Military Fighter Aircraft Selection Using Multiplicative Multiple Criteria Decision Making Analysis Method

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Abstract—Multiplicative multiple criteria decision making analysis (MCDMA) method is a systematic decision support system to aid decision makers reach appropriate decisions. The application of multiplicative MCDMA in the military aircraft selection problem is significant for proper decision making process, which is the decisive factor in minimizing expenditures and increasing defense capability and capacity.

Nine military fighter aircraft alternatives were evaluated by ten decision criteria to solve the decision making problem. In this study, multiplicative MCDMA model aims to evaluate and select an appropriate military fighter aircraft for the Air Force fleet planning. The ranking results of multiplicative MCDMA model were compared with the ranking results of additive MCDMA, logarithmic MCDMA, and regrettive MCDMA models under the L_2 norm data

normalization technique to substantiate the robustness of the proposed method. The final ranking results indicate the military fighter aircraft Su-57 as the best available solution.

Keywords—Military Fighter Aircraft Selection, Air Force Fleet Planning, Multiplicative MCDMA, Additive MCDMA, Logarithmic MCDMA, Regrettive MCDMA, Mean Weight, Sensitivity Analysis.

I. INTRODUCTION

MULTIPLICATIVE multiple criteria decision making analysis (MCDMA) method is proposed to evaluate military aircraft selection problem. The model aids decision makers to reach accurate decisions in decision making analysis problems. The decision making analysis methods may be categorized as compensatory (AHP, CP, MAUT, TOPSIS, VIKOR, and Fuzzy Applications) and noncompensatory approaches (ELECTRE, PROMETHEE, and Fuzzy Applications) [1-3].

The proposed technique is a mathematical decision analysis tool for solving complex real-life problems due to its intrinsic ability to judge diverse alternatives with reference to various decision criteria in order to choose to best alternative.

Military fighter aircraft selection problem involves conflicting multiple criteria to evaluate a set of alternative aircraft candidates. The decision criteria and alternatives were identified from the literature review.

Air Forces play an essential role in the national defense from policy & strategy to capability & capacity.

Rapid technology change necessitates modernization and

C. Ardil is with the National Aviation Academy, Baku, Azerbaijan. https://orcid.org/0000-0003-2457-7261 upgradation of military fighter aircraft fleet for modern air forces. The decision making for a military fighter aircraft selection must be methodical and systematic analytics to ensure a feasible and appropriate selection, which is critical for the Air Force fleet planning. The Air Force must have a fleet that provides the capability, capacity, and the affordability to achieve national defense strategy objectives and meet air superiority and global strike needs for the national joint force. However, the lack of new generation military fighter aircraft adversely impacts the capability and capacity of air forces.

The proposed method uses a varying set of decision criteria for military fighter aircraft evaluation problem, and supports decision makers in determining the best choice or alternative with great efficiency in the decision making process. In the systematic procedure, first, the proposed model is applied to the aircraft selection problem, and then, the proposed approach is employed for evaluation and ranking alternatives.

When employing multiplicative MCDMA for evaluation and ranking alternatives, the mean weight technique is employed to determine objective weight coefficients of criteria. Then, the objective criteria weights are applied to rank the military fighter aircraft alternatives.

From the literature review, it was determined that a number of compensatory MCDMA methods were used to solve military fighter aircraft selection problems. In that context, application of TOPSIS in evaluating initial training aircraft under a fuzzy environment was considered for the Taiwan Air Force. The fuzzy multiple criteria decision making analysis method was applied to determine the importance weights of evaluation criteria and to synthesize the ratings of candidate aircraft. Aggregated the evaluators' attitude toward preference; then TOPSIS was employed to obtain a crisp overall performance value for each alternative to make a final decision [4].

Evaluating military training aircrafts problem through the combination of multiple criteria decision making processes with fuzzy logic approach was used to solve a real-life decision problem of interest for the Spanish Air Force.

The Analytic Hierarchy Process (AHP) was used to obtain the weights of the criteria and, through the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), the alternatives were evaluated. The selection of the best military training aircraft was based on a set of decision criteria [5].

The selection of military aircraft problem for the Pakistan Air Force was considered using the Analytic Hierarchy Process (AHP) and Cost Benefit Analysis (CBA). A set of ten technical and economic criteria were applied over six alternative aircraft [6].

Multiple criteria decision making analysis helps a decision maker which quantifies particular criteria based on its importance in presence of other objectives. This work introduces some important features of the multiple criteria decision analysis, various algorithms available and highlights its various features in context to the military fighter aircraft evaluation. The multiple criteria decision making analysis technique presented here can be used to find out an apt solution to the military fighter aircraft analysis system design problems involving multiple and conflicting objectives. The decision analysis technique is formulated from a decision matrix of preferences, and the criteria weights. The weighted normalized values are then used to aggregate preference information, as well as to rank the order of decision alternatives [7-12].

Decision analyses for the military aircraft selection problems are based on the economic, environmental, and technical performance factors. Sensitivity analysis is performed to understand the effect of a set of independent variables on some dependent variable under certain specific conditions. For the robustness of the proposed method, the validity of the model is compared with the ranking results of additive MCDMA model, logarithmic MCDMA model, and regrettive MCDMA model under the L_2 norm data normalization technique [14-16].

This study presents a multiplicative MCDMA approach, which employs the L_2 norm data normalization technique, and supports precise decision making analysis process for the military fighter aircraft selection problem.

The remaining parts of this paper is organized as follows: Section 2 presents the methodology of the decision analysis problem. Section 3 indicates application of the L_2 norm data normalization technique to the multiplicative MCDMA model, additive MCDMA, logarithmic MCDMA, and regrettive MCDMA model. Validity analysis was performed for the robustness of the proposed model. Finally, conclusions and future directions are presented in section 4.

II. METHODOLOGY

Multiple criteria decision making techniques are used in complex decision situations where decision makers are tasked with choosing the best option among multiple alternatives. The proposed multiplicative MCDMA method is employed to reach an optimum decision solution when faced with multiple alternatives with multiple conflicting (i.e., "benefit" and "cost") and noncommensurable decision criteria.

The proposed multiplicative MCDMA model provides a conservative aggregative method that satisfies the principle of annihilation. The criteria weights reflect exponential relative importance, not proportional between decision criteria. The use of multiplicative MCMMA is pivotal to aggregate the performance values x_{ij} with the decision criteria weights ω_j in deriving the relative priorities in decision making.

The multiplicative MCMMA is sometimes called dimensionless analysis because its structure eliminates any units of measure. The method evaluates the decision matrix, which refers to *i* alternatives that are evaluated in terms of *j* decision criteria. The member x_{ij} denotes the performance measure of the *j*th alternative in terms of the *i*th criterion. The basic procedure of MMCDMA is presented as follows:

Step 1: Determine the decision matrix with the multiple criteria. In MCDMA with *J* decision criteria $g_j = (g_1, g_2, ..., g_n), j = 1, ..., J$, and each alternative *x* is represented by a vector $x_i = (x_1, x_2, ..., x_j)$ where x_j (j = 1, 2, ..., J) is a raw measure or description of the tangible or intangible impact of *x* in the criterion g_j .

$$X = \begin{pmatrix} x_1 \\ \vdots \\ x_i \end{pmatrix} \begin{pmatrix} g_1 & g_j \\ x_{11} & \cdots & x_{1j} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} \end{pmatrix}_{ixj}$$
(1)

Let $X_i = (x_1, x_2, ..., x_i)$ denote the set of all the alternatives under evaluation. Assume that the preference of the alternatives $(x_1, x_2, ..., x_i)$ with respect to a single criterion g_j is completely known and measured explicitly.

Let denote the criterion specific score of the alternative x_i with respect to the criterion g_j by $z_j(x_i)$.

The MCDMA problem is to evaluate $Z(x_i) = (z_1x_1, ..., z_jx_i)$, (i = 1, 2, ..., I) and determine the overall values of the alternatives or simply select the best alternative x^* .

The superfunction approach assumes that the overall value of $Z(x_i)$ can be explicitly represented by some unidimensional overall value function $U(x_i)$ so that x_i is preferred to the alternative x_j if and only if $U(x_i) > U(x_j)$ and the best alternative x^* has the largest $U(x^*)$ in $(U(x_i),...,U(x_i))$.

In general, the mathematical representation of the superfunction U can be complicated without any explicit notions of criteria weights [10]. However, it is common in many MCDMA models to assume the additive form as follows:

$$U(Z(x_i)) = \sum_{j=1}^{J} \omega_j z_j(x_i)$$
⁽²⁾

Step 2: Minimization procedure of the maximal regret in regrettive MCDMA model is given by

a. For each
$$g_i$$
 calculate $C_i = max(c_{ij})$ (3)

b. For each pair
$$x_i$$
 and g_i calculate $z_j(x_i) = C_j - c_{ij}$ (4)

c. For each
$$x_i$$
 calculate $U(Z(x_i)) = \sum_{j=1}^{J} \omega_j z_j(x_i)$ (5)

d. Select
$$x_{i^*}$$
 such that $U(Z(x_{i^*})) = \min \sum_{j=1}^J \omega_j z_j(x_i)$ (6)

where ω_j denotes the weight of the criterion g_j . Then, the decision maker needs only to determine the criteria weights $\omega = (\omega_1, ..., \omega_j)$ and the criterion specific scores in $Z(x_i)$ [17].

Step 3: Rank the alternatives according to the increasing values of assessment scores $U(Z(x_i))$. The alternative with the lowest with the lowest $U(Z(x_i))$, is the best choice among the alternatives.

Step 4: The multiplicative form of MCDMA model is given by

$$U(Z(x)_{i}) = \prod_{j=1}^{J} z_{j}(x_{i})^{\omega_{j}}$$
⁽⁷⁾

and the linear form of logarithmic MCDMA model is given by

$$\log\left(\prod_{j=1}^{J} z_{j}(x_{i})^{\omega_{j}}\right) = \sum_{j=1}^{J} \omega_{j} \log(z_{j}(x_{i}))$$
(8)

It follows that ω_j can be interpreted as the marginal contribution of one unit of $\log(z_j(x_i))$ into units of $\log[(z_i(x_i))]$. The overall ranking derived from using the

multiplicative form $\prod_{j=1}^{J} z_j(x_i)^{\omega_j}$ and the linear form $\sum_{j=1}^{J} \omega_j \log(z_j(x_i))$ would be similar since the logarithm

function log is monotone and increasing.

Evaluation of the alternatives under different criteria usually involves different and noncommensurate measuring scales. To truly assimilate the MCDMA problem, the decision criteria must be compared explicitly and allowed to compete with each other. It is useful to introduce intermediary information $Z(x_i)$ to explain the evaluation process from x_i to $U(Z(x_i))$. In context-dependent preferences, $Z(x_i)$ are extended to $Z(x_i, X)$ allow the criterion specific scores of x_i to depend on all the alternatives in X. Step 5: Normalization of the x_{ij} values is not required in the multiplicative MCDMA method. In this study, different data are used to analyze the effects of the L_2 norm data normalization technique on the multiplicative MCDMA model, additive MCDMA, logarithmic MCDMA, and regrettive MCDMA model . The decision matrix, $X = (x_{ij})_{mxn}$, is normalized. In the normalized decision matrix, the various criteria dimensions are converted into nondimensional criteria $x_{ij} \in [0,1]$.

Data normalization technique, L_2 norm is employed to normalize nocommensurate data in the decision matrix.

 L_2 norm data normalization technique:

Benefit criteria

$$L_{2} \operatorname{norm} || x_{ij} ||_{2} = |x_{ij}| / \left[\sum_{j=i}^{J} |x_{ij}|^{2} \right]^{1/2}$$
(9)

Cost criteria

$$L_{2} \operatorname{norm} || x_{ij} ||_{2} = 1 - \left| | x_{ij} | / \left[\sum_{j=i}^{J} | x_{ij} |^{2} \right]^{1/2} \right|$$
(10)

Step 6: Determine the weight ω_j of the criteria. Mean weight distributes weights equally to a set of decision criteria. It assumes that all criteria are of equal importance. It is used in MCDMA model when there is no information from the decision maker or there is not enough information to reach to a decision.

$$\omega_j = \frac{1}{J} \tag{11}$$

$$\begin{cases} \omega_{j} > 0 \\ \sum_{l=1}^{J} \omega_{j} = 1 \\ j = 1, 2, ..., J \end{cases}$$
(12)

where ω_j is objective criteria weight, and *J* is the number criteria. ω_j is positive for the beneficial criteria, and is negative for the cost criteria.

Step 7: Multiplicative MCDMA method uses multiplication to connect the decision criteria rating, where the rating of each criterion must first be paired with the weight of the decision criteria in question. Indeed, this process is the same as the data normalization process. The relation preferences of each alternative are given as follows:

$$R(Z(x_i)) = \frac{\prod_{j=1}^{J} z_j(x_i)^{\omega_j}}{\prod_{j=1}^{J} z_j(x_i^*)^{\omega_j}}$$
(13)

where $R(Z(x_i))$ is the preference degree, and the values $R(Z(x_i))$ are in the interval [0, 1]. Select $R(Z(x_i^*))$ such that the preference degree value of *i*th alternative is the highest. Step 8: Rank the alternatives according to their evaluation results $R(Z(x_i))$ in decreasing order. The larger the $R(Z(x_i))$, the better the alternative.

Step 9: Ranking consistency under the multiplicative MCDMA. The ranking irregularities do not occur when the multiplicative MCDMA procedure is used in decision making analysis. Theoretically to see this property consider any three alternatives, namely, x_1 , x_2 and x_3 , and let this ranking problem have *n* decision criteria. Next, suppose that alternative x_1 is more preferred than alternative x_2 , that is, $x_1 > x_2$. Then, the following relation must be true:

$$R_{i}((Z(x_{1})) / (Z(x_{2}))) = \prod_{j=1}^{J} \left(\frac{z_{j}(x_{1})}{z_{j}(x_{2})} \right)^{\omega_{j}} > 1$$
$$\prod_{j=1}^{J} z_{j}(x_{1})^{\omega_{j}} > \prod_{j=1}^{J} z_{j}(x_{2})^{\omega_{j}}$$
(14)

Similarly with above, now suppose that alternative x_2 is more preferred than alternative x_3 , that is, $x_2 > x_3$. Then, the following relation must be true:

$$R_{i}((Z(x_{2})) / (Z(x_{3}))) = \prod_{j=1}^{J} \left(\frac{z_{j}(x_{2})}{z_{j}(x_{3})} \right)^{\omega_{j}} > 1$$
$$\prod_{j=1}^{J} z_{j}(x_{2})^{\omega_{j}} > \prod_{j=1}^{J} z_{j}(x_{3})^{\omega_{j}}$$
(15)

The relations (10) and (11), when they are combined, yield:

$$\prod_{j=1}^{J} z_{j}(x_{1})^{\omega_{j}} > \prod_{j=1}^{J} z_{j}(x_{3})^{\omega_{j}}$$

$$\prod_{j=1}^{J} \left(\frac{z_{j}(x_{1})}{z_{j}(x_{3})} \right)^{\omega_{j}} > 1$$

$$R_{i}((Z(x_{1})) / (Z(x_{3}))) > 1$$

$$x_{1} > x_{3}$$
(16)

The above ranking consistency analysis demonstrates that if

 $x_1 > x_2$, and $x_2 > x_3$, then under the proposed multiplicative model, one always gets $x_1 > x_3$. That is, the transitivity property holds.

The above ranking consistency proof can easily be generalized to demonstrate that the proposed multiplicative MCDMA can never yield a ranking abnormality.

Step 10:The Spearman's correlation coefficient (ρ) is calculated by

$$\rho = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2 - 1)}$$
(17)

where $d_i = r_{1i} - r_{2i}$ is difference in paired ranks, r_{1i} is the rank of *i* in the first set of data, r_{2i} is the rank of *i* in the second set of data, and *n* is number of pair of observations.

III. APPLICATION

The proposed multiplicative MCDMA method is employed to a real-life problem in the Air Force. The specific aim is to analyze and evaluate possible alternative military fighter aircraft solutions, and aid the decision makers in terms of the Air Force fleet planning conditions and requirements.

Modern military weapons equipment can improve the defense capabilities and competencies of nations. Therefore, selecting the most appropriate military weapon, and military fighter aircraft, in particular, is vital for the Air Force fleet planning. However, selecting the most appropriate alternative among multiple alternatives is a hard challenge for the Air Force fleet planning. Thus, the main characteristic features and decision criteria of candidate fighter aircraft were identified for the military aircraft selection problem.

The problem of alternative military fighter aircraft selection was then addressed, and the initial decision making process, shortlisted nine military fighter aircraft candidates suitable for the needs considered, F-16 (a_1), Mig-35 (a_2), Su-35 (a_3), Rafale (a_4), Eurofighter (a_5), Gripen (a_6), Su-57 (a_7), F-35(a_8), and Chengdu J-10 (a_9), after determining the decision criteria.

A. Military fighter aircraft decision criteria definition

A set of ten main decision criteria are identified from the literature review and their definitions are given in Table 1: aircraft price (g_1) in million \$, the maximum takeoff weight (MTOW) (g_2) in kg, maximum payload (g_3) in kg, maximum speed (g_4) in Mach number (M), combat range (g_5) in km, ferry range (g_6) in km, service ceiling (g_7) in km, avionics (g_8) , beyond-visual-range (BVR) (g_9) , maneuverability (g_{10}) .

B. Calculation of the weights of main criteria using mean weight method

In the proposed model, the objective decision criteria were calculated by equations (11) and (12).

$$\omega_j = \frac{1}{J} = \frac{1}{10} = 0,1$$

where ω_j is objective criteria weight, and J is the number criteria.

Table 1. Military Fighter Aircraft Decision Criteria Definition

Decision Criteria	Definition
Aircraft price (g_1)	Aircraft price (million \$) is the total amount Buyer is to pay for the Aircraft at the time of delivery.
Maximum takeoff	Maximum takeoff weight (MTOW) (kg) is
weight (MTOW) (g_2)	the maximum weight allowed to attempt to take off, due to structural or other limits.
Maximum payload	Maximum payload capacity (kg) means the
(<i>g</i> ₃)	maximum certificated takeoff weight of an aircraft less the empty weight.
Maximum speed (g_4)	Maximum speed is the maximum operating speed of aircraft in Mach number.
Combat range (g_5)	Combat range (km) refers to the maximum distance an aircraft can travel away from its
	base along a given course with normal load and return without refueling. Combat range
Ferry range (g_6)	is always smaller than maximum range. Ferry range (km) means the maximum range the aircraft can fly without any passengers or cargo.
Service ceiling (g_7)	Service ceiling (km) means the maximum height at which a particular type of aircraft can sustain a specified rate of climb.
Avionics (g_8)	Avionics are the electronic systems used on aircraft.
Beyond-visual-range	Beyond-visual-range refers to radar-guided
$(BVR)(g_9)$	missile engagements without visual identification of the target.
Maneuverability (g_{10})	Maneuverability is defined as the ability to change the speed and flight direction of a military fighter aircraft.

C. The selection of military fighter aircraft using multiplicative MCDMA method

Following the identification of ten decision criteria, the selection of alternative aircraft problem was considered, and the initial decision making process determined nine suitable military fighter aircraft for the Air Force fleet planning shown in Table 2 using equation (1).

To compare the performance of the proposed multiplicative MCDMA method, the L_2 norm data normalization technique with multiplicative MCDMA, additive MCDMA, logarithmic MCDMA, and regrettive MCDMA model was also applied to the same military fighter aircraft selection problem with the same weights of the objectives and the data given in Table 2.

The normalized decision matrix, and weighted normalized decision matrices were constructed using the equations (2)-(13), and shown in Table 3 – Table 6. The rankings of the alternative solutions based on the evaluation scores given by these data normalization techniques and the proposed method are given in Table 7.

The multiplicative MCDMA aggregation of benefit and

cost criteria was calculated using equations (7) and (13) for the *i*th alternative and the final ranking results are given in Table 7.

The alternative aircraft evaluation results are illustrated in Table 7 and indicate that the Su-57 is the optimal military fighter aircraft for the Air Force. This multiplicative MCDMA assessment is based on economic constraints, and the technical characteristics, and real conditions in the Air Force.

The Su-57 is technically evaluated, and ranked first because of technical characteristics, economic and performance factors. Table 7 presents a clear view of the most suitable aircraft option derived in this study.

D. Validation analysis

Air defense purchases for the Air Force fleet planning are long-term investments of strategic importance for countries and depend on the country's geopolitical conditions and defense policy & strategy. Especially since the procurement or development of military fighter aircraft requires very large defense budget expenditures, it requires careful evaluation process and selection of appropriate aircraft. In the context of economic and geopolitical challenges associated with air defense procurement, the balance between requirements and constraints must be managed to ensure optimal selection.

In this study, the selection of military fighter aircraft for the Air Force was multidimensionally considered. Various aspects of the purchase of a military fighter aircraft, economic situation and technical characteristics were examined, and evaluated with multiple conflicting (i.e., "benefit" and "cost") decision criteria. Using multiplicative MCDMA approach for military fighter aircraft selection, the results indicated that the Su-57 aircraft was the best solution for the Air Force fleet planning. There is no rank reversal in multiplicative MCDMA model due to aggregation of cost and benefit criteria. The multiplicative aggregation has nonlinear properties that allow the selection of a superior compromise.

The final step in the decision process is sensitivity analysis, in which input data is slightly changed to observe its impact on results. Because complex decision models can be inherently unstable, they allow the creation of different scenarios that can lead to other rankings, and further discussion may be required to reach a consensus. If the ranking does not change, the results are said to be robust, otherwise it is sensitive.

The negative effects of normalization can also be eliminated by using the multiplicative MCDMA model. These properties of the multiplicative MCDMA model are theoretically and experimentally demonstrated in the military fighter aircraft selection problem.

Actual selection, sorting, and shortlisting problems often involve considering multiple criteria. Given a situation in which a selection (or ranking) must be made from a range of options or candidates, a commonly used approach is additive MCDMA model. Since there is no universal agreement on how to perform the normalization step, a change in the way data is normalized can lead to different rankings and therefore decision results [13].

A second unexpected aspect of the common scoring approach occurs when explicitly inadequate candidates are removed from a long list to create a shortlist. If the same scoring technique is reapplied to the shortlist, the rankings among the remaining candidates may be reversed, leading to a change in the final decision. In other words, when additive MCDMA model is used, the best in the shortlist may not be the best in the entire list. This problem occurs because normalizations depend on the data. Multiplication aggregate function prevents these problems [13].

Thus, it was also proved that most of the ranking irregularities which occurred when the additive MCDMA aggregation method was used do not occur with the multiplicative MCDMA method. Adding or removing weak alternatives from the decision matrix does not change the order of the best alternative [13].

A sensitivity analysis was carried out to verify the robustness of the proposed framework, and the evaluation results were compared with different data normalization techniques to indicate the impact of solutions for fighter aircraft selection on the evaluation process and ranking. The results of the sensitivity analysis are presented in Table 7. The given final ranking results indicate that all alternatives comply with multiplicative MCDMA model, the Su-57 is ranked first, has the same rank in additive MCDMA, logarithmic MCDMA, and regrettive MCDMA model. The correlation analysis indicates a significant correlation between the ranking results of applied MCDMA models shown in Fig. 1.

The impact of aggregating benefit and cost criteria using the multiplicative MDCMA model was considered under the L_2 norm data normalization technique. The main findings of this study are given as follows.

In multiplicative MCDMA model, for the decision problem of selecting the best alternative or ranking the alternatives when conflicting (i.e., "benefit" and "cost") criteria are present, it could make a difference which data normalization technique is used. In particular, if the L_2 norm data normalization technique is used, then the multiplicative MDCMA criteria aggregation method may yield slightly different ranking results.

The contradiction rates among rankings are more dramatic for problems with many alternatives, but a few criteria in MCDMA methods. The multiplicative MCDMA, additive MCDMA, logarithmic MCDMA, and regrettive MCDMA model performed the same way for the determination of the best alternative.

Also, there is no way to know which is the "right" ranking and which is not, under different criteria aggregation models. The multiplicative MDCMA model is immune to any ranking reversals as it always yields identical results for the selection of the best alternative. It is proved theoretically that multiplicative MDCMA criteria aggregation method is perfectly consistent in terms of the ranking tests performed. This result reinforces the merit of using the multiplicative MDCMA model in decision problems. This study further promotes the use of the multiplicative MDCMA model versus the additive MCDMA, logarithmic MCDMA, and regrettive MCDMA models in decision problems of selecting the best alternative or ranking the alternatives when conflicting (i.e., "benefit" and "cost") criteria are present. The findings of this study reinforce the opinion that the results of MCDMA methods should not be taken literally, but should be dealt with as decision support systems. Clearly, more research in this area is required to validate the MCDMA model decision results.

IV. CONCLUSION

The selection of military fighter aircraft is a very important and complex decision making process should be taken into account for optimal decision solutions. The selection of available alternatives and decision criteria is the starting point for studies focusing on military fighter aircraft fleet modeling. Available aircraft alternatives are those that better respond to the needs of the Air Force. The decision criteria for evaluating aircraft type alternatives are determined by literature review and are mainly based on operational, strategic, economic, technical and environmental factors. The method of evaluating military fighter aircraft alternatives based on specific decision criteria is important in decision making procedure. Since both contain an effective and efficient methodology, it allows the monitoring of the best alternatives using decision criteria that can be easily evaluated with a multiplicative MCDMA method, simple to understand and apply. According to the research findings, the uniform military fighter aircraft fleet structure is best suited for the optimum Air Force fleet to minimize strategic, tactical and operational risks. Although the current study examines a military fighter aircraft selection problem with certain characteristics, this study makes an important contribution to the optimization of the military fighter aircraft fleet selection process.

In this study, a multiplication MCDMA method was used to determine the best military fighter aircraft among a number of alternatives for the Air Force. The multiplicative MCDMA method has advantages over other techniques, since it uses a multiplicative aggregation of the benefit and cost criteria, and the result is more consistent.

Ten decision criteria were identified to evaluate aircraft, and mean weight method was used in the decision making process to acquire the weight values of the criteria while the multiplicative MCDMA method was used to derive the final ranking of military fighter aircraft with respect to decision criteria.

Sensitivity analysis was performed and the evaluation results were compared with the ranking results of different data normalization techniques to verify the robustness of the proposed method. The proposed method yielded reliable results. As a result of the decision making process, the Su-57 was selected the best suitable solution, followed by the Su-35, and the Mig-35 ranked third in the military fighter aircraft selection problem.

Therefore, the Su-57 can be considered a suitable military

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fighter aircraft, as it meets technical requirements, economic constraints and strategic real life conditions. The main contribution of multiplicative MCDMA model can be considered as a reference for future decision making analysis studies on determining the efficiency of the military fighter aircraft selection problem. literally, but should be dealt with as decision support systems when dealing with decision problems using the MCDMA methods.

Finally, the findings of this study should not be taken

	Decision Criteria										
Optimization		min	max	max	max	max	max	max	max	max	max
Alternatives		g_1	g_2	g_3	g_4	g_5	${g_6}$	$g_{_7}$	g_{8}	g_9	$g_{\scriptscriptstyle 10}$
F16	a_1	64000000	21772	7800	2,05	546	4217	15240	9	78	7,9
MiG 35	a_2	3000000	24500	6600	2,25	1000	3100	16000	8,3	83	9,6
Su 35	a_3	83000000	34500	8000	2,25	1600	4500	18000	8,3	87	9,9
Rafale	a_4	115000000	24500	9525	1,8	1850	3700	15240	8,4	90	9,3
Eurofighter	a_5	124000000	23500	6500	2	1389	3790	19812	8,6	90	9,5
Gripen	a_6	85000000	16500	5300	2	1500	4000	16000	7,9	81	9
Su 57	a_7	4000000	35000	10000	2,3	1500	3500	20000	9,3	97	10
F 35	a_8	78000000	31751	8160	1,6	1093	2200	15000	9	94	8,5
Chengdu J 10	a_9	40000000	20500	4500	1,8	900	3200	17000	8	75	8,6

Table 2. Decision Matrix for Aircraft Selection Problem

Table 3. L_2 Normalized Decision Matrix

	Decision Criteria												
	min	nin max max max max max max max											
	g_1	g_2	g_3	g_4	g_5	${g}_6$	g_{7}	g_{8}	g_9	$g_{\scriptscriptstyle 10}$			
a_1	0,7319	0,2734	0,3433	0,3386	0,1377	0,3865	0,2985	0,3511	0,3010	0,2872			
a_2	0,8743	0,3076	0,2905	0,3716	0,2523	0,2841	0,3134	0,3238	0,3202	0,3490			
a_3	0,6523	0,4332	0,3521	0,3716	0,4036	0,4125	0,3525	0,3238	0,3357	0,3599			
a_4	0,5183	0,3076	0,4192	0,2973	0,4667	0,3391	0,2985	0,3277	0,3473	0,3381			
a_5	0,4806	0,2951	0,2861	0,3303	0,3504	0,3474	0,3880	0,3355	0,3473	0,3454			
a_6	0,6440	0,2072	0,2333	0,3303	0,3784	0,3666	0,3134	0,3082	0,3125	0,3272			
a_7	0,8325	0,4395	0,4401	0,3799	0,3784	0,3208	0,3917	0,3628	0,3743	0,3636			
a_8	0,6733	0,3987	0,3591	0,2643	0,2757	0,2017	0,2938	0,3511	0,3627	0,3090			
a_9	0,8325	0,2574	0,1980	0,2973	0,2270	0,2933	0,3329	0,3121	0,2894	0,3127			

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Table 4. L_2	Weighted	Normalized	Decision	Matrix -	- Additive	MCDMA
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	Decision Criteria												
	min	max max max max max max max											
	g_1	g_2	g_{3}	g_4	g_5	${m g}_6$	g_{7}	${g_8}$	g_9	g_{10}			
a_1	0,0732	0,0273	0,0343	0,0339	0,0138	0,0387	0,0298	0,0351	0,0301	0,0287			
a_2	0,0874	0,0308	0,0290	0,0372	0,0252	0,0284	0,0313	0,0324	0,0320	0,0349			
a_3	0,0652	0,0433	0,0352	0,0372	0,0404	0,0412	0,0353	0,0324	0,0336	0,0360			
a_4	0,0518	0,0308	0,0419	0,0297	0,0467	0,0339	0,0298	0,0328	0,0347	0,0338			
a_5	0,0481	0,0295	0,0286	0,0330	0,0350	0,0347	0,0388	0,0335	0,0347	0,0345			
a_6	0,0644	0,0207	0,0233	0,0330	0,0378	0,0367	0,0313	0,0308	0,0313	0,0327			
a_7	0,0832	0,0439	0,0440	0,0380	0,0378	0,0321	0,0392	0,0363	0,0374	0,0364			
a_8	0,0673	0,0399	0,0359	0,0264	0,0276	0,0202	0,0294	0,0351	0,0363	0,0309			
a_9	0,0832	0,0257	0,0198	0,0297	0,0227	0,0293	0,0333	0,0312	0,0289	0,0313			

Table 5. L_2 Weighted Normalized Decision Matrix – Logarithmic MCDMA

	Decision Criteria												
	min	in max max max max max max max m											
	g_1	g_2	g_{3}	g_4	g_5	${m g}_6$	g_7	g_{8}	g_9	g_{10}			
a_1	0,0136	0,0563	0,0464	0,0470	0,0861	0,0413	0,0525	0,0455	0,0522	0,0542			
a_2	0,0058	0,0512	0,0537	0,0430	0,0598	0,0546	0,0504	0,0490	0,0495	0,0457			
a_3	0,0186	0,0363	0,0453	0,0430	0,0394	0,0385	0,0453	0,0490	0,0474	0,0444			
a_4	0,0285	0,0512	0,0378	0,0527	0,0331	0,0470	0,0525	0,0485	0,0459	0,0471			
a_5	0,0318	0,0530	0,0544	0,0481	0,0455	0,0459	0,0411	0,0474	0,0459	0,0462			
a_6	0,0191	0,0684	0,0632	0,0481	0,0422	0,0436	0,0504	0,0511	0,0505	0,0485			
a_7	0,0080	0,0357	0,0356	0,0420	0,0422	0,0494	0,0407	0,0440	0,0427	0,0439			
a_8	0,0172	0,0399	0,0445	0,0578	0,0560	0,0695	0,0532	0,0455	0,0440	0,0510			
a_9	0,0080	0,0589	0,0703	0,0527	0,0644	0,0533	0,0478	0,0506	0,0539	0,0505			

Table 6. L_2 Weighted Normalized Decision Matrix – Regrettive MCDMA

	Decision Criteria											
	min	min max max max max max max max										
	g_1	${m g}_2$	g_{3}	g_4	g_5	${g}_{6}$	g_7	g_8	g_9	$g_{\scriptscriptstyle 10}$		
a_1	0,0142	0,0166	0,0097	0,0041	0,0329	0,0026	0,0093	0,0012	0,0073	0,0076		
a_2	0,0000	0,0132	0,0150	0,0008	0,0214	0,0128	0,0078	0,0039	0,0054	0,0015		
a_3	0,0222	0,0006	0,0088	0,0008	0,0063	0,0000	0,0039	0,0039	0,0039	0,0004		
a_4	0,0356	0,0132	0,0021	0,0083	0,0000	0,0073	0,0093	0,0035	0,0027	0,0025		
a_5	0,0394	0,0144	0,0154	0,0050	0,0116	0,0065	0,0004	0,0027	0,0027	0,0018		
a_6	0,0230	0,0232	0,0207	0,0050	0,0088	0,0046	0,0078	0,0055	0,0062	0,0036		
a_7	0,0042	0,0000	0,0000	0,0000	0,0088	0,0092	0,0000	0,0000	0,0000	0,0000		
a_8	0,0201	0,0041	0,0081	0,0116	0,0191	0,0211	0,0098	0,0012	0,0012	0,0055		
a_9	0,0042	0,0182	0,0242	0,0083	0,0240	0,0119	0,0059	0,0051	0,0085	0,0051		

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				Ι	Decision Crit	eria				
	M-MCI	DMA	M-MCI	DMA	A-MCI	OMA	L-MCE	MA	R-MCDMA	
	$R_o^{Z(X_i)}$	R_o^M	$R_{L2}^{Z(X_I)}$	R_{L2}^M	$R_{L2}^{Z(X_I)}$	R^{A}_{L2}	$R_{L2}^{Z(X_{l})}$	R_{L2}^L	$R_{L2}^{Z(X_1)}$	R_{L2}^{R}
a_1	31,8032	8	0,3199	8	0,3449	7	0,4950	8	0,1056	7
a_2	36,3051	3	0,3446	5	0,3687	3	0,4627	5	0,0818	3
a_3	38,3782	2	0,3916	2	0,3997	2	0,4071	2	0,0508	2
a_4	34,8977	4	0,3596	3	0,3660	4	0,4442	3	0,0845	4
a_5	33,7002	5	0,3472	4	0,3506	5	0,4594	4	0,0999	5
a_6	32,0332	7	0,3272	7	0,3421	8	0,4851	7	0,1084	8
a_7	42,4637	1	0,4128	1	0,4283	1	0,3843	1	0,0222	1
a_8	32,6542	6	0,3322	6	0,3489	6	0,4786	6	0,1016	6
a_9	31,7736	9	0,3089	9	0,3353	9	0,5102	9	0,1153	9

Table 7. Ranking of Military Fighter Aircraft Alternatives

Table 8. Correlation Analysis for the Pearson Correlation Coefficient / the Spearman Rank Correlation Coefficient

	M-MCDMA	M-MCDMA-L2	A-MCDMA-L2	L-MCDMA-L2	R-MCDMA-L2
M-MCDMA	1				
M-MCDMA-L2	0,95	1			
A-MCDMA-L2	0,98	0,93	1		
L-MCDMA-L2	0,95	1	0,93	1	
R-MCDMA-L2	0,98	0,93	1	0,93	

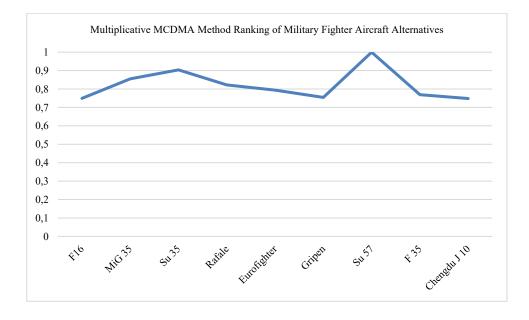


Fig. 1 Multiplicative MCDMA Method Ranking of Military Fighter Aircraft Alternatives

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