Comparison of Conventional Control and Robust Control on Double-Pipe Heat Exchanger

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***Abstract*—**Heat exchanger is a device used to mix liquids having different temperatures. In this case, the temperature control becomes a critical objective. This research work presents the temperature control of the double-pipe heat exchanger (multi-input multi-output (MIMO) system), which is modeled as first-order coupled hyperbolic partial differential equations (PDEs), using conventional and advanced control techniques, and develops appropriate robust control strategy to meet stability requirements and performance objectives. We designed the proportional–integral–derivative (PID) controller and H-infinity controller for a heat exchanger (HE) system. Frequency characteristics of sensitivity functions and open-loop and closed-loop time responses are simulated using MATLAB software and the stability of the system is analyzed using Kalman's test. The simulation results have demonstrated that the H-infinity controller is more efficient than PID in terms of robustness and performance.

***Keywords*—**Heat exchanger, multi-input multi-output system, MATLAB simulation, partial differential equations, PID controller, robust control.

I. Introduction

THE double-pipe HE is widely used in food and oil refinery industries and it is an important element for various types of installations like steam power labs, air conditioning systems, and heating. A counter-flow HE is one of the most popular double-pipe HE types because of its high effectiveness [1]. Besides that, the double pipe HE is very suited for low flow rates and high temperature or pressure applications. Due to the small amount of heat transfer per section of the double pipe HE, it can be used in the applications of small heat transfer surface [2]. References [3]-[6] have developed different methods on the functioning and the mathematical models of the HE and concluded that the second-order differential equation with delay time is the most significant representation while this type of HE is a complex system, MIMO system, modeled by two first-order coupled hyperbolic PDEs. To overcome this problem and develop a proper mathematical model, Euler discretization method was used to get the state-space model of the HE system decreasing its mathematical complexity, hence easing the design of a real-time control strategy.

HEs are the main components that are present in any refrigeration system. The control of such units is vital to have low rates of energy consumption while maximizing the heat transfer rates. Temperature control is an important aspect in the process industry such as light bulbs, the dairy industry, pharmaceutical industry, incubators, and others. Current research on the HE focuses on the control of the temperature which is the main parameter to be controlled. References [3], [7]-[12] studied the dynamic behavior of these mathematical models with different conventional PID, predictive and robust control approaches, based on the internal model control, and with/without uncertainties. References [13]-[19] have proposed the design strategies and examples and mechanisms in some practical applications, reducing the controllers' order. Reference [20] applied PI and H-infinity controllers to a HE system described by a second-order mathematical model with delay time, taking into account the effect of uncertainties on the two-time constants of the transfer function and the un-modeled dynamics in the proportional factor and concluded that the H-infinity controller has better performance than PI controller. The behavior of the output and the control input with these controllers w.r.t different external inputs (reference, disturbance, noise) has not been studied, especially in [20], on the considered HE system. Therefore, this paper presents the design and comparison of conventional and robust controller for a MIMO HE system, in which the behavior of the output and the control input with these controllers w.r.t different external inputs and the effect of the observability of the system states w.r.t to the two controllers on the performance can be explicitly and quantitatively studied, which makes the study more real and easy to be validated experimentally. The first conventional controller is tuned by Ziegler-Nichols's criteria and the second robust controller is tuned by the H-infinity theory with improved results compared to [20].

The following section contains the mathematical modeling of the HE system including the problem statement and the state-space description of HE. Then the design description of the PID controller and the robust controller is presented. In the final part of the paper, the frequency characteristics of sensitivity functions, open-loop and closed-loop time responses performed using MATLAB software, the analysis of the system stability using Kalman's test, and also some conclusions are presented.

II. Problem Statement of the HE

The following assumptions from [21] are considered:

- The flow is one-dimensional (along an axis called x), with the hot fluid direction considered as positive;
- Heat diffusion through the tubes is neglected;
- Advection between the two fluids is the only heat exchange taking place;
- The external walls of the tubular structure are adiabatic and

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no heat exchange between it and its surrounding environment takes place;

- Heat transfer coefficient is constant.

Under these assumptions, the complete model of HE, shown in Fig. 1, is presented by the following coupled hyperbolic first-order PDEs:

$$\frac{\partial T^H(x,t)}{\partial t} + c_1 \frac{\partial T^H(x,t)}{\partial x} = -d_1\left(T^H(x,t) - T^C(x,t)\right) \quad (1)$$

$$\frac{\partial T^C(x,t)}{\partial t} - c_2 \frac{\partial T^C(x,t)}{\partial x} = d_2\left(T^H(x,t) - T^C(x,t)\right) \quad (2)$$

The system parameters are considered known and constant. This system has boundary conditions:

$$T^H(0,t) = T^H_{in}(t) = U_1(t),\ T^C(1,t) = T^C_{in}(t) = U_2(t) \quad (3)$$

and initial conditions:

$$T^H(x,0) = T^H_0(x),\ T^C(x,0) = T^C_0(x) \quad (4)$$

The outputs of the system are:

$$T^H(1,t) = T^H_{meas}(t) = y_1(t),\ T^C(0,t) = T^C_{meas}(t) = y_2(t) \quad (5)$$

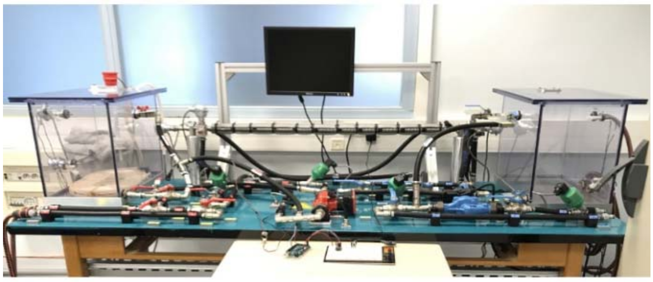

Fig. 1 HE test-bench in GIPSA-lab [21]

III. State-Space Description

The state-space model of the system was obtained by discretization of (1) and (2) w.r.t space. Because the flows are in opposite directions as shown in Fig. 2, the discretization methods will be backward Euler for the hot flow and forward Euler for the cold flow. With boundary conditions (3), the space-model for the system will be:

$$\dot{X} = A.X + B.U \quad (6)$$

$$Y = C.X + D.U \quad (7)$$

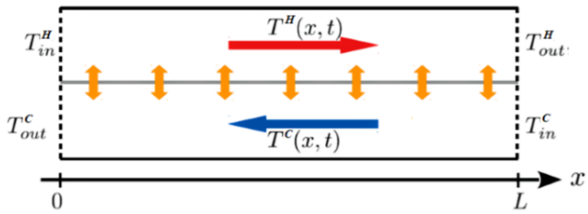

Fig. 2 Scheme of simplified HE [21]

IV. PID Control Design

The PID controller is used in industrial control applications to regulate temperature and other process variables. PID controllers use a control loop feedback mechanism to control the variables of the process and are the most stable and accurate controllers. The continuous transfer function of the PID controller is obtained through Laplace transform as:

$$G_c(s) = K_p + \frac{K_i}{s} + K_d\, s \quad (8)$$

The initial tuning constants, K_p, K_i, and K_d, of PID controller are found using Ziegler-Nichols (which is based on updating the gains starting from its initial nominal values and then tune the controller parameters by the response simulation until the performance would be acceptable) and based on the state-space model of the HE system which is presented in (6) and (7), the controller gain of PID is computed and attests for a closed-loop control system and fine-tuning is conducted using a MATLAB Simulink shown in Fig. 3.

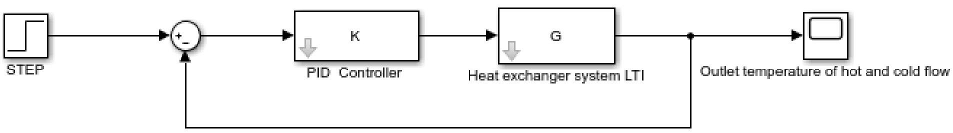

Fig. 3 PID control Implementation

V. H-Infinity Control Design

In this section, we use a robust control technique based on the H-infinity synthesis, which includes the mixed sensitivity with sub-optimal solution. The control synthesized using the H-infinity framework provides inherent advantages to attenuate the effect of noises and disturbance on the system. The general control structure is given in Fig. 4 (system states are not fully observable for 2-DOF H-Infinity controller), with the main objectives of the control design (time-domain specifications), good tracking performance (equivalently steady-state error must be small), convenient module margin, small rise time (controller must be fast), disturbance attenuation, and noise rejection.

To synthesize the controller using H-infinity framework, the following steps must be followed:

A. Time-Domain Specifications to Templates

The first step in the control design of the H-infinity controller is to convert the objectives specifically, time domain specifications into templates on the appropriate functions. Based on our goals, we will be employing two templates (mixed sensitivity), one for the sensitivity function and the other for the controller sensitivity function.

The sensitivity function (S) is mainly concerned with particular performance specifications such as tracking performance, module margin, attenuating the effect of measurement noise on output (indirectly), and attenuating the effect of input disturbance on output. The template on the sensitivity function is given as:

$$\frac{1}{W_e(s)} = \frac{s + w_b \, \epsilon_1}{\frac{s}{M_s} + w_b} \tag{9}$$

To achieve good tracking performance, alternatively, to make steady-state error small, we choose $\epsilon_1 = 0.001$ and to have a convenient module margin, we choose M_s to be ≤ 2 (M_s= 1.8), as explained by [19]. Lastly, W_b is chosen to be 0.14.

The controller sensitivity function (KS) is mainly concerned with certain performance specifications such as input saturation, attenuating the effect of measurement noise on the input, and attenuating the effect of output disturbance on input. The template on the input sensitivity function is given by:

$$\frac{1}{W_u(s)} = \frac{\epsilon_2 s + w_{bc}}{s + \frac{w_{bc}}{M_u}} \tag{10}$$

To ensure that the controller would not saturate for maximum change in the reference, we can impose the following specification on the template:

According to the typical templates in [19],

$$M_u \leq \frac{u_{max}}{r_{max}} = 2 \tag{11}$$

It must be noted that the range of frequencies, for which the controller must act, must be greater than the operating range of the system. Hence, we choose a large value for w_{bc}. Lastly, w_{bc} is chosen to be 100 and $\epsilon_2 = 0.001$.

B. Control Structure

To perform the controller synthesis, a control structure needs to be defined. We define a control structure having three external inputs (the reference input, disturbance, measurement noise) and two weighting matrices, one on the tracking error W_e and the other on the control input W_u, which are responsible for the performance and control effort respectively. Fig. 5 shows the generalized control scheme for the H-infinity control problem which includes the plant together with the controller.

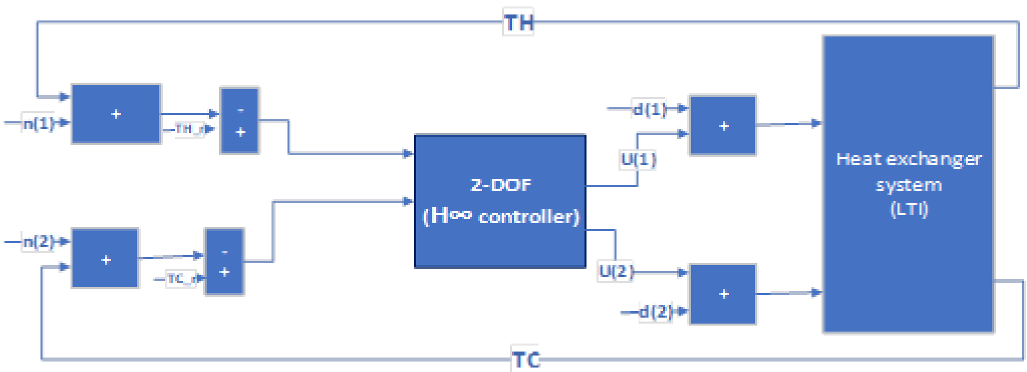

Fig. 4 2-DOF H-Infinity control structure

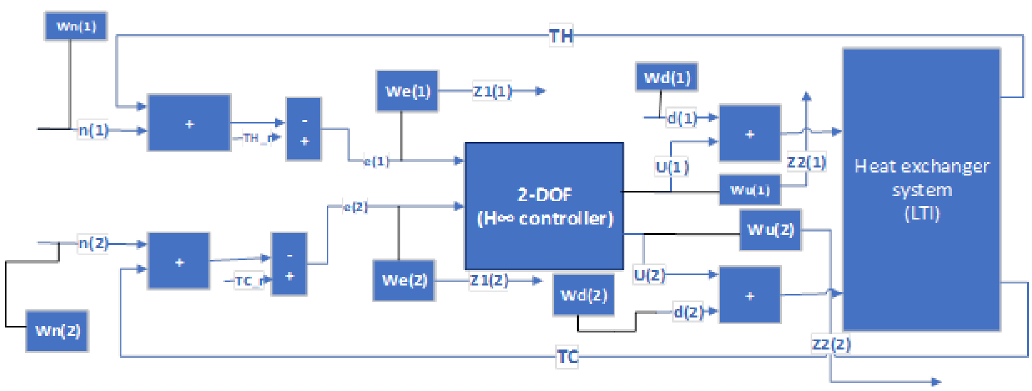

Fig. 5 H-Infinity control scheme

C. Generalized Plant (P)

To proceed to controller synthesis, we first need to get the generalized plant for the above control structure. Now, to get the generalized plant P for the mixed sensitivity H-infinity control problem as shown in Fig. 6, we defined the variables as follows:

The external input vector includes, respectively, the reference input, input disturbance, and measurement noise:

$$W(t) = [r(1)\ r(2)\ d(1)\ d(2)\ n(1)\ n(2)]^T \tag{12}$$

The control input vector contains the control input signals as:

$$U(t) = [U(1)\ U(2)]^T \tag{13}$$

The output vector includes the measured output signals:

$$Y(t) = [Y_1(t) \quad Y_2(t)]^T \tag{14}$$

where $Y_1(t) = [r - y - n]$ (1) and $Y_2(t) = [r - y - n]$ (2).

The controlled output vector contains, respectively, the error signal and the control signal as:

$$Z(t) = [Z_1(1) \quad Z_1(2) \quad Z_2(1) \quad Z_2(2)]^T = [We(1)e(1) \quad We(2)e(2) \quad Wu(1)u(1) \quad Wu(2)u(2)]^T \tag{15}$$

where r and y are the reference and the measured temperature respectively and the indices (1) and (2) refer to the hot and the cold flow respectively.

$$p: \begin{cases} \dot{X} = AX + B_1 w + B_2 U \\ Z = C_1 x + D_{11} w + D_{12} U \\ Y = C_2 x + D_{21} w + D_{22} U \end{cases} \tag{16}$$

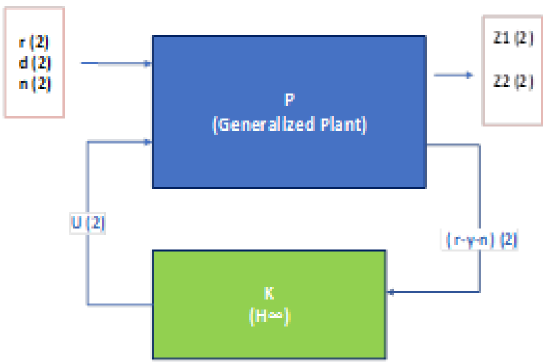

Fig. 6 Generalized plant

The "group of four" sensitivity transfer functions is considered as [14], [15], [17] and [19]:

$$S = (I + KG)^{-1} \tag{17}$$

$$T = KG(I + KG)^{-1} \tag{18}$$

$$KS = K(I + KG)^{-1} \tag{19}$$

$$GS = G(I + KG)^{-1} \tag{20}$$

from Fig. 5, the following set of relationships result:

$$e = Tr - SGd$$
$$U = KS(r - n) - Td$$
$$Y = S(r - n) + SGd \tag{21}$$

Once we have the generalized control configuration, the control objective is to find a state feedback control law:

$$U = -KX \text{ s.t: } \|T_{ZW}(s)\|_\infty \leq \gamma \tag{22}$$

where

$$\|T_{ZW}(s)\|_\infty = [P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}] = \left\| \begin{matrix} W_e S \\ W_u KS \\ W_t T \end{matrix} \right\|_\infty,$$
$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{23}$$

$T_{ZW}(s)$ denotes the close loop system and γ is a positive real number. The lesser the value of the γ, the better is the controller and W_t is the weight function of the complementary sensitivity function T.

D. Control Synthesis

After getting the generalized plant, we can move towards the synthesis of the controller. There are two main methods to synthesize the controller:

- RICATI Method.
- Linear Matrix Inequality (LMI) Method.

We will stick to RICATI based approach to synthesize the controller. The dynamic output feedback, in this case, can be represented as:

$$\dot{X}_K(t) = A_K X_K(t) + B_K Y(t) \tag{24}$$

$$U(t) = C_K X_K(t) + D_K Y(t) \tag{25}$$

It must be noted that, A_K, B_K, C_K, D_K can be found by following the Theorem-1 and Theorem-2 under the assumptions A-1 to A-5 in the RICATTI method approach for control synthesis in [19]. The interconnected system (P) and H-infinity (H_∞) procedures are performed in Robust Toolbox of MATLAB (using sysic and hinfsyn), with the implementation of the control as shown in Fig. 7, the resulted full-controller $K_{H_\infty}(s)$ has the 8th order transfer function. The value of gamma achieved is: $\gamma = 1.6937$, when the system states are not fully observable for 2-DOF and equal to 0.8629 when system states are fully observable for 2-DOF, which refers to the effect of the observability of the system states w.r.t the H-infinity controller.

VI. Simulation Results

A. Stability Analysis

Stability analysis is carried out on the system model before considering the implementation of PID and H-Infinity controller. The values of the parameters and variables are substituted in (6) and (7) to obtain the eigenvalues of the model as shown in Fig. 8.

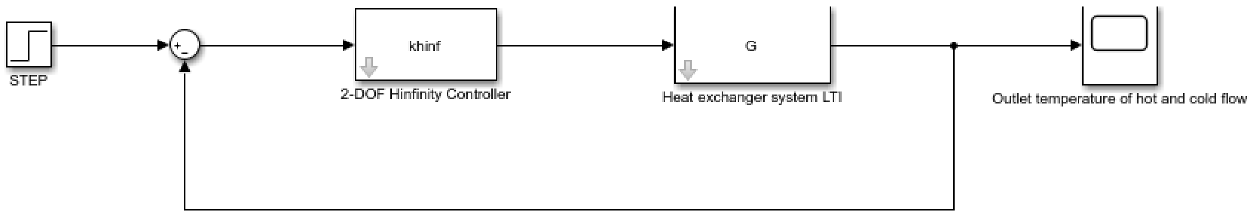

Fig. 7 2-DOF H-infinity control implementation

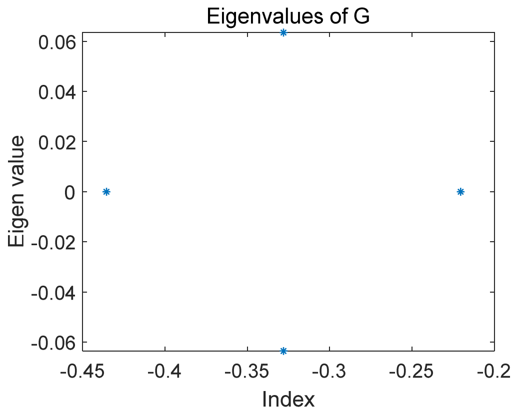

Fig. 8 Stability plot (eigenvalues)

Controllability and observability tests are carried out on the model using Kalman's test with (26) and (27) respectively.

$$M_c = [B \quad AB \quad \cdots \quad A^3B] \tag{26}$$

$$M_o = [C^T \quad A^TC^T \cdots (A^T)^3C^T] \tag{27}$$

Based on Fig. 8, all the eigenvalues have a negative real part, $Re(\lambda) \leq 0$ and it is found that Rank $[M_c] \neq 0$ and Rank $[M_o] \neq 0$ and the rank of the matrices is 4, which is equal to the number of states of the system. Thus, the system is completely state controllable and observable.

B. Performance Analysis Using Frequency Response (Sensitivity Functions)

The obtained shapes of sensitivity functions (Z_1, Z_2) of the closed-loop system with PID (Zeigler-Nichols) controller and 2-DOF H-infinity controller (which can be considered as a combination of two 1-DOF H-infinity controllers, one for the hot flow and the other for the cold flow), over selected frequency range, for the hot flow are presented in Figs. 9 and 10.

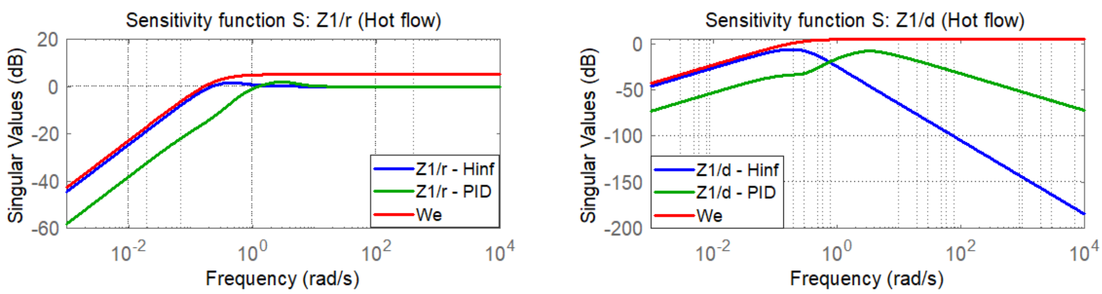

Fig. 9 The frequency responses of the sensitivity function Sr(jw) and Sd(jw)

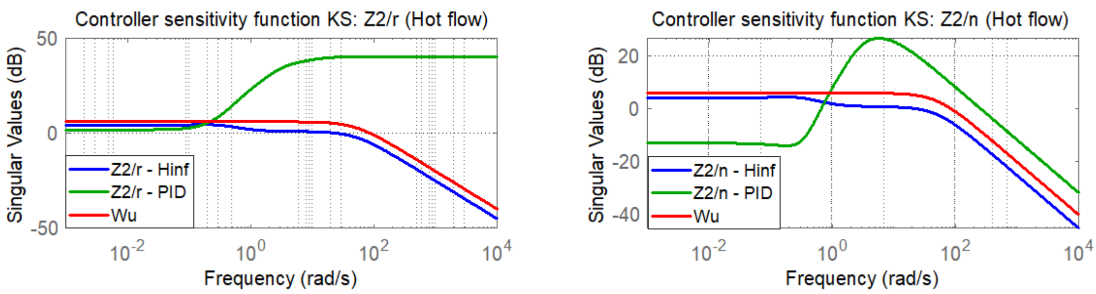

Fig. 10 The frequency responses of the control sensitivity function KSr(jw) and KSn(jw)

1. Analysis on the Sensitivity Function

Fig. 9 shows the sensitivity function with the PID controller and 2-DOF H-infinity controller. The defined performance specification is given by w_e. The results show that PID controller has a faster tracking response and a better disturbance rejection in the frequency range of interest (low frequency) than H-infinity controller.

2. Analysis on the Control Sensitivity Function

Fig. 10 gives the control sensitivity function. The defined performance specification is given by w_u. The results show that the H-infinity controller has more robustness to input saturation than the PID controller as visualized in Fig. 10. This is because the peak value with the PID controller overshoots beyond the specifications. Furthermore, the results show that H-infinity controller has better attenuation to measurement noises as KS with H-infinity controller takes small values in high frequencies. In summary, based on the time-domain specifications mentioned in Section V and the analysis on the sensitivity function and the control sensitivity function, the H-infinity controller meets the stability requirements and performance objectives which makes the closed-loop HE system more stable and robust. Therefore, the H-infinity controller is more robust and has a better overall performance than PID Controller.

3. Performance Analysis Using Time Response

The open-loop behavior of the system is presented in Fig. 11 and to test the performance in the time domain, a closed-loop system is developed for HE in Figs. 3 and 7 with PID controller and H-infinity controller respectively. A servo problem is considered using MATLAB Simulink via unit step change in the set point of controlled variable which is the outlet temperature of cold and hot flow. The response of outlet temperature of the cold and the hot flow respectively for both controllers using step change in the setpoint is shown in Fig. 12.

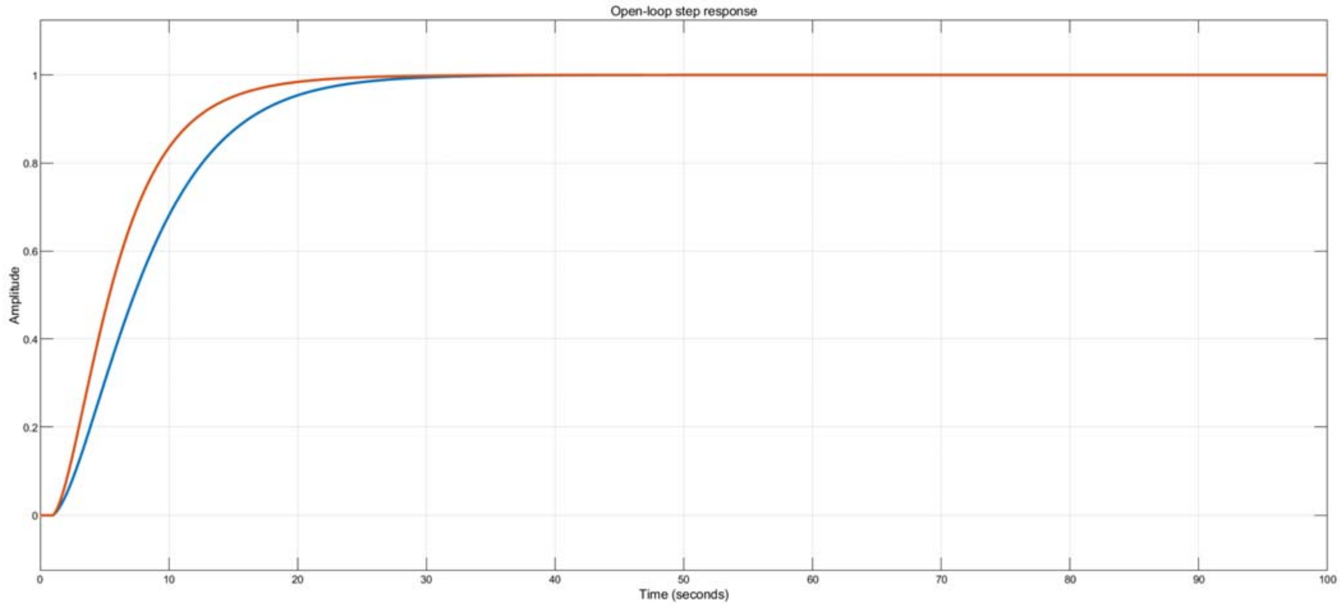

Fig. 11 Open -loop step responses of the system

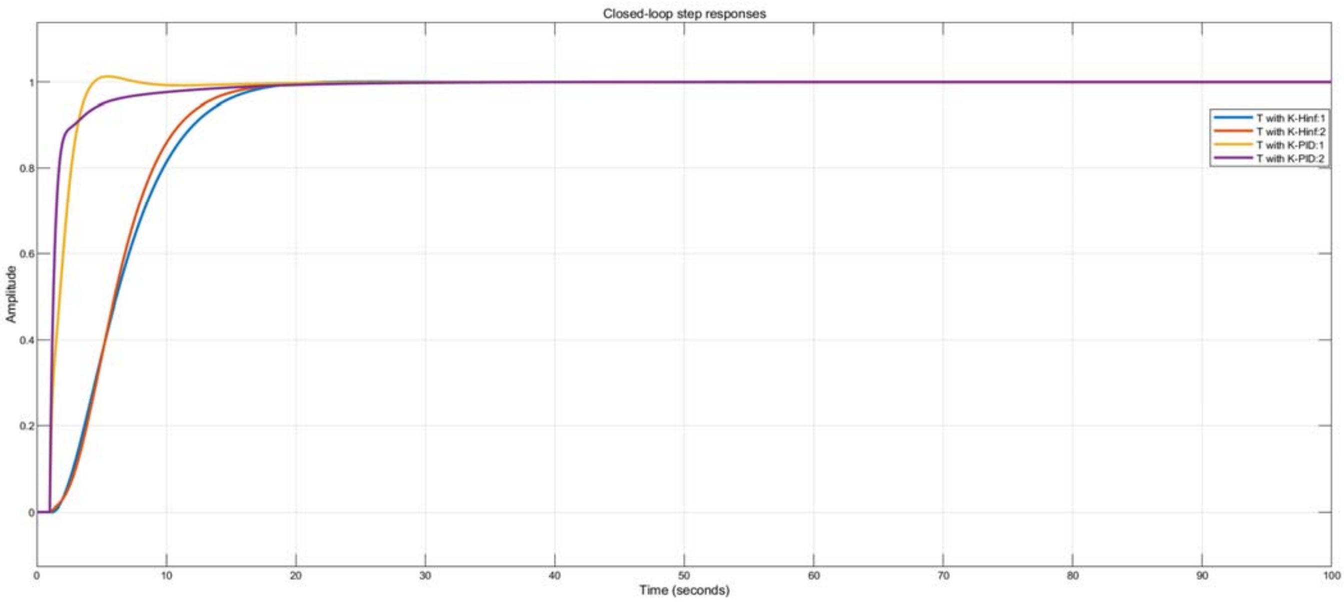

Fig. 12 Closed-loop step responses of the system

The results indicate that the PID controller has a faster response and less settling time response (10 sec) compared to the H-infinity controller (20 sec) which agrees the results obtained from the frequency analysis of the system performance with both controllers and the outlet temperature of the cold flow (purple for PID and red for H-infinity) reaches the steady-state faster than the outlet temperature of the hot flow (orange for PID and blue for H-infinity), as obtained by [22].

VII. Conclusion

In this paper, frequency characteristics of sensitivity functions are analyzed, and steady-state and transient response of tubular HE (MIMO system) using the PID controller and the H-infinity controller in Simulink model using MATLAB is studied for a unit step change in the set point of the controlled variable. The analysis of the results shows that both controllers guarantee the stability of the system and track the step responses of the HE system exactly in a steady-state with a very good transient time. However, the H-infinity controller is more robust and achieves a superior performance compared to the PID controller and it is found, from the mathematical analysis of frequency characteristics of sensitivity functions of the system with H-infinity controller, that the gain margin and the phase margin are positive ($\|S\|_\infty = 1.53\text{ dB} < 6\text{ dB}$), as discussed in [19] which confirms the stability of this MIMO system and the developed model of the HE may be considered for various studies. On the other hand, considering the computational requirements, the PID controller is easy to tune and implement while the H-infinity procedure is difficult in some applications and the resulted robust controller has a higher-order which opens the door for future scope of this research work lies in reducing the order of the robust controller using some techniques based on absolute error, etc. Also, appropriate w_e and w_u ensure sufficient module margin, faster tracking response, and better rejection of disturbance and noise effects as the selection of the weight functions is an art and science.

Fig. 1 shows the experimental HE developed in GIPSA-lab of Grenoble Alpes University, one of the top universities in France. This experimental HE will be used to validate the reported simulation results in this work, which were performed successfully, and testing the performance of the PID and the H-Infinity controller in practice.

An interesting future work would be to decouple the two equations representing the HE system and apply the proposed control approaches for comparison studies.

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