

Multiple Criteria Decision Making Analysis for Selecting and Evaluating Fighter Aircraft

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Abstract—In this paper, multiple criteria decision making analysis technique, is presented for ranking and selection of a set of determined alternatives - fighter aircraft - which are associated with a set of decision factors. In fighter aircraft design, conflicting decision criteria, disciplines, and technologies are always involved in the design process. Multiple criteria decision making analysis techniques can be helpful to effectively deal with such situations and make wise design decisions. Multiple criteria decision making analysis theory is a systematic mathematical approach for dealing with problems which contain uncertainties in decision making. The feasibility and contributions of applying the multiple criteria decision making analysis technique in fighter aircraft selection analysis is explored. In this study, an integrated framework incorporating multiple criteria decision making analysis technique in fighter aircraft analysis is established using entropy objective weighting method. An improved integrated multiple criteria decision making analysis method is utilized to aggregate the multiple decision criteria into one composite figure of merit, which serves as an objective function in the decision process. Therefore, it is demonstrated that the suitable multiple criteria decision making analysis method with decision solution provides an effective objective function for the decision making analysis. Considering that the inherent uncertainties and the weighting factors have crucial decision impacts on the fighter aircraft evaluation, seven fighter aircraft models for the multiple design criteria in terms of the weighting factors are constructed. The proposed multiple criteria decision making analysis model is based on integrated entropy index procedure, and additive multiple criteria decision making analysis theory. Hence, the applicability of proposed technique for fighter aircraft selection problem is considered. The constructed multiple criteria decision making analysis model can provide efficient decision analysis approach for uncertainty assessment of the decision problem. Consequently, the fighter aircraft alternatives are ranked based their final evaluation scores, and sensitivity analysis is conducted.

Keywords—Fighter Aircraft, Fighter Aircraft Selection, Multiple Criteria Decision Making, Multiple Criteria Decision Making Analysis, MCDMA.

I. INTRODUCTION

MULTIPLE criteria decision making analysis (MCDMA) is used in many areas of human activities including in solving complex technical problems. Multiple criteria decision making analysis refers to making decisions in the presence of multiple, usually conflicting criteria. MCDMA method is aiming to choose the prior one among the considered several alternatives, and contributing for evaluation, ordering, classification and choosing processes. MCDMA approach may be described as the choice made by

using at least two criteria from decision maker in problem solving. In multiple criteria decision context, these problems are more complicated, and usually of large scale in nature.

MCDMA techniques are aimed to make the best decision whilst considering the contrast among the criteria. There are many methods available for solving complex MCDMA problems. Choosing the best alternative from a set of alternatives is rather complex decision procedure for decision maker. As a matter of fact, when making an optimal choice among the several conflicting alternatives, decision makers usually apply MCDMA methods, which are categorized into two groups as compensatory methods which permit tradeoffs among criteria, small changes in one criterion can be offset by opposing changes in any other criterion and noncompensatory methods which do not permit tradeoffs among attributes, a disadvantage in one criterion cannot be offset by an advantage in other criterion [2-44].

Each alternative in multiple criteria decision making problem can be described by a set of evaluation criteria. Decision criteria can be qualitative and quantitative. These criteria usually have different units of measurement and different optimization direction. The data normalization aims at obtaining comparable scales of criteria values. The normalization of criteria values is not always needed, but it may be essential for decision analysis. In multiple criteria decision making analysis, different data normalization methods are possible: vector, linear scale, non-linear and logarithmic techniques. The present study is focused on linear normalization method for decision making matrix normalization. The deficiency of information is often ignored in standard decisions made in engineering, management and economy. A decision is often made by comparing costs and benefits of the available alternatives using a set of evaluation criteria under various environmental conditions.

The evaluation of all possible actions is usually conducted using the decision criteria taking into consideration all possible results. Therefore, multiple criteria decision making analysis becomes extremely important. An alternative in multiple criteria evaluation is usually described by quantitative and qualitative criteria. These criteria have different units of measurement. Data normalization is aimed at obtaining the comparable scales of the criteria values. Different techniques of criteria value normalization are used. Normalization of the criteria values is not always necessary. The impact of the decision matrix normalization methods on the decision results is investigated [14-15]. One particular

problem solution method or approach is chosen to decision making matrix normalization. There are still no rules determining the application of multiple criteria evaluation methods and interpretation of the results obtained.

From the literature review, it was determined that several compensatory MCDMA methods were used to solve military fighter aircraft selection problems [2-44]. In that context, application of TOPSIS in evaluating initial training aircraft under a fuzzy environment was considered for the Taiwan Air Force. The fuzzy multiple criteria decision making analysis method was applied to determine the importance weights of evaluation criteria and to synthesize the ratings of candidate aircraft. Aggregated the evaluators' attitude toward preference; then TOPSIS was employed to obtain a crisp overall performance value for each alternative to make a final decision [47].

Evaluating military training aircrafts problem through the combination of multiple criteria decision making processes with fuzzy logic approach was used to solve a real-life decision problem of interest for the Spanish Air Force.

The Analytic Hierarchy Process (AHP) was used to obtain the weights of the criteria and, through the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), the alternatives were evaluated. The selection of the best military training aircraft was based on a set of decision criteria [48].

The selection of military aircraft problem for the Pakistan Air Force was considered using the Analytic Hierarchy Process (AHP) and Cost Benefit Analysis (CBA). A set of ten technical and economic criteria were applied over six alternative aircraft [49].

Also, military fighter aircraft selection problem was considered using multiplicative multiple criteria decision making analysis method for evaluating nine alternatives under ten decision criteria. Robustness of the proposed model was tested by using other MCDMA techniques [45].

Fighter aircraft selection problem using technique for order preference by similarity to ideal solution (TOPSIS) was handled by functioning multiple criteria decision making analysis, considering three real and two test (best, worst) aircraft candidates. Sensitivity analysis was conducted using six objective weighting methods [46].

This study is investigating the application of multiple attribute decision making principles to aircraft engineering technology and performance analysis problems. The study is created for evaluating various processes in aircraft systems design and performance analysis.

In the multiple criteria decision making analysis, a combined entropy index and additive MCDMA model using linear normalization method is implemented. This approach enables decision maker to find an optimal solution under the conditions of risk and uncertainty and to compare the results.

In this paper, the concept of integrated entropy objective weighting index and additive MCDMA technique is presented to develop a multiple criteria decision making analysis method for solving MCDMA problems. Hence, the proposed technique will be extended on the multiple criteria decision making analysis theory. Then, the proposed method

is illustrated with a numerical example.

The rest of the paper is organized as follows. In section 2, multiple criteria decision making analysis method is presented. In section 3, a mathematical experiment to illustrate the application of the described multiple criteria decision making analysis method is presented. In section 4, finally conclusion is pointed out for the MCDMA problem.

II. METHODOLOGY

A. Multiple Criteria Decision Making Analysis

MCDMA approach is a discipline aimed at supporting decision makers who are faced with numerous and conflicting alternatives to make an optimal decision. To achieve this purpose, two critical questions should be unlocked: preference structure and criteria weights. Therefore, various functions are proposed to represent the true preference structure of a decision maker and the correct weights of criteria.

Therefore, a multiple criteria decision making analysis technique is introduced to tackle with preference structure and criteria weights in decision making process. Multiple criteria decision making analysis is very useful method in dealing with decision problems in the real life. It helps decision makers to organize the problems to be solved, and carry out analysis, comparisons and rankings of the alternatives. It has been successfully applied to the areas of diverse research disciplines; ranging from engineering to economics, human resources management, location analysis, quality control, water management, manufacturing, product design, purchasing and outsourcing, financial performance measurement and transportation. In addition, the concept of integrated entropy index and additive MCDMA technique has also been connected to multiple objective decision making analysis and group decision making.

A relative advantage of multiple criteria decision making is its ability to identify the best alternative with subjective/objective or combined methods of criteria weights quickly. It can utilize both quantitative and qualitative data in decision making.

Multiple criteria decision making analysis has four advantages: (1) a scalar value that accounts for both the best and worst alternatives simultaneously; (2) a sound logic that represents the rationale of human choice; (3) the performance measures of all alternatives on attributes can be visualized on a polyhedron, at least for any two dimensions; and (4) a computation process that can be mathematically programmed.

In many fields, such as economics, engineering, environment, involve data that contain uncertainties. To understand and manipulate the uncertainties, there are many approaches such as probability theory, fuzzy set theory, intuitionistic fuzzy sets, rough set theory, soft set theory, and each of these theories has its own difficulties [35-37].

There exists to alternative sets due to the different problem settings: one set contains a finite number of elements (alternatives), and the other has an infinite number. The

problems of MCDMA can be broadly classified into two categories in this respect: Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making (MODM). MODM problem solving is for selection (evaluation), MODM is for design. The fundamental procedural steps of MCDMA methods are as follows: (1) to determine the evaluation criteria of the problem, (2) to determine the alternatives, (3) to evaluate the alternatives according to the decision criteria, (4) to apply the MCDMA technique, and (5) to choose an alternative according to the essentials of the MCDMA technique. MCDMA problems share some common characteristics:

Multiple objectives/attributes: Each problem setting has multiple objectives/attributes.

Conflict among criteria: Multiple criteria usually conflict with each other (i.e., benefit, or cost).

Incommensurable units: Each objective/attribute has a different unit of measurement.

Design/selection: Solutions to these problems are either to design the best alternative or to select the best one among previously specified finite alternatives. The MCDMA process involves designing/searching for an alternative which is the most attractive over all criteria (dimensions).

The terms for MCDMA environment are introduced as follows:

Criteria: A criterion is a measure of effectiveness. It is the basis for evaluation. Criteria are emerging as a form of attributes or objectives in the actual problem setting.

Goals: Goals (targets) are a priori values or levels of aspiration. These are to either achieved or surpassed or not exceeded. Often goals are referred as constraints because they are designed to limit and restrict the alternative set.

Attributes: Performance parameters, components, factors, characteristics, and properties are synonyms for attributes. An attribute should provide a means of evaluating the levels of an objective. Each alternative can be characterized by a number of attributes chosen by a decision maker's conception of criteria.

Objectives: An objective is something to be pursued to its fullest. An objective generally indicates the direction of change desired.

Decision matrix: A MCDMA problem can be concisely expressed in a matrix format. A decision matrix $D = (x_{ij})_{m \times n}$ is a matrix whose elements (x_{ij}) indicate evaluation or value of alternative i , (x_i) , with respect to j , x_j . Hence (x_i) , $i = 1, \dots, m$ is denoted by $x_i = (x_{i1}, \dots, x_{in})$, and the column vector $c_i = (c_{1j}, \dots, c_{mj})^T$ shows the contrast of each alternative with respect to attribute j , c_j . Decision matrix is also called goal achievement matrix, or project impact matrix.

An optimal MCDMA solution is one which results in the maximum value of each of the objective functions

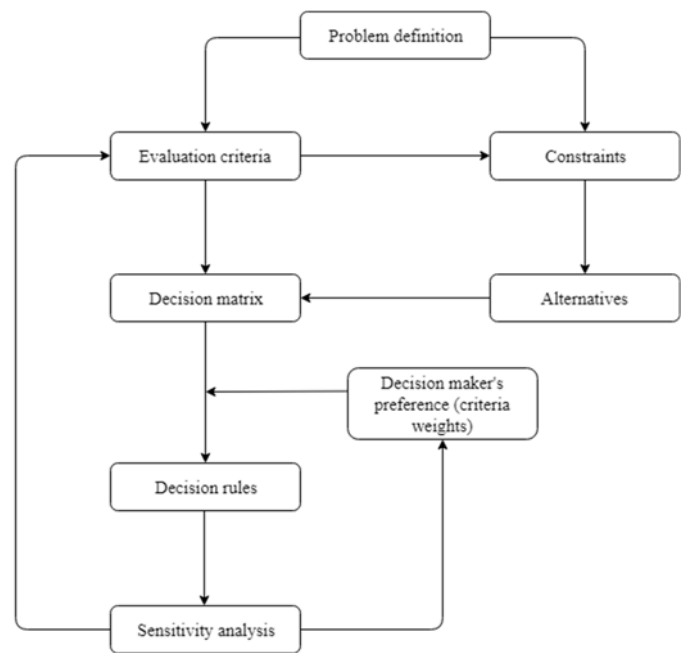


Fig. 1 Schematic procedure for multiple criteria decision making analysis

simultaneously. But it is the nature of MCDMA problems to have conflicting objectives/attributes, usually there is an optimal solution to a MCDMA problem.

A compromise MCDMA solution is one which the minimum value of each of the objective functions simultaneously. It is used to identify solutions that are closest to the ideal solution as determined by some measure of distance. It consists of identifying the different attributes or indicators or performance objectives that contribute to overall performance index compromise solution [23, 34].

A combined multiple criteria evaluation framework incorporating MCDMA techniques in the fighter aircraft selection analysis is established, as given in Fig. 1.

B. Definition of Multiple Criteria Decision Making Analysis Problem

Multiple criteria decision making analysis problem is focused on the fighter aircraft selection and evaluation analysis. The proposed decision analysis approach is the assessment of the contribution of incorporating MCDMA technique in the fighter aircraft selection analysis. Therefore, due to complexity involved in the fighter aircraft evaluation process, and to keep the decision making process transparent, the complexity of the fighter aircraft selection problem is limited with ten decision variables for evaluating fighter aircraft models. Therefore, ten decision variables for fighter aircraft models are considered in this study: maximum cruising speed (mph) (c_1), service ceiling (feet) (c_2), rate of climb (fpm) (c_3), maximum takeoff weight (lbs) (c_4), maximum payload (lbs) (c_5), power (lbf) (c_6), fuel tank capacity (gallon) (c_7), fuel economy (NM per gallon) (c_8),

minimum take off distance (feet) (c_9), minimum landing distance (feet) (c_{10}). The proposed MCDMA technique is applied to the assessment of a set of Sukhoi family of seven fighter aircraft [1] using the multiple criteria decision making analysis technique.

C. Decision Criteria for Fighter Aircraft Selection Problem

The acquisition of an aircraft is usually based on a detailed evaluation of the models which fit the requirements of the industry. In order to pass the selection and evaluation process, the fighter aircraft design should be critically evaluated against the requirements as well as against competing aircraft concepts. Consequently, the fighter aircraft design should meet the future selection criteria and industry requirements including operational advantages compared to its competitors. The evaluation of competing products usually turns on economic, environmental, and technical factors whilst the differences between novel designs and in production fighter aircraft are so small, other criteria have to be considered as well, to win the fierce competition for the Air Force purchase decision.

The identification and selection of appropriate fighter aircraft design criteria is essential to determining an optimal design. The conceptual aircraft design as an approach to fulfill the given requirements is evaluated against its competitors. The design criteria of the fighter aircraft must be a nontrivial and calculable indication of the value of the concept, it should be significantly affected by the design variables and constraints.

The fighter aircraft design characteristics are stability, maneuverability, and controllability qualities. These significant qualities are governing fighter aircraft's performance capabilities.

Stability is the inherent quality of fighter aircraft to correct conditions that may disturb balance and return to the initial flight path or continue. The flight paths and attitudes of fighter aircraft are limited by the aerodynamic characteristics of the aircraft, its propulsion system, and its structural strength. These limitations indicate the maximum performance and maneuverability of the fighter aircraft. If the fighter aircraft is to provide maximum utility, it must be safely controllable within the limits of these limitations without exceeding the pilot's strength or requiring exceptional flight capabilities. If fighter aircraft is to fly straight and stable along an arbitrary flight path, the forces acting on it must be in static equilibrium. The reaction of any body when its equilibrium is disturbed is called stability.

Maneuverability is the quality of an aircraft that allows it to be maneuvered easily and withstand the stresses imposed by maneuvering. It depends on the weight of the aircraft, its inertia, the size and location of the flight controls, the structural strength and the power unit.

Controllability is the ability of an aircraft to respond to pilot's control, including flight path and attitude. This is the quality of the aircraft's response to the pilot's control application when maneuvering the aircraft, regardless of its stability characteristics.

In view of aircraft design characteristics, in order to select a rational alternative of fighter aircraft design and performance analysis, a thorough research of outstanding technological design alternatives is conducted. In this case study, a set of Sukhoi fighter aircraft [1], served as an example of fighter aircraft design and performance decision analysis. The initial data for evaluation of the fighter aircraft alternatives are given in Table 1.

Evaluating fighter aircraft according to qualitative criteria such as reliability, accessibility of operation, communication and command, logistics support and material limits is frequently quite difficult. To improve understanding, technological performance data are used to assess an optimal fighter aircraft. The performance criteria for evaluating fighter aircraft are derived through widespread literature investigation, and the research findings are used to yield the major evaluation criteria, which are briefly described as follows [47, 48]:

Fuel capacity (US gallon): To ensure that the aircraft carries sufficient fuel for mission, considering reserve for diversions, contingencies and safety.

Power plant (shp): To generate the propulsive force directly by increasing the momentum of the airflow through the engine(s).

Service ceiling (ft): The highest operating altitude at which the aircraft can bear the atmosphere and operate efficiently.

Maximum G limits: G (gravity) forces are the acceleration forces that pull on pilots changing the plane of motion. Pilots encounter these forces when engaged in high speed dog fighting. G forces can be either positive or negative, and both types may be dangerous to a pilot. A pilot's weight increases correspondingly as he pulls more Gs. The maximum G limits is the largest positive G force that a pilot can endure.

Minimum G limits: If a pilot is flying straight and pushing the nose of the plane down, then the negative forces of gravity reduces his weight. A pilot who pushes too many negative Gs sees the world through bloodshot eyes. The minimum G limits means the strongest negative force of gravity that a pilots can tolerate.

Maximum operating speed (kt/h): The aircraft is damaged if the maximum operating speed is exceeded.

Economy cruising speed (kt/h): The aircraft must be flown at an economic speed and in a manner that optimizes the fuel cost. Generally, the smaller the fuel consumption per unit distance in cruising flight, the greater the mission distance for a given fuel load.

Maximum operating speed with landing gears down (kt/h): The maximum speed at which the landing gears can be put down safely.

Maximum operating speed with flaps down (kt/h): The maximum speed to ensure safe braking when the aircraft puts the flaps down.

Stalling speed when flameout (kt/h): The minimum speed at which the wings maintain lift at flameout.

Maximum cruising speed (kt/h): The maximum speed of an aircraft being flown in cruising flight.

Maximum climbing rate at sea level (kt/m): The climb is the increase in aircraft height up to the cruising altitude, and descent means the fall in height from end of cruising until landing. The maximum rate climb allows the aircraft to reach its operating height in the minimum time to enable cruising to commence.

Take-off distance (ft): The distance required for the aircraft to accelerate along the runway until it reaches a speed at which it can generate sufficient aerodynamic lift to overcome its weight.

Landing distance (ft): The distance for an aircraft to achieve touchdown, after which the nose is lowered onto the runway and the aircraft is brought to a halt.

Distance required for takeoff and reaching 50 ft height (ft): The distance needed for the aircraft to takeoff and fly over a 50 ft height obstacle.

Distance required to descend from 50 ft height to land and make a full stop (ft): The distance required for the aircraft to fly over a 50 ft height obstacle when landing and bringing to a halt.

In this study, the ten selected decision criteria were used to evaluate a set of Sukhoi fighter aircraft by considering technical performance aspects based MCDMA analysis.

D.Determination of Weights of Criteria

a.Mean Index

The mean index assumes that all criteria are of equal importance, and thus weights are assigned to criteria equally. the mean weight index is used when there is no information from decision maker or information is not sufficient to reach a decision. The mean weight index requires minimal knowledge about priorities of criteria and minimal input of decision maker. In mean weight method, criteria weights are derived objectively.

$$\omega_j = \frac{1}{n} \quad (1)$$

$$\sum_{j=1}^n \omega_j = 1$$

$$\omega_j \geq 0, \quad j = 1, \dots, n$$

where ω_j is the objective weight of the j th criterion which mean weight method assigns.

b. Entropy Index

Entropy is used as a criterion for measuring of represented disorder by a discrete probability distribution. The assumption of entropy is that a wide data distribution shows more disorder than a packed distribution. Entropy method is a useful for seeking for contrast between sets of data. In multiple criteria decision analysis, entropy relates to the degree of diversity within an attribute dataset. The greater the degree of the diversity, the higher the weight of that attribute. In another words, the smaller the entropy within the data associated to an attribute, the greater the discrimination power of the attribute in changing ranks of alternatives. Entropy relates to incomplete information because it relates to the number of possible alternative results for a physical system after all the macroscopically observable information is recorded. The steps for calculation of entropy weights are as follows.

Step 1. Normalizing the decision matrix

Since measured data under different criteria can be of different units or scales, a given decision matrix should be first transformed into a dimensionless space:

$$n_{ij}(x) = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad (2)$$

where x_{ij} is an element of the decision matrix corresponding to the i th alternative and the j th criterion. m is the total number of alternatives, and n is the number of criteria.

Step 2. Calculation of the entropy (e_j) and the degree of diversity (d_j)

Entropy within the datasets of the normalized decision matrix for the j th criterion can be calculated

$$e_j = -\frac{1}{\ln(m)} \sum_{i=1}^m p_{ij} \ln p_{ij} \quad (3)$$

The degree of diversity (d_j) is then calculated as

$$d_j = 1 - e_j \quad (4)$$

Step 3. Calculation of objective weights (ω_j)

The linear normalization of d_j to find the relative objective weight of each criterion:

$$\omega_j = \frac{d_i}{\sum_{j=1}^n d_j} \quad (5)$$

$$s.t. \begin{cases} \omega_j \in e_j \\ \sum_{j=1}^n \omega_j = 1 \\ \omega_j \geq 0 \end{cases}$$

$$i = 1, 2, \dots, m, \quad j = 1, \dots, n$$

where ω_j is the objective weight of the j th criterion which entropy method assigns.

F. Additive MCDMA Model

The additive MCDMA model is utilizing linear combination function when dealing with multiple criteria decision making analysis problems. Proposed method is developed for solving a multiple criteria decision making analysis uncertainty with criteria weights. In a multiple attribute decision making problem, let $x_i = \{x_i, \dots, x_m\}$ be the set of alternatives, $c_j = \{c_1, \dots, c_n\}$ be a set of criteria (attributes), $p_i = \{p_i, \dots, p_m\}$ be set of solution. The procedural algorithm is given as follows:

Step 1: Construction of Decision Matrix

The decision matrix $D = (x_{ij})_{m \times n}$ of performance value x_{ij} is constructed.

Step 2: Construction of Normalized Decision Matrix

Compute the information of each performance value in the judgment matrix and get the information matrix of judgment matrix as $D = (x_{ij})_{m \times n}$. The information values ($n_{ij}(x)$) of the i th alternative with respect to the j th criterion in the decision matrix is normalized by

a. For benefit criteria (larger is better)

$$n_{ij}(x) = \frac{x_{ij} - x_j^-}{x_j^+ - x_j^-} \quad (6)$$

b. For cost criteria (smaller is better)

$$n_{ij}(x) = \frac{x_j^+ - x_{ij}}{x_j^+ - x_j^-} \quad (7)$$

Step 3: Determination of Weights of Criteria

Attribute weights are determined using objective entropy information method.

Step 4: Calculation of Weighted Normalized Decision Matrix

The weighted normalized values $u_{ij}(x)$ in the decision matrix are calculated using the values of the weight coefficients of the criteria.

$$u_{ij}(x) = \omega_j n_{ij}(x) \quad (8)$$

The combined utility of the multiple objectives is the sum of the single utility functions multiplied by attribute weight which reflects the importance of each objective within the decision context.

$$p_i(x) = \sum_{j=1}^n \omega_j n_{ij}(x) = \sum_{j=1}^n u_{ij}(x) \quad (9)$$

where $p_i(x)$ is a synthesizing performance value of the i th alternative; ω_j denotes the weights of the j th criterion; $n_{ij}(x)$ is the normalized preferred ratings of the i th alternative with respect to the j th criterion.

Step 5: Preference Ranking Index

Alternatives are ranked based their final performance scores. Choose the optimal value $p_i(x_o)$ among the values $p_i(x_i)$; $i = 1, \dots, m$, $j = 1, \dots, n$, and hence $p_i(x_o)$ is the optimal choice.

$$q_i(x_i) = \frac{p_i(x_i)}{p_i(x_o)} \quad (10)$$

where $q_i(x_i)$ is preference ranking index.

Step 6: Correlation test is used to evaluate the association between two or more variables.

Pearson Correlation: The Pearson's correlation coefficient is a common measure of association between two continuous variables. It is defined as the ratio of the covariance of the two variables to the product of their respective standard deviations, commonly denoted by the Greek letter ρ (rho):

$$\rho = \frac{Cov(X, Y)}{\sigma_x \sigma_y} \quad (11)$$

The Pearson correlation coefficient $r \in [-1, 1]$ is used to measure the strength of a linear association between two variables, where the value $r = 1$ means a perfect positive correlation and the value $r = -1$ means a perfect negative correlation. The sample correlation coefficient, r , can be obtaining by plugging-in the sample covariance and the sample standard deviations into the previous formula:

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} \quad (12)$$

where

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} ; \bar{y} = \frac{\sum_{i=1}^n y_i}{n}$$

The Pearson's correlation coefficient ranges from -1 to +1. A positive monotonic association (two variables tend to increase or decrease simultaneously) results in $\rho > 0$, and negative monotonic association (one variable tends to increase when the other decreases) results in $\rho < 0$. ρ of 0 corresponds to the absence of the monotonic association, or absence of any association in the case of bivariate normal data. However, for bivariate distributions other than bivariate normal distribution, the Pearson's correlation can be zero for dependent variables. For example, it can be '0' for the variables with non-monotonic relationship, $Y = X^2$, ($x \in (-1, 1)$). The absolute value of ρ indicates the strength of the monotonic relationship between the two variables. ρ of 1 indicates a perfect linear relationship, $Y = a + bX$.

Spearman Correlation: The Spearman correlation method computes the correlation between the rank of x and the rank of y variables. Spearman's rank-order correlation coefficient (denoted ρ_s) is a rank-based version of the Pearson's correlation coefficient. Its estimate or sample correlation coefficient (denoted r_s), can be written as follows:

$$\rho_s = \frac{\sum_{i=1}^n (r(x_i) - r(\bar{x}))(r(y_i) - r(\bar{y}))}{\sqrt{\sum_{i=1}^n (r(x_i) - r(\bar{x}))^2} \sqrt{\sum_{i=1}^n (r(y_i) - r(\bar{y}))^2}} \quad (13)$$

where $r(x_i)$ and $r(y_i)$ are the ranks of the observation in the sample. Spearman's correlation coefficient varies from -1 to +1 and the absolute value of ρ describes the strength of the monotonic relationship. The closer the absolute value of ρ_s to 0, the weaker is the monotonic relationship between the two variables. However, similar to the Pearson product moment correlation coefficient, Spearman's correlation coefficient can be 0 for variables that are related in a non-monotonic manner. At the same time, unlike the Pearson's correlation coefficient, Spearman's coefficient can be 1 not only for linearly related variables, but also for the variables that are related according to some type of non-linear but monotonic relationship.

Kendall's Tau Correlation: The Kendall correlation method measures the correspondence between the ranking of x and y

variables. The total number of possible pairings of x with y observations is $n(n-1)/2$, where n is the size of x and y .

The procedure starts by ordering the pairs by the x values. If x and y are correlated, then they would have the same relative rank orders. Now, for each y_i , count the number of $y_j > y_i$ (concordant pairs (c)) and the number of $y_j < y_i$ (discordant pairs (d)). Kendall correlation distance is defined as follow:

$$\tau = \frac{n_c - n_d}{n(n-1)/2} \quad (14)$$

where

n_c is total number of concordant pairs

n_d is total number of discordant pairs

n is size of x and y

$$\tau = \frac{\sum_{i=1}^n \sum_{j=1}^n \text{sgn}(x_i - x_j) \text{sgn}(y_i - y_j)}{n(n-1)} \quad (15)$$

where

$$\text{sgn}(x_i - x_j) = \begin{cases} 1 & \text{if } (x_i - x_j) > 0 \\ 0 & \text{if } (x_i - x_j) = 0 \\ -1 & \text{if } (x_i - x_j) < 0 \end{cases}$$

$$\text{sgn}(y_i - y_j) = \begin{cases} 1 & \text{if } (y_i - y_j) > 0 \\ 0 & \text{if } (y_i - y_j) = 0 \\ -1 & \text{if } (y_i - y_j) < 0 \end{cases}$$

This coefficient quantifies the discrepancy between the number of concordant and discordant pairs. Any two pairs of ranks (x_i, y_i) and (x_j, y_j) are said to be concordant when $x_i < x_j$ and $y_i < y_j$, or when $x_i > x_j$ and $y_i > y_j$, or when $(x_i - x_j)(y_i - y_j) > 0$. Correspondingly, any two pairs of ranks (x_i, y_i) and (x_j, y_j) are said to be discordant when $x_i > x_j$ and $y_i < y_j$, or when $x_i < x_j$ and $y_i > y_j$ or when $(x_i - x_j)(y_i - y_j) < 0$. Similar to the two previous correlation coefficients, Kendall's tau ranges from -1 to +1, with the absolute value of τ indicating the strength of the monotonic relationship between the two variable. However, Kendall's tau can be 1 for even a wider range of scenarios than Spearman's correlation coefficient.

III. APPLICATION

In this study, to illustrate the application of the described multiple criteria decision making analysis method, the problem of the selection of a rational option of fighter aircraft selection analysis is considered.

In the problem solution, the multiple criteria decision making analysis theory of the discrete decision problem solution is used. As described in multiple criteria decision making analysis technique, any problem to be solved is represented by a decision matrix, containing the alternatives

(rows) and the criteria (columns). Usually, the criteria have different dimensions. In order to avoid the difficulties caused by different dimensions of the criteria, the ratio to the optimal value is used for data transformation. There are various theories describing the ratio to the optimal value. However, the values are mapped either on the interval $[0;1]$ or the interval $[0; \infty]$ by applying the normalization of a decision making matrix. When the linear normalization method is completed, it is possible to evaluate the criteria with weighting factors $0 \leq \omega_j \leq 1$. The sum of the weighting factors should be equal to 1. Here, only objective weighting factors are used because weighting factors influencing the solution are always subjective.

In order to apply the described MCDMA method, first the dataset is preprocessed, and then the normalization of the decision matrix is considered. The initial data of alternatives for normalization are designed and normalized in the decision matrix using linear normalization technique. The criteria weights are determined by the entropy information method. The obtained weight vector of attributes is $\omega_j = (0,06; 0,03; 0,06; 0,02; 0,21; 0,01; 0,13; 0,21; 0,10; 0,18)$. Also, the priority analysis of objective criteria weights is given in Fig. 2.

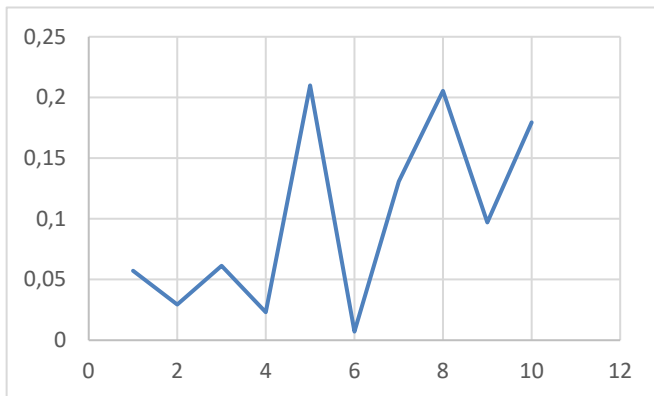


Fig. 2 Priority analysis of criteria weights

In the present study, the linear method of initial decision making matrix normalization is used. The problem solution method of multiple criteria decision making theory is also applied. Tables 3, 4 and 5 provide the solution results and graphical representation of their decision analysis.

The use of linear normalization in multiple criteria decision making analysis technique improves the quality of decision matrix normalization in solving complex technical decision problems.

Sensitivity Analysis: Sensitivity analysis produces a ranking of alternatives based on priority value score. It gives an indication of the relative amount of decision maker value or benefit derived from whatever ratings inputs and priority weights are chosen. Alternative priority value scores fall in a range of $[0;1]$.

An individual alternative's priority value score is not as relevant as its priority value score in relation to other alternatives. Sensitivity analysis enables decision maker to

test "what if" scenarios to see the scoring and ranking impact of changing priority weights and/or ratings inputs.

The sensitivity analysis displays from where alternatives draw most of their priorities in terms of the criteria. If the criteria weights are changed, the resulting impact on the alternatives is seen. If the alternative set is sensitive to changing priorities, then the shifts in the most preferred alternatives are seen. Sensitivity analysis also provides an excellent mechanism for handling objections to the group's criteria priorities. When briefing the results of a decision session, stakeholders often question the priority values derived by the group. A situation can be mimicked where one criteria is increasing in value to see if it changes the most preferred alternative and what level of change is required to impact the alternatives.

Sensitivity analysis is performed using three different scenarios. Therefore, normalized alternatives set is tested with; (1) no criteria weights, (2) mean (equal) criteria weights, and (3) criteria weights obtained from entropy information index. Consequently, the following decision solutions are obtained:

- when weights of the criteria are not included in the evaluation process, the most effective option according to the linear normalization is the alternative "seven" (Table 3). The priority order of the alternatives is presented as $x_4 \succ x_7 \succ x_5 \succ x_3 \succ x_1 \succ x_2 \succ x_6$
- when the equal criteria weights are considered in the process of alternative assessment, the similar ranking solution set is obtained. (Table 4). The priority order of the alternatives is presented as $x_4 \succ x_7 \succ x_5 \succ x_3 \succ x_1 \succ x_2 \succ x_6$
- when the weights of the criteria resulting from entropy information method are included in calculation, linear normalization used in solving the problem determines the alternative "seven" as the most effective (Table 5). The priority order of the alternatives is presented as $x_4 \succ x_7 \succ x_1 \succ x_5 \succ x_3 \succ x_6 \succ x_2$

On the other hand, it should be noted that different results may be obtained if different data normalization techniques, criteria weighting procedures, and multiple criteria decision making methods are used.

The decision analysis results of problem solution are almost stable under the three different scenarios. According to the results obtained in the decision analysis, the most effective seventh alternative is chosen as the best alternative.

Correlation Tests: Correlation tests are used to test the association between two quantitative variables. These tests compute different kinds of correlation coefficients, between two or more variables, and to determine if the correlations are significant or not. This paper proposes three correlation coefficients to compute the correlation between a set of quantitative variables obtained from MCDMA solutions, and the results of three correlation tests are given in Table 1.

Table 1. Correlation Analysis Test Results

Correlation Test Type	Correlation Coefficient	p-value
Pearson correlation coefficient	0,8571	0,0137
Spearman's rank correlation rho	0,8571	0,0238
Kendall's tau correlation	0,7143	0,0302

This certainty value (p-value) shows, how likely it is, that the observed correlation coefficient came out only by coincidence. A low p-value (below 0.05) means that one can be sure about the fact that there is a correlation between the two kind of values, i.e. they move together on the diagram. A high p-value (above 0.05) means that one cannot be sure whether there is correlation between your numbers or not. Usually above 95% (p-value above 0.05) or 99% certainty level (p-value below 0.05 or 0.01) is considered to be high. It's important that despite of the certainty being high, it only means that there is a relationship between the two values, but the strength of the connection between the two data pairs may be minimal or negligible. Therefore, one must also check the experienced strength of the correlation as shown in Table 1.

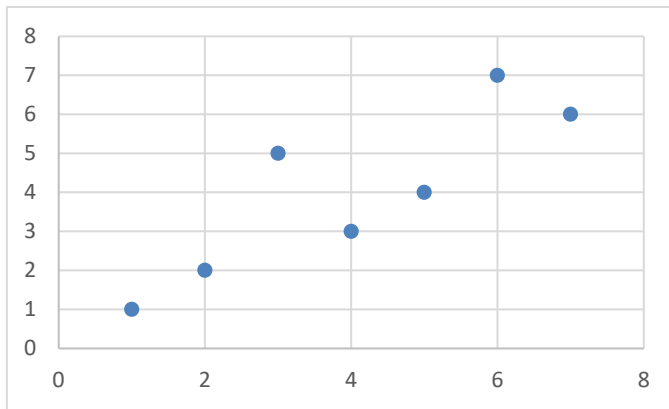


Fig. 5 Scatterplots and Linear Correlation

IV. CONCLUSION

This paper develops an evaluation approach based on the multiple criteria decision making analysis (MCDMA) technique, to help the decision maker choose optimal fighter

aircraft using conflicting decision variables. This study applies the multiple criteria decision making analysis method to determine the importance weights of evaluation criteria and to synthesize the ratings of optimal candidate fighter aircraft. Aggregated the aircrafts' performance scores toward preference; then MCDMA method is employed to obtain an overall performance value for each alternative to make a final decision. This approach was demonstrated with a real case study involving ten evaluation criteria for seven fighter aircrafts.

In complex decision problems, it is difficult to evaluate the effect of different methods of normalizing a decision matrix on the numerical results obtained. However, these problems can be solved by applying the theory of multiple criteria decision making analysis. The linear normalization procedure of decision making matrix is used for data transformation. The entropy index procedure and the linear normalization of decision making matrix provide more stable results for solving multi criteria decision problems.

In this paper, the feasibility, and the contributions of applying MCDMA technique in aircraft selection problems are explored. An integrated framework incorporating entropy index and MCDMA technique for fighter aircraft selection process is established. An integrated multiple criteria decision support system is developed to select the most appropriate MCDMA solution. It is demonstrated that the chosen MCDMA method with improvement provides an efficient objective function for the decision analysis problem. Furthermore, the weighting factors of the decision criteria have significant impacts on the optimal solution. A set of Sukhoi fighter aircraft models for the multiple decision criteria in terms of the weighting factors are constructed. The constructed seven fighter aircrafts models can enable efficient uncertainty assessment for the weighting factors. The application of the MCDMA techniques can be extended to assess air transportation systems, and aircraft for balancing social, economic, ecological, and technical constraints.

Table 2. Fighter Aircraft Evaluation and Selection Problem

Alternatives x_i	Decision Criteria (C_j)									
	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
x_1	815	36090	29530	96462	17637	24675	3650	0,41	3281	5085
x_2	1554	60700	64000	72752	8818	27600	1060	0,68	1805	2198
x_3	1780	61000	65000	67130	9921	27500	3080	0,62	1476	2034
x_4	1554	59000	70000	85600	17600	29400	3165	1,36	1805	2461
x_5	1181	49200	64000	99428	17637	30845	3980	0,54	1969	2133
x_6	1429	55800	64350	72752	14330	27560	3196	0,51	1312	1476
x_7	1678	59100	55100	76059	34171	31900	4410	0,44	1805	2198
Weights	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
Optimization	max	max	max	max	Max	max	max	min	min	min

Table 3. Normalized Decision Making Matrix and Ranking Results of Alternatives without Criteria Weights

Alternatives x_i	Decision Criteria (C_j)										p_i	q_i
	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}		
x_1	0,0000	0,0000	0,0000	0,9082	0,3478	0,0000	0,7731	0,0000	1,0000	1,0000	4,0292	5
x_2	0,7658	0,9880	0,8517	0,1741	0,0000	0,4048	0,0000	0,2842	0,2504	0,2001	3,9191	6
x_3	1,0000	1,0000	0,8765	0,0000	0,0435	0,3910	0,6030	0,2211	0,0833	0,1546	4,3729	4
x_4	0,7658	0,9197	1,0000	0,5719	0,3464	0,6540	0,6284	1,0000	0,2504	0,2729	6,4094	1
x_5	0,3793	0,5263	0,8517	1,0000	0,3478	0,8540	0,8716	0,1368	0,3337	0,1820	5,4833	3
x_6	0,6363	0,7912	0,8604	0,1741	0,2174	0,3993	0,6376	0,1053	0,0000	0,0000	3,8216	7
x_7	0,8943	0,9237	0,6318	0,2765	1,0000	1,0000	1,0000	0,0316	0,2504	0,2001	6,2083	2
Weights	-	-	-	-	-	-	-	-	-	-	-	-
Optimization	max	max	max	max	max	max	max	min	min	min	-	-

Table 4. Weighted Normalized Decision Making Matrix Using Mean Index and Ranking Results of Alternatives

Alternatives x_i	Decision Criteria (C_j)										p_i	q_i
	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}		
x_1	0,0000	0,0000	0,0000	0,0908	0,0348	0,0000	0,0773	0,0000	0,1000	0,1000	0,4029	5
x_2	0,0766	0,0988	0,0852	0,0174	0,0000	0,0405	0,0000	0,0284	0,0250	0,0200	0,3919	6
x_3	0,1000	0,1000	0,0876	0,0000	0,0044	0,0391	0,0603	0,0221	0,0083	0,0155	0,4373	4
x_4	0,0766	0,0920	0,1000	0,0572	0,0346	0,0654	0,0628	0,1000	0,0250	0,0273	0,6409	1
x_5	0,0379	0,0526	0,0852	0,1000	0,0348	0,0854	0,0872	0,0137	0,0334	0,0182	0,5483	3
x_6	0,0636	0,0791	0,0860	0,0174	0,0217	0,0399	0,0638	0,0105	0,0000	0,0000	0,3822	7
x_7	0,0894	0,0924	0,0632	0,0276	0,1000	0,1000	0,1000	0,0032	0,0250	0,0200	0,6208	2
Weights	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	-	-
Optimization	max	max	max	max	max	max	max	min	min	min	-	-

Table 5. Weighted Normalized Decision Making Matrix Using Entropy Index and Ranking Results of Alternatives

Alternatives x_i	Decision Criteria (C_j)										P_i	q_i
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}		
x_1	0,0000	0,0000	0,0000	0,0908	0,0348	0,0000	0,0773	0,0000	0,1000	0,1000	0,4029	3
x_2	0,0766	0,0988	0,0852	0,0174	0,0000	0,0405	0,0000	0,0284	0,0250	0,0200	0,3919	7
x_3	0,1000	0,1000	0,0876	0,0000	0,0044	0,0391	0,0603	0,0221	0,0083	0,0155	0,4373	5
x_4	0,0766	0,0920	0,1000	0,0572	0,0346	0,0654	0,0628	0,1000	0,0250	0,0273	0,6409	1
x_5	0,0379	0,0526	0,0852	0,1000	0,0348	0,0854	0,0872	0,0137	0,0334	0,0182	0,5483	4
x_6	0,0636	0,0791	0,0860	0,0174	0,0217	0,0399	0,0638	0,0105	0,0000	0,0000	0,3822	6
x_7	0,0894	0,0924	0,0632	0,0276	0,1000	0,1000	0,1000	0,0032	0,0250	0,0200	0,6208	2
Weights	0,06	0,03	0,06	0,02	0,21	0,01	0,13	0,21	0,10	0,18		
Optimization	max	max	max	max	max	max	max	min	min	min		

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