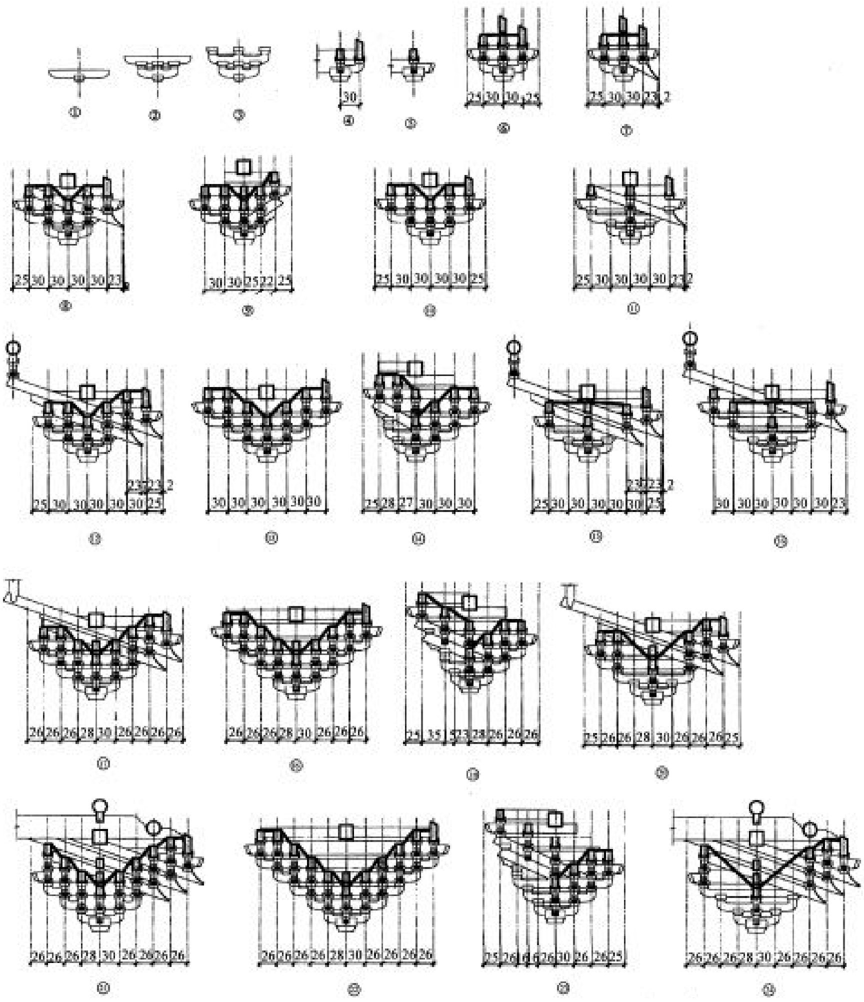


Fig. 6 Generating dougong system through irritation of algorithm [29]

The dougong technique has four major functions in a building structure: 1) transfer the structural weight of the roof and beams to pillars. The dougong system dissipates most of the implemented energy by the building structure through the sliding, distribution and movements between each element [21]. 2) perform as an anti-seismic element by absorbing seismic energy through their fractal-based structure and convert the tension to pillars. This performance is due to the re-centering capability and energy dissipation capacity of the dougong structure [21]. 3) provide extension to the eaves of the roofs to restrain rainwater permeability and structural elements deterioration caused by humidity, and 4) provide aesthetic features of elements on the overall structure of the building, which could be recognized as justification for using this technique in palatial structures and religious temples [19], [23].

Borujerdi House

Broujerdi's house, located in Kashan, Iran, was completed in 1892 after a long construction process. Due to the geographical and ecological conditions of Kashan, restrictions such as hot-dry climate and seismic conditions, construction was a perpetual issue in order to fulfill the requirements of its owners and community.

Fig. 7 Borujerdi house exterior façade [30]

Surprisingly, specific techniques had been adopted by architects through which buildings had survived for decades and were durable against several earthquakes. Various ecological aspects had been considered in this project, and environmental characteristics were more important than aesthetic aspects [31]. Borujerdi house materiality consists of brick, mud and adobe. Benefiting from the characteristics of these materials and their integration, the structural walls distribute the vertical load evenly in this project [32]. The main focus of this research is the specific technique used in the main hall of the project, known as “karbandi”.

Fig. 8 Section of Borujerdi house, elaborating the height and placement of skylight holes [33]

Studies show that from the 10th century AD, we get acquainted with dome structures with a karbandi brick bearing system with gypsum mortar [34]. Karbandi is known as one of the mathematical and precise coating modules and also one of the most important construction techniques in Iranian architecture [35]. This technique is considered as a pattern and a practical model in the traditional Iranian architecture due to its structural and geometric characteristics, creation of a systematic structure and high formal and functional capabilities [36].

Fig. 9 Karbandi fractal system in the interior [37]

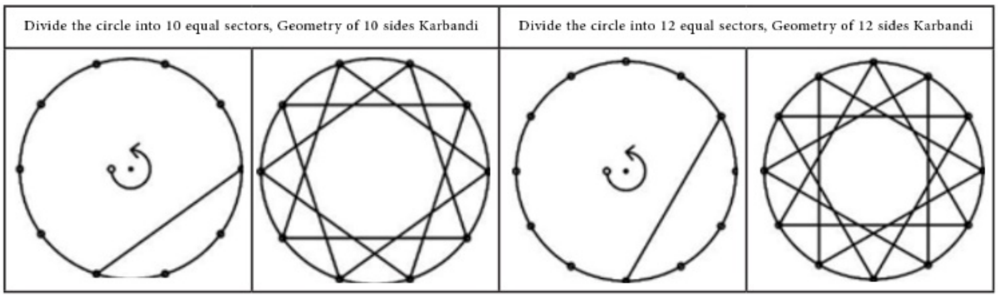


Fig. 10 Plan geometry of Karbandi [35]

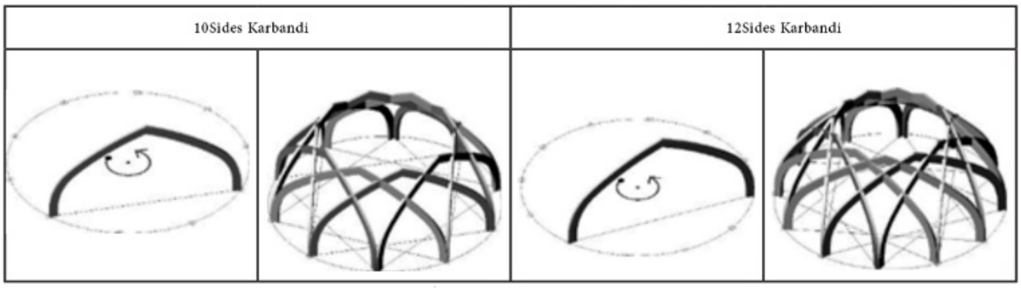


Fig. 11 Three-dimensional geometry of Karbandi [35]

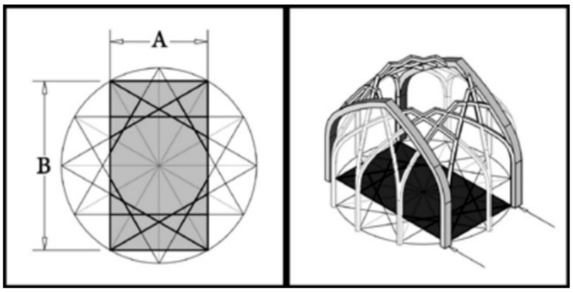


Fig. 12 Three-dimensional geometry of Karbandi generated by the plan [38]

One of the most important features of karbandi is structural behavior and increased bearing capacity based on precise mathematical rules and drawings [35].

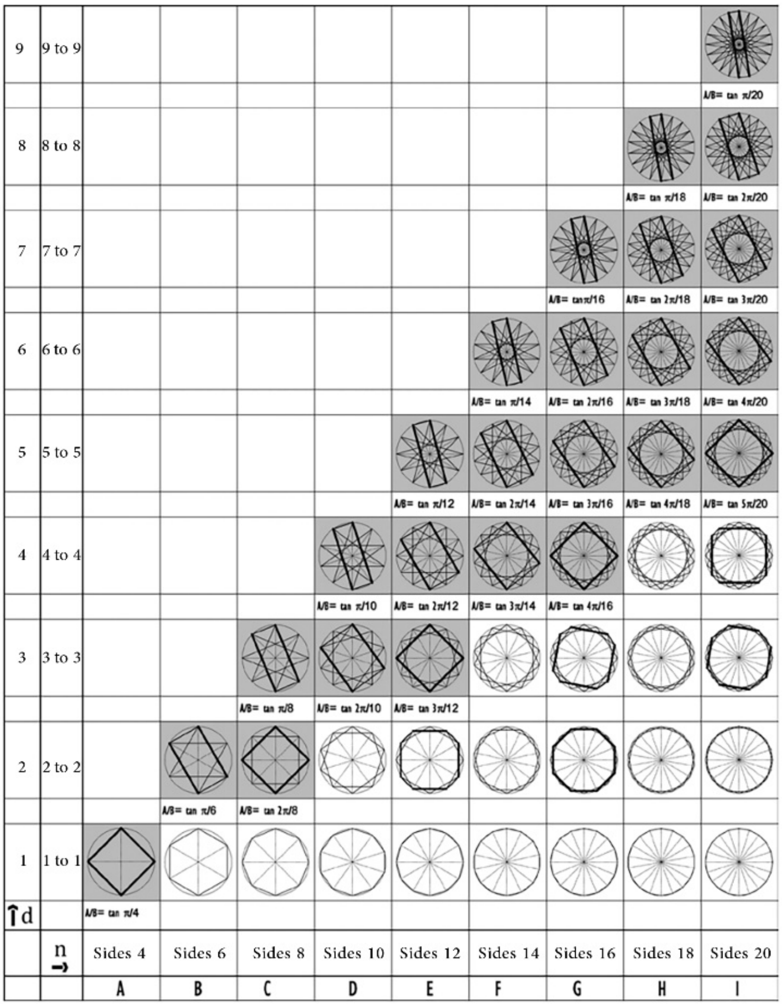


Fig. 13 Generating of karbandi system based on different divisions on the circle and proportions [38]

Research shows that karbandi is based on two principles: 1) Plan geometry: based on dividing a circle into equal segments and drawing intersecting and equal chords between the points of division. According to the studies in the scrolls belonging to the Qajar period, in the past, the plan drawings were used to introduce karbandi [38]. 2) The structure of three-dimensional geometry is based on the rotation of an arch around the center of the circumference [35]. Due to the development of this technique over time, various outcomes have been generated based on karbandi. In this regard, numerous researches and studies have been done in the direction of typology and geometric classification of this technique [33]-[36].

The roof of the Borujerdi house was constructed with an arch and dome system by karbandi technique, which was one of the best structural systems in Iran construction in that era [32]. One of the most important features of karbandi is the distributing of large openings with the help of narrow and cross arches and the application of smaller arches, which is more efficient and optimized compared to the method of installing integrated arches [39]. Another unique feature of this technique is the maximum use of natural light and illumination, and therefore it is in the category of lighting and lighting techniques of Iranian architecture [40]. Perhaps the most important reason for the emergence of skylights and the use of karbandi for the installation of light in interior spaces can be considered the expansion of buildings with a deep plan and depth of interior spaces [40], [41].

This karbandi roof of borujedi house reflects a significant portion of sunlight and operates as a complex ventilation system by caring significant height in the space [31], [32]. The invention of the karbandi technique caused the load of the roof to be transferred directly to the rafters, and due to the lightening of the arches and walls, the main gaps and skylights were created in the roofs and a lot of light penetrated indirectly into the building [40]. For this reason, many researchers consider karbandi a technique related to light and, based on this, they consider the geometry of space to be influenced by the geometry and shape of skylights [41].

Aali Qapu Palace

Ali Qapu palace recognized as one of the most important buildings of Safavid architecture, located on the western side of Naqshe Jahan Square, is an imperial palace completed in 1597 in Isfahan, Iran. In the sixth floor of this 48-meter-high palace, the music hall with a complicated acoustic system caused wonder and appreciation of architects and researchers in the history of Iranian architecture [42].

Muqarnas could be defined as a mathematical and algorithmic transition element from a two-dimensional geometry to three-dimensional form based on simple geometrical and mathematical relations. This technique, known as a “geometric generator” could be traced back to the 10th-century architecture buildings in Middle Eastern civilizations due to the close relation between architects and mathematicians [45]. Although Muqarnas forms carry a significant level of complexity and detail in the structural, geometrical and spatial aspects, the generative algorithm and mathematical principles of this technique are simple and comprehensible [46].

Fig. 14 Ali Qapu palace in Naghshe Jahan Square [43]

Fig. 15 Interior view of music hall designed by muqarnas fractal system [44]

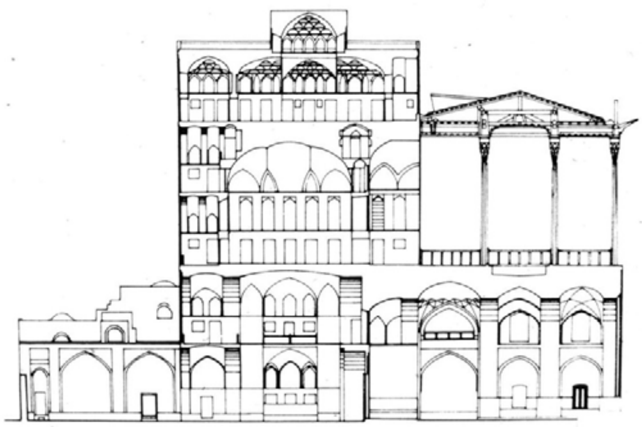


Fig. 16 Section of Ali Qapu palace, elaborating music hall on the top level [47]

Muqarnas is an organized fractal object which alongside its formal complexity consists of internal organizations and internal order in the pattern of growth and emergence [46].

This fractal property could describe muqarnas as a group of small prismatic pieces, organized on a complex patter and sequence to shape and layout the whole set [48]. Muqarnas block which is the one single component of the whole system is a three-dimensional artifact which by following a pattern of generation would shape the muqarnas system. These blocks are categorized based on their degree of classification (Fig. 18), due to the significant effect of buckets degree on the generated system [49]. Family blocks have the capacity to generate various systems based on the juxtaposition, repetition and irritation, following the defined pattern of growth [49]. In the process of the generation of the blocks, the components get smaller over the layers of growth.

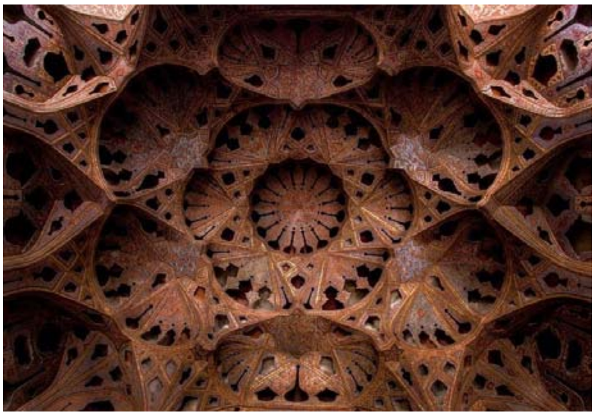


Fig. 17 Ceiling of music hall designed by muqarnas fractal system [50]

Muqarnas compositions perpetually fascinated audiences due to its inherent order and hierarchical relations between individual parts and the whole system [49]. The main characteristics of muqarnas could be discussed in four major categories: 1) three-dimensional form, 2) variable size of muqarnas blocks, 3) specific generative algorithm, and 4) two-dimensional pattern and plan [51].

In the music hall of Ali Qapu Palace, the muqarnas are located for the use of interior acoustics. Using the art of plastering and creating interior geometry based on the muqarnas technique, the reflections from the musicians' songs and music are combined with the geometry of the room, and natural sounds are heard without unpleasant reflections [52].

Curved surfaces behave differently to sound reflection, depending on whether they are convex or concave. When reflected on a convex surface, the angle of propagation of the sound beam and its direction change and the frequency of the sound diverges, while this process is reversed on concave surfaces [54]. The purpose of creating muqarnas in the interior of the music hall is to create concave surfaces, eliminate unnecessary and annoying noises in the hall, isolate the sound of the hall and create suitable space for sound transmission [52], [54]. Acoustically, the atmosphere of the music hall can be considered as a complex intensifier that creates several permissible and special vibration modes in the space. When the sound source is placed in such an atmosphere, the space creates a constant intensifying function and vibration with the sound source [52]. The interior of the music hall of Ali Qapu palace, due to the use of muqarnas, which are considered as a kind of concave space cavities, intensifies the sound frequencies and the pleasant reflection of the sound in the interior [54].

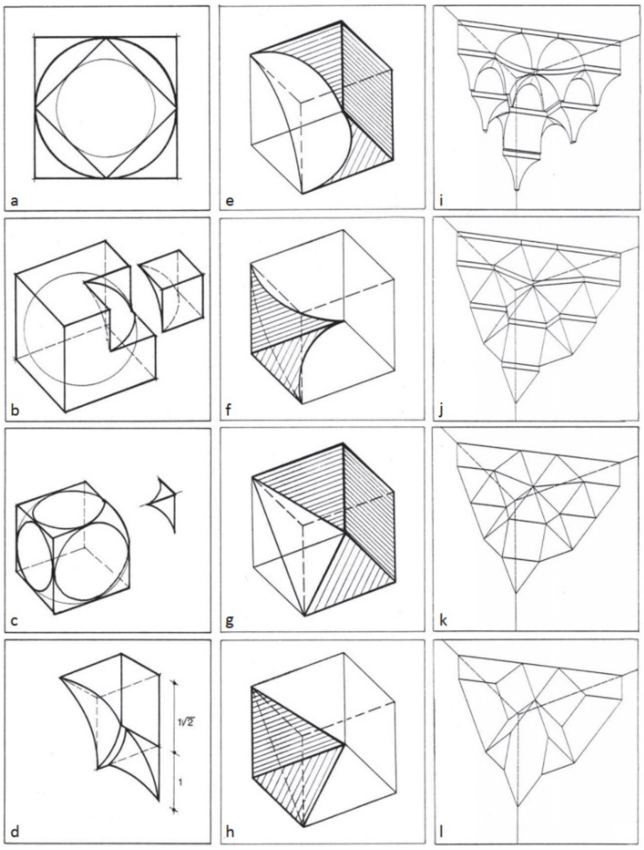


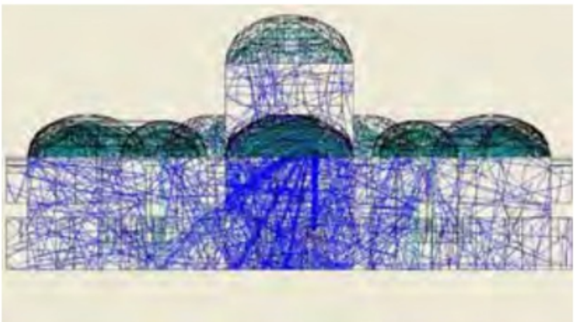
Fig. 18 Sample geometry of the muqarnas block generation [49]

IV. Discussion and Conclusions

By addressing the issue of functionality of fractal geometry in three cases from the Asia continent, we could argue that the deployment of fractal geometry had specific application in architecture. What we witnessed in modern and post-modern architecture movements was a literal understanding regardless of proper attention to potentials and features of this term of knowledge. Meanwhile, classic architects used fractals to address serious issues and vital challenges in their architecture designs. Fractal geometry provides the opportunity of rereading architecture history and reveals the geometric advents in different eras and geographies. This perspective could be used in future studies and research on fractal geometry and its application in the history of architecture.

Adopting a critical approach towards results and findings of the research, we could conclude that fractal geometry had been developed and practiced through different eras by architects with different perspectives and approaches with specific techniques to solve serious challenges. These developed techniques played an important role in the architecture of the Asia continent and could be recognized as impressive advents of their time and geography. Meanwhile, what we witnessed in modern and post-modern attempts at using fractals was merely restricted to aesthetic values and compositional properties regardless of the potentials and opportunities of fractal geometry in architecture.

(a)



(b)

Fig. 19 (a) Ceiling plan of Ali Qapu music hall, (b) Simulation of propagation of rays in the music hall space [53]

Fractal geometry is a generative geometry which develops and grows based on irritation of a fractal object through a fractal pattern. In fact, a fractal system would emerge based on the predefined issues and forces. Briefly classified in Table I, we could state that fractal properties are geometric features regardless of materiality.

TABLE I
CLASSIFICATION OF CASE STUDIES AND APPLICATIONS OF FRACTAL GEOMETRY AND DEVELOPED TECHNIQUES

	Sakyamuni Pagoda of Fogong Temple	Borujerdi house	Aali Qapu Palace
Year	1056	1857	1597
Country	China	Iran	Iran
Technique	Dougong	Karbandi	Muqarnas
Material	Wood	Adobe & Brick	Plaster
Fractal Pattern	Vertical	Rotational	Spatial
Function	Structure	Structure/Luminaire	Acoustics

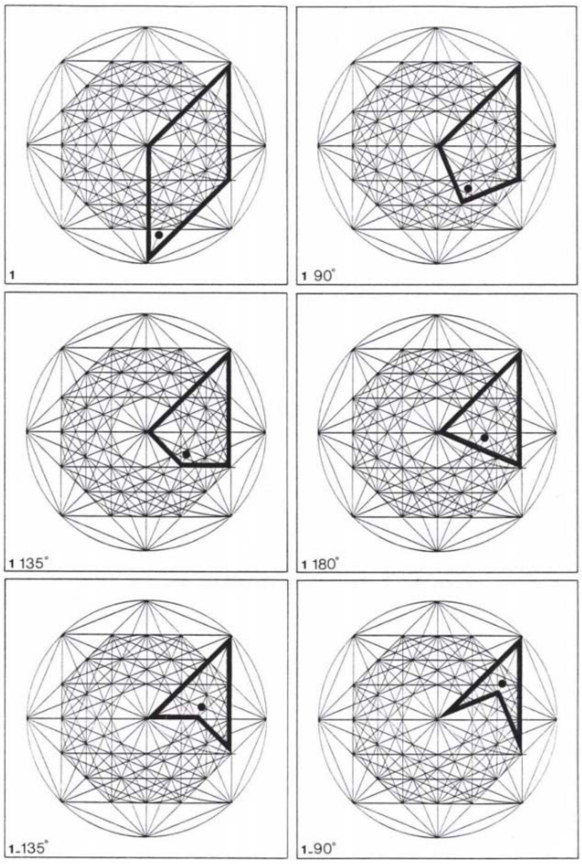


Fig. 20 Base forms of muqarnas block families [49]

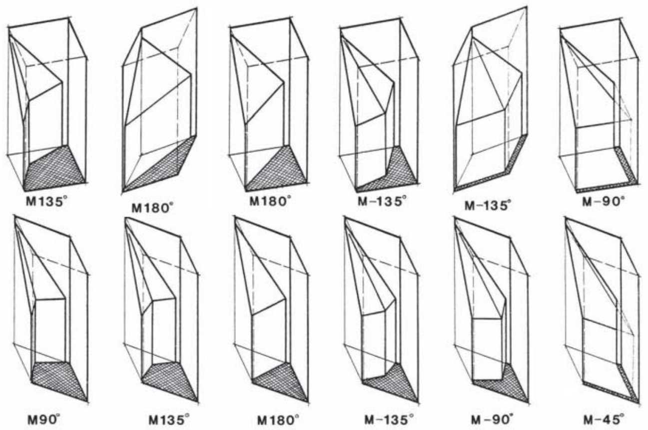


Fig. 21 Muqarnas block families based on the angle degree [49]

In the Sakyamuni Pagoda of Fogong Temple in China, Chinese architects cover the force and load transfer issue to achieve a high-rise building by developing the Dougong technique. We could find contemporary developments of the dougong technique in the work of contemporary Japanese architect, Kengo Kuma.

Fractal geometry was developed through collaboration between mathematics and architecture in Iranian examples. In two case studies, fractal geometry was developed in order to cover the issues of structure, climatic challenges and acoustics of space by different techniques and the inventions of Iranian architects. Fractal geometry has the property of distribution of applied forces. This effect will occur confronting forces and energy of any kind. Distribution of load in dougong example, distribution of sound frequency in Ali Qapu example and distribution of load and light in Borujerdi house example would bring the conclusion that these techniques were developed in order to cover a massive load of force by distributing and focusing through the geometry.

One of the fractal properties is the scaling of a fractal object according to a fractal pattern. This phenomenon would reduce the load transfer through the geometry and also enhance the adoptability of fractal geometry in architecture spaces.

The mentioned techniques of fractal application could be practiced in contemporary forms and materials. As we discussed, properties and characteristics of fractals is a geometric phenomenon regardless of materiality. Fractal geometry has numerous potentials which could be discovered and practiced in contemporary architecture.

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