# A Review on Bearing Capacity Factor Nγ of Shallow Foundations with Different Shapes

S. Taghvamanesh, R. Ziaie Moayed



***Abstract*****—**There are several methods for calculating the bearing capacity factors of foundations and retaining walls. In this paper, the bearing capacity factor Nγ (shape factor) for different types of foundation have been investigated. The formula for bearing capacity on c–φ–γ soil can still be expressed by Terzaghi's equation except that the bearing capacity factor Nγ depends on the surcharge ratio, and friction angle φ. It is apparent that the value of Nγ increases irregularly with the friction angle of the subsoil, which leads to an excessive increment in Nγ of foundations with larger width. Also, the bearing capacity factor Nγ will significantly decrease with an increase in foundation`s width. It also should be highlighted that the effect of shape and dimension will be less noticeable with a decrease in the relative density of the soil. Hence, the bearing capacity factor Nγ relatively depends on foundation`s width, surcharge and roughness ratio. This paper presents the results of various studies conducted on the bearing capacity factor Nγ of: different types of shallow foundation and foundations with irregular geometry (ring footing, triangular footing, shell foundations and etc.) Further studies on the effect of bearing capacity factor Nγ on mat foundations and the characteristics of this factor with or without consideration for the presence of friction between soil and foundation are recommended.

***Keywords*****—**Bearing capacity, Bearing capacity factor, irregular foundation, shallow foundation.

## I. Introduction

THE foundation is a term used to describe the lowest section of a building. The purpose of this structure is to transmit the load to the earth on which it rests. A well-constructed foundation distributes the weight over the earth without overstressing the soil. Overpressure of the soil may lead to either excessive settlement or shear fracture of the soil, which cause structural damage. Geotechnical and structural engineers designing foundations thus need to assess the bearing capacity of soils. Various kinds of foundations are adopted depending on the structure and soil encountered. The most prevalent kinds of foundations are shown in Fig. 1. A spread footing is essentially an extension of a load-bearing wall or column that allows the structure's weight to be distributed across a wider area of the soil. The size of the spread footings needed in low-load-bearing soil is impractically enormous. In such scenario, constructing the whole structure upon a concrete pad is more cost-effective which is called mat or raft foundation [1].

Shallow foundations are defined as spread footings and mat foundations, while deep foundations are defined as the pile and drilled shaft foundations. Shallow foundations, in a more general sense, are foundations with a depth-of-embedment-to-width ratio of less than four. A foundation may be characterized as a deep foundation if the depth-of-embedment-to-width ratio is higher than four [1].

Among the standard form footing, irregular shaped foundations may be required in certain cases owing to specific circumstances. This classification includes ring footing, circular footing, and triangular footing. Ring footings are mostly used for supporting large and tall structure that the geometry is axisymmetric. Some examples of the ring footing applications are tower soils, oil and water storage tank, bridge piers and offshore structures [16].

A big challenge in foundation design has been determining the bearing capacity and predicting load-displacement behavior. In the past century, a number of traditional techniques for predicting foundation bearing capacity were established. The following studies on the characterization of soil bearing capacity are referred to here: [4], [22], [26], [88], and [36].

The Terzaghi equation is frequently used in foundation design to estimate the bearing capacity of a shallow strip footing [3]:

$$q_u = qN_q + cN_c + \frac{1}{2}B\gamma N_\gamma \tag{1}$$

where $q_u$ is the ultimate bearing capacity; $c$ is the cohesion of the soil beneath the footing; $q$ is the surcharge above the footing's base level; $\gamma$ is the soil's unit weight; $B$ is the footing's width; $N_c$ is the bearing capacity factor related to cohesion $c$; $N_q$ is the bearing capacity factor related to surcharge $q$; and $N_\gamma$ is the bearing capacity factor related to $\gamma$. [4]

Various investigations of the values of the bearing-capacity factors have been conducted since this method was introduced. The technique of characteristics [88], in which plasticity solutions are created by assuming that the soil acts as a material with an associated flow rule, has become the most widely recognized approach for assessing these coefficients. The self-weight of the soil must be taken into account when calculating the third coefficient, $N_\gamma$.

The stress-strain relationship of rigid-perfect plasticity is considered in the technique of characteristics. Only plastic

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zones satisfy the equilibrium, yield criteria, and stress boundary requirements. The bearing capacity of a footing determined using the technique of characteristics is not always a lower-bound or upper-bound solution for soil with self-weight. This method can be used to obtain the combined bearing capacity factor $N_{\gamma q}$ due to the contributions of surcharge q and soil weight, which can be expressed as

$$N_{\gamma q} = \frac{\bar{q}N_q + N_\gamma}{1+\bar{q}} \quad (2)$$

where

$$\bar{q} = \frac{q}{0.5\gamma B} \quad (3)$$

For $\bar{q} = 0$, $N_{\gamma q}$ becomes $N_\gamma$; $\bar{q} =$ infinity, $N_{\gamma q}$ represent $N_q$.

Stress characteristics fields may be generated numerically in this instance, but it is unclear if the resultant slip mechanisms (which include curved slip lines) are also kinematically acceptable, or whether the stress field can be extended beyond the indicated plastic zone. As a result, it is unclear if the solutions found using this method (e.g., Bolton and Lau's (1993) smooth footing solutions [47]) are precise or even lower bounds to the exact answer [5].

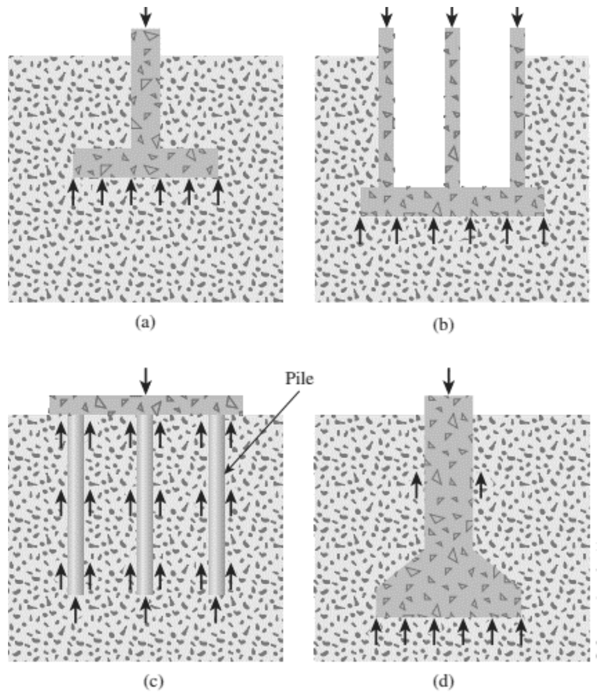


Fig. 1 Common types of foundations: (a) spread footing; (b) mat foundation; (c) pile foundation; (d) drilled shaft foundation [1]

## II. Calculating the Bearing Capacity Factor $N_\gamma$

After Terzaghi's bearing-capacity equation was developed, many researchers worked on it and improved it [87], [26], [83], [36], 1970 [33]. The limit equilibrium technique [4], [87], the method of characteristics [86], the limit analysis method [85], and the finite element method have all been used to calculate $N_\gamma$. These techniques provide a wide range of $N_\gamma$ values, and different formulas based on the limit equilibrium method are used in different countries' design codes to compute $N_\gamma$ [3]. The bearing-capacity factors $N_c$ and $N_q$ do not vary significantly as a result of different solutions. However, the values of $\phi'$ found by various researchers for a given value of $N_\gamma$ vary greatly. This discrepancy is due to a variance in the assumption of the wedge form of soil right under the footing [1] and the different procedures used in the computation of passive earth pressure acting on the edge of the active wedge [3].

The active wedge's shape is specified by the base angle $\psi$ (Fig. 2), which has been considered to be equal to $\phi$ [4], the friction angle; 45°+ $\phi$/2 [26], [36]; or a chosen value so that $N_\gamma$ is a minimum [33], [84].

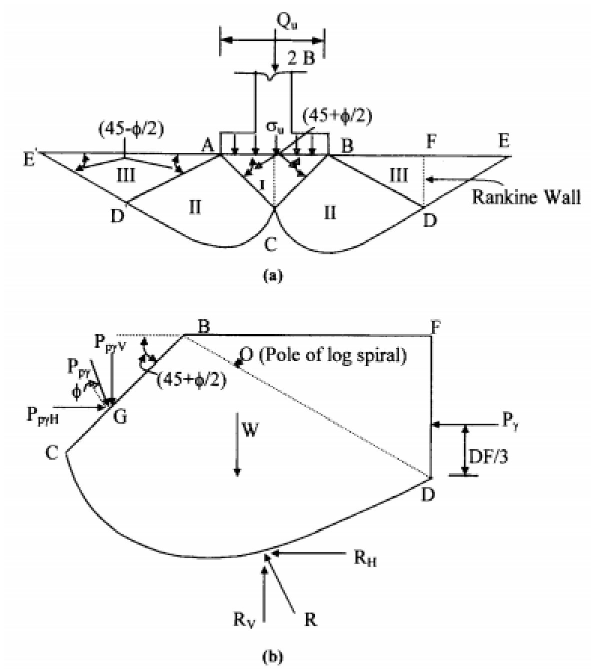


Fig. 2 (a) Failure mechanism-Prandtl`s analysis and (b) free body diagram of Wedge CDFB [6]

Table I provides sixty models for estimating $N_\gamma$ in terms of the author of each approach and the theory on which it is based.

Han et al. [8] computed the bearing capacity factor $N_\gamma$ of strip footings on c–φ–γ soil using the method of characteristics. As mentioned, before it is widely accepted that the bearing capacity factor $N_c$ and $N_q$ are correlated with each other through the following formula and could be computed without any sophistication:

$$N_c = (N_q - 1)\cot\phi \tag{4}$$

When the bearing capacity is calculated without superposition on generic $c-\phi-\gamma$ soil and the result is still stated in the form of (1), the bearing capacity factor $N_\gamma$ is not the same as when the bearing capacity is computed using the superposition technique.

Combining (4) and (1) gives

$$q_u + c\cot\phi = (q + c\cot\phi)N_q + \frac{1}{2}\gamma B N_\gamma \tag{5}$$

By dividing both sides of (5) by $\gamma B$ and setting $p_u = (q_u + c\cot\phi)/\gamma B$ and $\lambda = (q + c\cot\phi)/\gamma B$, the bearing capacity formula is further transformed to

$$p_u = \lambda N_q + \frac{1}{2}N_\gamma \tag{6}$$

where $p_u$ defined as the normalized bearing capacity and $\lambda$ is the surcharge ratio. Equation (6) is a generic solution for strip footing bearing capacity that is equal to (1).

In the precise bearing capacity technique, $N_\gamma$ values are equivalent to those derived using superposition method only when $\lambda = 0$. To get an accurate solution of $N_\gamma$ on generalized $c-\phi-\gamma$ soil, a technique other than Terzaghi's superposition approximation must be used to determine the bearing capacity in the actual failure mechanism. If the bearing capacity is computed without superposition on general $c-\phi-\gamma$ soil, as deduced in the preceding section, the value of $N_\gamma$ relies solely on at a determined $\phi$.

The value of $N_\gamma$ approaches the theoretical upper bound stated by [37], in the Hill mechanism when $\lambda$ equals $10^4$ or even larger:

$$N_\gamma = \frac{1}{4}\tan u\{(\tan u e^{1.5\pi f} - 1) + \frac{3\sin\phi}{1+8\sin^2\phi} \tag{7}$$
$$[(\tan u - \frac{\cot\phi}{3})e^{1.5\pi f} + \tan u\frac{\cot\phi}{3} + 1]\}$$

where $u = \frac{\pi}{4} + \frac{\phi}{2}$ and $f = \tan\phi$.

The suggested formula can offer an accurate estimate with an error of less than $\pm 2\%$, according to comparisons with precise answers derived from numerical findings [8].

## III. Different Types of Foundations

The foundation is a crucial element of the structure since it is responsible for transferring the structure's load to the subsoil. The weight is distributed across a wide area by the foundation. As a result, the pressure on the soil does not exceed its permitted bearing capacity, and the structure settles within acceptable bounds. The foundation improves the structure's stability. The settlement of the structure should be as consistent as possible while staying below permissible limits.

The following are the main functions of foundations:

1- Distribution of loads
2- Resistance to sliding and overturning
3- Keeping the difference in settlement to a minimum
4- Protected from undermining
5- Providing level surface
6- Reducing the amount of disturbance caused by soil movement.

Different kinds of footing are chosen and executed depending on the soil bearing capacity of a certain site.

Foundations is basically classified into two major types such as:

1. Shallow foundation
2. Deep foundation

Shallow foundation is a type of foundation in which the structural load is transferred to the earth's surface, which is quite near to the ground. The ground depth in shallow foundations ranges from 1.5 m to 3 m.

### *A. Spread Footing Foundation*

Spread footing, which is commonly used in residential building, has a broader bottom part than the weight bearing foundation walls it supports. This broader bottom section distributes the structure's weight across a larger area, resulting in increased stability.

### *B. Pad Foundation*

Pad foundation is a flawless sub-category that settled and extended securely into the soil. Pad foundations are preferred when the soil is sufficiently strong and not too deep. The pad foundation thickness is usually consistent. The foundation is securely extended to the bearing stratum above the concentrated load. A schematic overview of pad foundation is shown in Fig. 3.

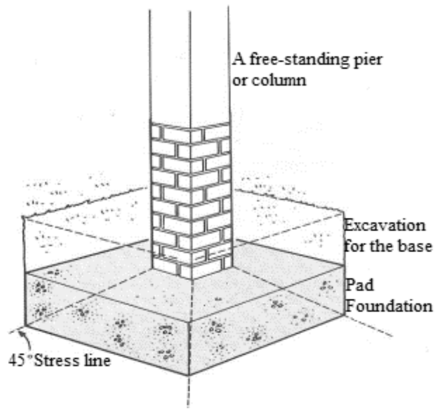


Fig. 3 Schematic view of a pad foundation [37]

TABLE I
EXPRESSIONS FOR THE ESTIMATION OF THE $N_{\gamma}$ FACTOR [32]

| No. | Authors | Expression |
|---|---|---|
| 1 | Terzaghi: fitted expression; limit equilibrium[4] | $N_{\gamma}=[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)+3.0]\tan(1.34\phi)$ |
| 2 | Taylor: limit equilibrium [22] | $N_{\gamma}=[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)-1]\tan(\frac{\pi}{4}+\frac{\phi}{2})$ |
| 3 | Caquot and Kérisel; (obtained from Ukritchon et al. [66]; method of characteristics) [23] | $N_{\gamma}=[1.413\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)+1.794]\tan(1.27\phi)$ |
| 4 | Biarez et al.: equilibrium limit [24] | $N_{\gamma}=1.8[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)-1]\tan\phi$ |
| 5 | Feda: empirical method [25] | $N_{\gamma}=0.01\exp(\frac{\phi}{4})$ (for $\phi<35^{\circ}$; $\phi$ in degrees) |
| 6 | Meyerhof: semi-empirical approach based on limit equilibrium [26] | $N_{\gamma}=[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)-1]\tan(1.4\phi)$ |
| 7 | Hu: fitted expression; equilibrium limit [27] | $N_{\gamma}=[1.901\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)+0.27]\tan(1.285\phi)$ |
| 8 | Krizek: empirical method [28] | $N_{\gamma}=\frac{6\phi}{40-\phi}$ (for $\phi<35^{\circ}$; $\phi$ in degrees) |
| 9 | Booker: method of characteristics [29] | $N_{\gamma}=0.1045\exp(9.6\phi)$ |
| 10 | Hansen and Christensen: fitted expression; method of characteristics [30] | $N_{\gamma}=[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)-1]\tan(1.33\phi)$ |
| 11 | Muhs and Weiss: (Eurocode 7); semi-empirical expression [31] | $N_{\gamma}=2[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)-1]\tan\phi$ |
| 12 | Abdul-Baki and Beik: fitted expression; limit equilibrium [32] | $N_{\gamma}=[1.752\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)+0.186]\tan(1.32\phi)$ |
| 13 | Brinch-Hansen: semi-empirical method (based on Lundgren-Mortensen failure mechanics [83]) [33] | $N_{\gamma}=1.5[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)-1]\tan\phi$ |
| 14 | Davis and Booker: fitted expression; limit equilibrium [34] | $N_{\gamma}=[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)+2.33]\tan(1.316\phi)$ |
| 15 | Chummar: fitted expression; semi-empirical approach [35] | $N_{\gamma}=[7.12\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)+65.5]\tan(0.27\phi)$ |
| 16 | Vesic: approximation (based on Caquot and Kérisel [23] analysis using the method of characteristics) [36] | $N_{\gamma}=2[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)+1]\tan\phi$ |
| 17 | Chen: upper bound limit analysis [37] | $N_{\gamma}=2[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)+1]\tan\phi\tan(\frac{\pi}{4}+\frac{\phi}{5})$ |
| 18 | Chen: fitted from mechanics two values; upper bound limit analysis [37] | $N_{\gamma}=[1.45\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)+0.754]\tan(1.41\phi)$ |
| 19 | Salençon et al.: fitted expression; limit equilibrium [38] | $N_{\gamma}=[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)-1]\tan(1.405\phi)$ |
| 20 | Steenfelt: empirical fitting from N values obtained from Lundgren and Mortensen [39], [40] | $N_{\gamma}=[0.08705+0.3231\sin(2\phi)-0.04836\sin^2(2\phi)]$ $[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(1.5\pi\tan\phi)-1]$ |
| 21 | Craig and Pariti: fitted expression; limit equilibrium [41] | $N_{\gamma}=[2.22\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)+0.222]\tan\phi$ |
| 22 | Spangler and Handy: approximation from Terzaghi's Mechanism [42] | $N_{\gamma}=1.1[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)-1]\tan(1.3\phi)$ |
| 23 | Ingra and Baecher: statistical analysis of footing load test data [43] | $N_{\gamma}=\exp(0.173\phi-1.464)$ ($\phi$ in degrees) |
| 24 | Simone and Restaino:; fitted expression; method of characteristics [44] | $N_{\gamma}=[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi)-1]\tan(1.341\phi)$ |
| 25 | Hettler and Gudehus: empirical approach [45] | $N_{\gamma}=\exp[5.71(\tan\phi)^{1.15}]-1$ |

| No. | Authors | Expression |
|---|---|---|
| 26 | Saran and Agarwal: fitted expression; limit equilibrium [46] | $N_\gamma = \exp[\frac{0.757}{\ln\phi} + 15.286\phi - 3.452]$ |
| 27 | Bolton and Lau: method of characteristics [47] | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 1]\tan(1.5\phi)$ |
| 28 | Bolton and Lau: fitted expression from original values; method of characteristics [48] | $N_\gamma = [1.274\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 3.736]\tan(1.367\phi)$ |
| 29 | Kumbhojkar: fitted expression; numerical solution by graphical method [49] | $N_\gamma = [1.2\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 1.324]\tan(1.417\phi)$ |
| 30 | Zadroga: empirical expression [50] | $N_\gamma = 0.657\exp(1.141\phi)$ ( $\phi$ in degrees) |
| 31 | Manoharan and Dasgupta: fitted expression; finite element nonassociated flow rule | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 3.464]\tan(1.279\phi)$ |
| 32 | Bowles: fitted expression from $K_{py}$ values; limit equilibrium [51] | $N_\gamma = \frac{\tan\phi}{2}(\frac{K_{py}}{\cos^2\phi} - 1) \quad K_{py} = \exp(1.708 + 3.287\phi - \frac{0.34}{\ln\phi})$ |
| 33 | Frydman and Burd: fitted expression; finite difference analysis [52] | $N_\gamma = \frac{1}{4}\{[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 1]\cos\phi\}^{1.5}$ |
| 34 | Michalowski: upper bound limit analysis [53] | $N_\gamma = \exp(0.66 + 5.11\tan\phi)\tan\phi$ |
| 35 | Paolucci and Pecker: fitted expression; upper bound limit analysis [54] | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 1]\tan(1.71\phi)$ |
| 36 | Danish standard DS415 (Danish Standards Association 1998); empirical fitting (from N values obtained from Lundgren and Mortensen [83]) [55] | $N_\gamma = \frac{1}{4}\{[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 1]\cos\phi\}^{1.5}$ |
| 37 | Soubra: fitted expression; upper bound limit analysis [56] | $N_\gamma = [1.374\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 0.162]\tan(1.343\phi)$ |
| 38 | Coduto: approximation from Terzaghi's Mechanism [57] | $N_\gamma = \frac{2[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 1]\tan\phi}{1 + 0.4\sin(4.0\phi)}$ |
| 39 | Perkins and Madson: upper-bound analysis (based on Chen [37]) [58] | $N_\gamma = \frac{1}{2}\tan(\frac{\pi}{4}+\frac{\phi}{2})[\tan(\frac{\pi}{4}+\frac{\phi}{2})\exp(1.5\pi\tan\phi) - 1] + \frac{\sin\phi\cos\phi}{(1+8\sin^2\phi)(1-\sin\phi)}[(\tan(\frac{\pi}{4}+\frac{\phi}{2}) - \frac{\cot\phi}{3})\exp(1.5\pi\tan\phi) + \tan(\frac{\pi}{4}+\frac{\phi}{2})\frac{\cot\phi}{3} + 1]$ |
| 40 | Poulos et al.: solution (based on Davis and Booker [34]) [59] | $N_\gamma = 0.1054\exp(9.6\phi)$ |
| 41 | Ueno et al.: fitted expression; method of characteristics [60] | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 1]\tan(1.461\phi)$ |
| 42 | Wang et al.: fitted expression for mechanics one; upper bound limit analysis [61] | $N_\gamma = 1.2[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 4.6]\tan(1.436\phi)$ |
| 43 | Wang et al.: fitted expression for mechanics two; upper bound limit analysi [61] | $N_\gamma = [1.234\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 4.151]\tan(1.394\phi)$ |
| 44 | Zhu et al.: case 1; limit equilibrium [62] | $N_\gamma = [2\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 1](\tan\phi)^{1.35}$ |
| 45 | Zhu et al.: case 2; limit equilibrium [62] | $N_\gamma = [2\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 1]\tan(1.07\phi)$ |
| 46 | Cassidy and Houlsby: fitted expression; method of characteristics [63] | $N_\gamma = [0.85\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 3.884]\tan(1.716\phi)$ |
| 47 | Dewaikar and Mohapatra: fitted expression; limit equilibrium — Terzaghi's mechanism [64] | $N_\gamma = [1.626\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 2.019]\tan(1.373\phi)$ |
| 48 | Kumar: fitted expression; method of characteristics [65] | $N_\gamma = [0.96\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 0.508]\tan(1.352\phi)$ |
| 49 | Kumar: fitted expression; upper bound analysis — both sides failure mechanism [65] | $N_\gamma = [1.379\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 0.461]\tan(1.337\phi)$ |

| No. | Authors | Expression |
|---|---|---|
| 50 | Ukritchon et al.: fitted expression from mean values; lower and upper bound analysis [66] | $N_\gamma = [1.279\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 3.057]\tan(1.219\phi)$ |
| 51 | Hjiaj et al.: lower and upper bound analysis [67] | $N_\gamma = \exp[\frac{\pi}{6}(1+3\pi\tan\phi)](\tan\phi)^{\frac{2\pi}{5}}$ |
| 52 | Martin: fitted expression method of characteristics [68] | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 1]\tan(1.338\phi)$ |
| 53 | Smith: method of characteristics [69] | $N_\gamma = 1.75[\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 1]\tan\phi$ |
| 54 | Kumar and Kouzer: fitted expression; upper bound limit analysis [70] | $N_\gamma = [1.012\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 0.226]\tan(1.426\phi)$ |
| 55 | Lyamin et al.: lower and upper bound analysis [71] | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 0.6]\tan(1.33\phi)$ |
| 56 | Kumar and Khatr: fitted expression; lower bound finite elements and linear programming [72] | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 1]\tan(1.264\phi)$ |
| 57 | Salgado: approximation expression (from N values of Martin [78] and Lyamin et al. [71]) [73] | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 1]\tan(1.32\phi)$ |
| 58 | Yang and Yang: fitted expression; upper bound limit analysis [74] | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 1]\tan(1.396\phi)$ |
| 59 | Jahanandish et al.: fitted expression; zero extension lines method [75] | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) + 1]\tan(1.5\phi)$ |
| 60 | Kumar and Khatri: fitted expression; lower bound with finite element and linear programming [76] | $N_\gamma = [\tan^2(\frac{\pi}{4}+\frac{\phi}{2})\exp(\pi\tan\phi) - 5.115]\tan(1.577\phi)$ |

### *C. Strip Foundation*

Strip foundations are used to give a continuous level or stepped strip of support to a linear structure, such as walls or closely spaced rows or columns, which are constructed in the center above them. Strip foundations may be built in almost any subsoil, although they need a soil with good bearing capacity. This foundation is often utilized in the construction of medium and low-rise domestic buildings. Fig. 4 illustrated a schematic view of a strip foundation.

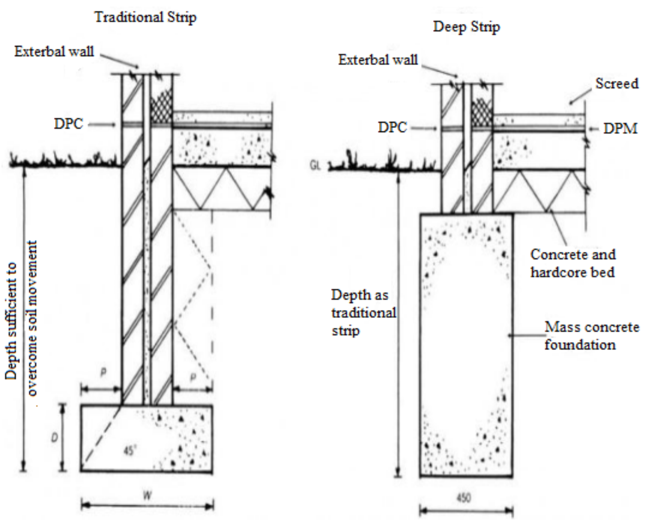


Fig. 4 Schematic view of a strip foundation [37]

### *D. Combined Foundation*

A combined foundation is constructed when two or more columns are close to each other and their foundations overlap. Generally, it is carried out on fields that have low soil bearing capacity. This type of foundation is again subdivided into three categories such as:

1. Rectangular Foundation

When one of the footing projections or the width of the footing is limited, a rectangular footing is built. It operates as an upwardly loaded beam spanning between columns and cantilevering beyond them in the longitudinal direction.

2. Trapezoidal Foundation

In trapezoidal footing two columns carry unequal load and the distance outside the column of the heaviest load is limited. Fig. 5 depicted a schematic view of both above foundations.

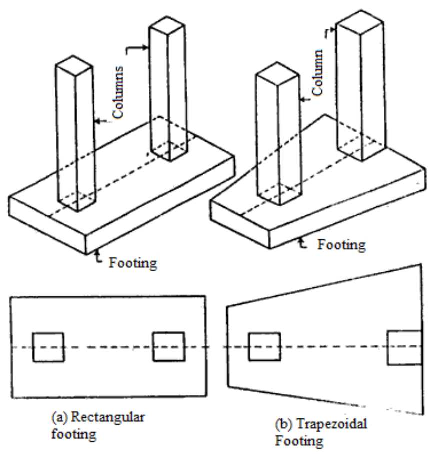


Fig. 5 Schematic view of (a) rectangular footing; (b) trapezoidal footing [37]

3. Strap Foundation

Strap foundation is also known as cantilever foundation. A strap foundation is required when two columns with separate footing bases are linked by a beam. Two or more columns footing are linked by a concrete beam in a strap footing. The weight of a heavy or eccentrically loaded column footing is distributed to neighboring footing using this kind of footing. Fig. 6 shows schematic view of a strap footing.

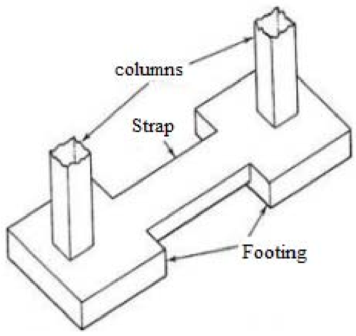


Fig. 6 Schematic view of a strap foundation [9]

*E. Mat Foundation (Rat Foundation)*

A mat or raft foundation is a continuous slap lying on the soil that runs the length of the building's footprint, supporting the structure and transmitting its weight to the ground. Mat footing is a thick concrete slab reinforced with steel that acts as a thick floor, covering the whole contact surface of the building. They are huge concrete slabs that serve as the foundation for a number of walls and columns [9].

It should be noted that every other types of foundation are classified under the categories mentioned above.

## IV. Previous Studies

Table IV summarizes previous studies conducted on the bearing capacity factor $N_\gamma$ on foundation with different shapes.

Zhu et al. [10] investigated scale effect of strip and circular footings resting on dense sand. Using the method of characteristics and the centrifuge modeling methodology, this study presents theoretical and practical analyses of the scale effect of strip and circular footings laying on dry dense sand. For the analysis of the bearing capacity, it is necessary to choose an appropriate boundary condition using the characteristic approach. The stress-dependent soil friction angle obtained from the triaxial test was used. The friction angle of the sand is lowered by around $5^\circ$ for a log-cycle rise in stress. The highest friction angle from triaxial testing was used in the numerical analysis of the circular footing. To validate the numerical results, centrifuge tests of footings were performed. The bearing capacity of the footings obtained using the method of characteristics fits well with the experimental data.

The study of strip and circular footings using the characteristics approach shows that bearing capacity increases exponentially with footing dimension. Accordingly, the bearing capacity factor $N_\gamma$ decreases with increasing dimension, for the dry sand with a unit weight of 15.4 $kN/m^3$ , when the bearing capacity is from 0.62 to 12.3 MPa for strip footing and is from 0.29 to 6.73 MPa for circular footings. A tenfold increase in footing size leads in a 55% decrease in the bearing capacity factor $N_\gamma$. In centrifuge modeling of footings, consistent findings have been found [10].

Sargazi and Hosseininia [11] studied the bearing capacity of ring footings on cohesionless soil under eccentric load. In this study, three-dimensional simulations were employed. The soil model was modeled as a linear elastic perfectly-plastic Mohr-Coulomb model with a flow rule. The material used for the footings was expected to be elastic. The numerical calculations were carried out for a wide range of soil friction angles ( $25-45^\circ$ ). The value of bearing capacity factor for both eccentric $N^*_{\gamma(e=0)}$ and eccentric $N^*_{\gamma(e)}$ loading conditions were measured.

In contrast with two-dimensional analyses performed in the previous studies [14], [80]-[82], where the $N^*_{\gamma(e=0)}$ value decreases as the footing shape become narrower (bigger n value), the $N^*_{\gamma(e=0)}$ value in this study was obtained the biggest for n = 0.25. This tendency was also seen for ring footings subjected to eccentric loading [11].

Gholami and Hoseininia [12] investigated the bearing capacity factors of ring footings by using the method of characteristics. The method of characteristics transforms a set of hyperbolic differential equations into a system of ordinary differential equations. These equations are then solved by the finite difference method. This study presents a comprehensive series of bearing capacity factors for various ratios of internal radius to external radios of ring footings and a wide range of internal friction angle. Fig. 7 indicates the plan of ring footings.

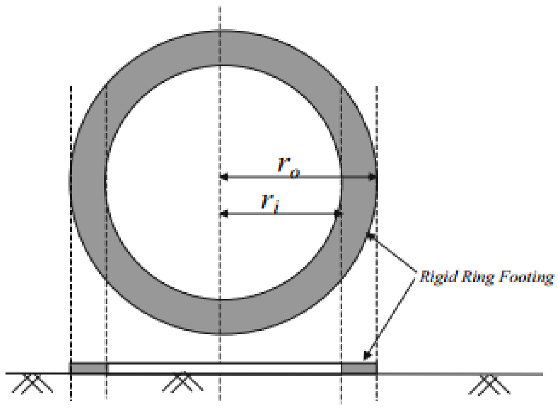


Fig. 7 The plan of ring footings [12]

The results show that the bearing capacity of a ring foundation may be estimated using the calculated bearing capacity components in the superposition equation. The average difference in bearing capacity between these techniques is 10%. Table II shows $N_\gamma$ for smooth and rough ring footings.

TABLE II
VALUES OF $N_\gamma$ FOR SMOOTH AND ROUGH RING FOOTING [12]

| $\phi(^\circ)$ | $N_\gamma$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Smooth, $r_i/r_o$ | | | | | Rough, $r_i/r_o$ | | | | |
| | 0 | 0.25 | 0.5 | 0.7 | 0.9 | 0 | 0.25 | 0.5 | 0.7 | 0.9 |
| 5 | 0.06 | 0.05 | 0.04 | 0.03 | 0.01 | 0.07 | 0.06 | 0.05 | 0.03 | 0.01 |
| 10 | 0.21 | 0.18 | 0.13 | 0.08 | 0.03 | 0.27 | 0.24 | 0.19 | 0.12 | 0.05 |
| 15 | 0.53 | 0.47 | 0.33 | 0.21 | 0.07 | 0.76 | 0.69 | 0.54 | 0.36 | 0.14 |
| 20 | 1.27 | 1.10 | 0.78 | 0.47 | 0.16 | 1.96 | 1.79 | 1.41 | 0.97 | 0.39 |
| 25 | 2.97 | 2.60 | 1.80 | 1.07 | 0.36 | 4.99 | 4.56 | 3.63 | 2.55 | 1.05 |
| 30 | 7.11 | 6.06 | 4.13 | 2.44 | 0.80 | 12.76 | 11.68 | 9.38 | 6.69 | 2.81 |
| 35 | 18.11 | 15.27 | 10.27 | 6.15 | 2.50 | 36.86 | 34.15 | 28.35 | 21.01 | 9.19 |
| 40 | 49.87 | 41.97 | 28.77 | 16.00 | 5.13 | 113.01 | 105.16 | 88.74 | 66.99 | 29.76 |
| 45 | 159.90 | 129.00 | 84.00 | 45.00 | 14.95 | 450.49 | 430.10 | 380.66 | 304.86 | 142.48 |
| 50 | 615.39 | 508.45 | 319.66 | 170.18 | 48.77 | 2008.99 | 1824.40 | 1725.60 | 1145.0 | 708.70 |

Stones et al. [17] conducted an investigation of the bearing capacity of irregular shaped (triangular) footings. The bearing capacity of a triangular footing is calculated in this study using traditional bearing capacity theory applied to an equivalent rectangle obtained from the footing's modified geometry (contact area). However, it should be noted that the bearing capacity estimate is quite sensitive to the angle of friction employed in the computation. The bearing capacity factor $N_\gamma$ is obviously just a function of the stimulated angle of frictional resistance $\phi$.

It should be noted that for smooth footings, the values of $N_\gamma$ given by different authors are very similar. However, for rough footings, there is still a significant variation in the values of $N_\gamma$ reported by different researchers [17].

A numerical study of the bearing capacity factor $N'_\gamma$ of ring footings was carried out by Benmebarek et al. [14]. For both smooth and rough footings, FLAC is utilized to determine the soil bearing capacity factors $N'_\gamma$. In a second research, the dilation angle was used to evaluate soil Non-associativity, which was empirically demonstrated. During the assessments, the following results were found:

- The magnitude of $N'_\gamma$ is found to decrease continuously with an increase in $r_i/r_o$. In addition, the decrease is more pronounced for $r_i/r_o$ beyond 1/3 for rough footing and high friction soils ( $\varphi > 30^\circ$ ).
- The soil dilation angle has a major influence on the value of $N'_\gamma$ when the soil displays high Non-associativity for $\varphi > 30^\circ$. In addition, $N'_\gamma$ decreases significantly when the value of $\psi/\phi$ decreases from $3/4$ to 0. Beyond this limit, the decrease seems to be insignificant.

Moreover, especially for greater values of $\varphi$, the magnitude of $N'$ for a rough footing is found to be significantly higher than the values for a smooth one. For $\varphi = 45^\circ$, the ratio reaches 257% and 317% for circular and ring footings with $r_i/r_o = 0.75$ respectively. As expected, the present values of $N'_\gamma$ for smooth footing are found to match quite well with the various available results in the literature and they are all very close to each other. However, for rough footings, the authors note some discrepancy [14].

Abadi and Hanifi [18] conducted a numerical evaluation of bearing capacity factor $N_\gamma$ for ring footings on layered soil. This article focuses on evaluating effects of soil and also parameters related to geometry of ring footing on variation of bearing capacity factor $N_\gamma$. The modeling of the ring footing has been done in two-dimensional forms and in axial symmetry (axisymmetric elements). The soil profile is consisting of sand and clay. For this study FLAC software is utilized. The following results are observed:

- The values of bearing capacity factor $N_\gamma$ increase substantially when the internal friction angle of the sand layer is increased.
- The failure mechanism shifts from the local to the general phase when the internal friction angle of the sand layer increases.
- When the ratio of internal radius to external radius is increased, the values of bearing capacity factor $N_\gamma$ decrease dramatically.
- Consequentially, raising the dilatation angle increases the bearing capacity factor of the ring foundation [18].

Yang et al. [19] investigated the bearing capacity of ring foundations on sand overlying clay. $R_i/R_0$ is taken into account in five distinct ways in this article, i.e. 0, 0, 25, 0.33, 0.5 and 0.75. From 0.25 to 4, the value of $H/R_0$ is varied, corresponding to various values of $c_u/\gamma R_0$, i.e. It is set at 25, 30, 35, 40, 45. In the numerical simulations using FELA method, both upper and lower bound limit analyses are considered.

The values of $N_\gamma$ for a rough footing on homogeneous sand are presented in Table III. The present values for $N_\gamma$ correspond very closely to the results of other researchers. The lower bound values for $N_\gamma$ obtained are up to 9% higher than other researcher results, while the upper bound values obtained in this study are up to 25% lower than other researchers result [19].

Cerato and Lutenegger [20] studied the bearing capacity of square and circular footing on a finite layer of granular soil underlain by a rigid base. Traditional bearing capacity theories for the ultimate capacity of shallow foundations imply that the thickness of the bearing stratum is limitless. The existence of a hard layer at a particular depth below the foundation may substantially affect the unit load sustained by the soil. Therefore, the original bearing capacity calculations should be changed to account for this situation in calculating the ultimate bearing capacity. In order to examine this phenomenon further, model square and circular footing test were conducted on a bed of well-graded soil.

TABLE III
COMPARISON OF BEARING CAPACITY FACTOR $N_\gamma$ FOR A ROUGH CIRCULAR FOUNDATION ON HOMOGENOUS SAND [19]

| | $\varphi = 30^\circ$ | $\varphi = 35^\circ$ | $\varphi = 40^\circ$ | $\varphi = 45^\circ$ |
|---|---|---|---|---|
| De Simone [76] | 15.73 | 42.38 | 124.46 | 418.93 |
| Erickson and Drescher [77] | - | 45.00 | 130.00 | 456.00 |
| Martin [78] | 15.54 | 41.97 | 124.10 | 419.44 |
| Lyamin et al. [77] | 14.10 | 37.18 | 106.60 | 338.00 |
| Lyamin et al. [71] | 19.84 | 52.51 | 157.20 | 539.20 |
| Loukidis and Salgado [79] | 15.80 | 42.00 | 122.20 | 405.50 |
| Kumar and Khatri [72] | 14.65 | 39.97 | 116.20 | 379.79 |
| Kumar and Charkraborty [80] | 45.80 | 40.10 | 116.57 | 380.08 |
| Present study | 15.06 | 40.15 | 115.03 | 373.37 |
| Present study | 15.62 | 42.21 | 124.97 | 422.68 |

TABLE IV
BRIEF SUMMARY OF THE MAIN DISCUSSION OF THE PAPER

| No. | Title | Authors | Description |
|---|---|---|---|
| 1 | Bearing capacity factors of ring footings by using the method of characteristics | Ghoolami et al. [12] | The bearing capacity of a ring footing with a horizontal ground surface is coded in this research using the stress characteristics method. Friction at the soil-foundation interface is taken into account in the calculations. The soil in this study follows the Mohr–Coulomb yield criteria and is cohesive–frictional-weighted when surcharge pressure is applied. A written code based on the technique of characteristics was used to compute the bearing capacity factors $N_c$, $N_q$, and $N_\gamma$ for ring footings. In contrast to the findings of the principle of superposition, bearing capacity was calculated for various soil conditions and varied ratios of radii. The results demonstrate that the concept of superposition may be used to determine a ring footing's bearing capacity. |
| 2 | Bearing capacity of ring footings on cohesionless soil under eccentric load | Sargazi et al. [11] | The bearing capacity of an eccentrically loaded rough ring footing lying on cohesionless soil was investigated in this study. A series of 3D numerical simulations utilizing the finite difference method were carried out to achieve this purpose. The reduction factor method is used to account for the impact of load eccentricity. The bearing capacity of an eccentrically loaded footing is compared to the bearing capacity of the same footing under vertical load using this technique. When the findings of numerical simulations are compared to those of analytical solutions and experimental data, they show excellent agreement. |
| 3 | The bearing capacity factor $N_\gamma$ of strip footing on $c-\phi-\gamma$ soil using the method of characteristics | Han et al. [8] | The numerical solution of $N_\gamma$ is compatible with full solutions reported in the literature based on cohesionless soil with no surcharge load. The value of $N_\gamma$ for a smooth footing is only half or more of that for a rough footing, according on the relationship of $N_\gamma$ between smooth and rough foundations. It is resulted that both the surcharge ratio $\gamma$ and the roughness of the footing have a substantial effect on $N_\gamma$. |
| 4 | Numerical and experimental evaluation of bearing capacity factor Nγ of strip footing on sand slopes | Taha and Altalhe [13] | This article investigates and presents the results of laboratory model testing and numerical analyses on the behaviour of a strip footing adjacent to a sand slope. The effects of the initial reinforcement layer's depth, vertical spacing, the number of reinforcement layers, and the distance between the edges of footings on bearing capacity were studied. The impacts of each parameter were determined by analysing the results. The use of a strip footing at the top of a sand slope improved bearing capacity significantly. When relative density decreased, the improvement increased. When the distance between the footing edge and the slope crest grew, the depth of the first layer reduced with further improvement. |
| 5 | Numerical evaluation of the bearing capacity factor $N'_\gamma$ of ring footings | Benmebarek et al. [14] | In this paper, numerical computation using FLAC code are carried out to evaluate the soil bearing capacity factors $N'_\gamma$ for both smooth and rough ring footings for low and high friction associated and Non-associated Mohr-Coulomb soils. The findings show that when the ratio of internal radius to exterior radius of the ring increases, the value of $N'_\gamma$ decreases significantly. They also suggest that when the soil exhibits strong non-associativity for large internal friction angle values, the soil dilation angle affects the value of $N'_\gamma$. |
| 6 | Scale effect of strip and circular footings resting on dense sand | Zhu et al. [10] | The findings of a programmed study on strip and circular footings sitting on dry thick sand are presented in this article. The computational and experimental investigation of the scale impact on the bearing capacity and the shape factor $s_\gamma$ of the footings. The characteristics technique is used to examine the footings. It has been decided to use a wedge failure mechanism. The friction angle of thick sand decreases with stress level in a triaxial compression test performed under confining pressure up to 2,500 kPa. In the study of the characteristic, the stress-dependent friction angle of soil is used. The numerical findings show that the bearing capacity grows exponentially with the size of the footing. The bearing capacity factor $N_\gamma$ decreases as the footing size increases, but the shape factor $s_\gamma$ increases. |
| 7 | An investigation of the bearing capacity of irregular shaped (triangular) footings | Stones et al. [15] | The findings of a study of the response of triangle footings exposed to both centric and eccentric loads are presented in this article. To examine the footing reaction, a series of small-scale single gravity experiments were performed on model footings. In addition, a simple elastic-perfectly plastic soil model was used to conduct a basic numerical analysis. |

| No. | Title | Authors | Description |
|---|---|---|---|
| 8 | $N_\gamma$ for rough strip footing using the Method of Characteristics | Kumar [16] | By using the method of characteristics, the bearing capacity factor Nγ was computed for a rough strip footing. The analysis was performed by considering a curved nonelastic wedge under the foundation base bounded by curved slip lines being tangential to the base of the footing at its either edge and inclined at an angle π/4 – φ/2 with the vertical axis of symmetry. The existing theories in the literature for rough footings, which usually employ a triangular wedge below the footing base, were generally found to provide greater values of Nγ as compared with the results obtained in this contribution. |

When the layer between the footing and the rigid base, H, is sufficiently thin, the bearing capacity factor, $N_\gamma$ should be modified to $N^*_\gamma$. Large model footing test was performed in a 2.44 m by 2.44 m $\times$ 1.22 m test pit with walls and floors constructed of reinforced concrete and the loading frame constructed of steel. All tests were performed under saturated conditions with the footing resting on the sand surface ($D_f = 0$). Square footings were tested at all three relative densities in this study ($D_r$ =24, 57 and 87%), while the circular footings were tested only at the highest relative density ($D_r$ =87%) [20].

The values of $N^*_\gamma$ were back-calculated for each test assuming Terzaigh`s [4] definition of shape factors on an infinite layer, because the $q_{ult}$ results from [20] showed that there actually was bearing capacity differences between square and circular footings. Table V presents results of the ultimate bearing capacity and back-calculated values of $N^*_\gamma$ for the large model footing tests.

TABLE V
LARGE MODEL FOOTING TEST RESULTS [20]

| $D_r$ (%) | H/B | B (m) | Square $q_{ult}$ (kPa) | Square $N^*_\gamma$ | Circular $q_{ult}$ (kPa) | Circular $N^*_\gamma$ |
|---|---|---|---|---|---|---|
| 24 | 1 | 0.152 | 208 | 328 | - | - |
| | 0.5 | 0.152 | 320 | 505 | - | - |
| | 1 | 0.305 | 42 | 33 | - | - |
| 57 | 1 | 0.152 | 290 | 429 | - | - |
| | 0.5 | 0.152 | 378 | 559 | - | - |
| | 3 | 0.305 | 83[a] | 62 | - | - |
| | 1.5 | 0.305 | 82 | 61 | - | - |
| | 1 | 0.305 | 110 | 81 | - | - |
| | 0.5 | 0.305 | 190 | 140 | - | - |
| 87 | 3 | 0.152 | 225 | 313 | 170 | 252 |
| | 1 | 0.152 | 342 | 476 | - | - |
| | 0.5 | 0.152 | 403 | 561 | - | - |
| | 3 | 0.305 | 207 | 144 | 203 | 188 |
| | 3 | 0.457 | 217 | 101 | - | - |
| | 1.5 | 0.305 | 180 | 125 | 210 | 195 |
| | 1 | 0.305 | 240 | 167 | 210 | 195 |
| | 0.5 | 0.305 | 512 | 356 | 256[b] | 237 |
| | 0.25 | 0.305 | - | - | 1,230 | 1.141 |

[a] average of two tests
[b] average of three tests

Results of the model footing test show that the modified bearing capacity factor, $N^*_\gamma$, is dependent on relative density $D_r$, $H/B$ and footing width, B. The tests indicate that $N^*_\gamma$ decreases as footing size B, and $H/B$ increases and $N^*_\gamma$ increases as relative density, $D_r$ increases. It was observed that the modification of the bearing capacity factor, $N^*_\gamma$, should extend to $H/B \le 3$, for circular and square footings. It was seen that $N^*_\gamma = N_\gamma$ at $H/B \ge 3$ [20].

Yang and Yang [21] proposed a revised failure mechanism of strip footings for upper bound solution. Based on the suggested failure mechanism, a formula for calculating the upper limit solution for bearing capacity factor $N_\gamma$ is derived and a method is constructed. When compared to previous rigid block upper bound limit analysis solutions, the findings produced in this research may offer better values of $N_\gamma$, which are closer to the precise values given by Martin utilizing the characteristics approach. Table VI represents the comparison of $N_\gamma$ for rough footing using upper bound limit analysis.

TABLE VI
COMPARISON OF $N_\gamma$ FOR ROUGH FOOTING USING UPPER BOUND LIMIT ANALYSIS [21]

| $\phi$ (°) | This study [21] | Chen [37] | Michalowski [53] | Soubra [56] | Zhu [75] | Hjiaj [67] | Martin [68] |
|---|---|---|---|---|---|---|---|
| 5 | 0.150 | 0.38 | 0.18 | - | - | 0.1196 | 0.1134 |
| 10 | 0.543 | 1.16 | 0.71 | - | 0.71 | 0.4552 | 0.4332 |
| 15 | 1.442 | 2.73 | 1.94 | - | - | 1.2378 | 1.1814 |
| 20 | 3.396 | 5.87 | 4.47 | 4.49 | 4.47 | 2.9612 | 2.8389 |
| 25 | 7.617 | 12.4 | 9.77 | 9.81 | - | 6.7379 | 6.4913 |
| 30 | 16.968 | 26.7 | 21.39 | 21.51 | 21.38 | 15.2372 | 14.7543 |
| 35 | 39.155 | 60.2 | 48.70 | 49.00 | - | 35.6491 | 34.4761 |
| 40 | 96.795 | 147.0 | 118.83 | 119.84 | 118.75 | 88.3901 | 85.5656 |
| 45 | 265.412 | 401.0 | 332.0 | 326.59 | - | 240.8801 | 234.213 |

The value of bearing capacity factors $N_\gamma$ for rough footings is clearly overestimated via upper bound limit analysis. Compared to previous rigid block limit analyses, the findings given in this article give better estimates of $N_\gamma$ and accord well with finite element upper bound limit analysis methods [21].

## V. CONCLUSION

An overview of the bearing capacity factor $N_\gamma$ of foundations with different shapes is presented in this paper. It is widely accepted that the bearing capacity factors $N_c$ and $N_q$ are correlated with each other and could be computed without any sophistication. However, there are arguments on calculating the bearing capacity factor $N_\gamma$.

The different types of foundations are discussed in this article, as well as the calculation and simulation of bearing capacity factor $N_\gamma$. It should also be highlighted that, out of all the methods discussed in the text, the characteristics method has received the most attention.

Footing width B, internal friction angle of subsoil $\phi$, dilatation angle $\psi$ and relative density $D_r$ are parameters that have major impact on bearing capacity factor $N_\gamma$. Future research is required, in order to evaluate the bearing capacity factor $N_\gamma$ of T-shaped footing.

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