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Discrete Breeding Swarm for Cost Minimization of Parallel Job Shop Scheduling Problem

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***Abstract*—**Parallel Job Shop Scheduling Problem (JSSP) is a multi-objective and multi constrains NP-optimization problem. Traditional Artificial Intelligence techniques have been widely used; however, they could be trapped into the local minimum without reaching the optimum solution. Thus, we propose a hybrid Artificial Intelligence (AI) model with Discrete Breeding Swarm (DBS) added to traditional AI to avoid this trapping. This model is applied in the cost minimization of the Car Sequencing and Operator Allocation (CSOA) problem. The practical experiment shows that our model outperforms other techniques in cost minimization.

***Keywords*—**Parallel Job Shop Scheduling Problem, Artificial Intelligence, Discrete Breeding Swarm, Car Sequencing and Operator Allocation, cost minimization.

I. Introduction

PARALLEL JSSP has been described as a given a set of jobs containing a set of operations to be carried out by a set of operators, and the objective is to find the best operators' allocation for operations execution to minimize the cost [1]. During the last decade, the JSP problem has become one of the most challenging optimization problems [2]. It is widely acknowledged as one of the most difficult NP-complete problems and also well known for its practical applications in many manufacturing industries where the objective of this problem is to find a schedule of minimum length or cost [3].

AI has been widely used in solving JSP. In this context, Genetic Algorithm (GA) is an evolutionary process to solve optimization problems in theoretical computer science to evolve solutions to the problems of the real world [4]. Thus, GA mutation and crossover concepts have been used to schedule the jobs and assign the operators [5]. To enhance GA efficiency in solving JSP, tabu search was introduced in selection operation [6], new chromosome representation and different methods for crossover operation were introduced [7], and entropy principals and immunes was added [8].

Particle Swarm Optimization (PSO) is a population-based stochastic optimization technique where each particle in the swarm represents a potential solution of the optimization problem in the search space [9]. Since the search space of the JSP is discrete, a modified PSO was proposed where particle movement is changed based on swap operator while particle velocity is changed based on the tabu list concept [10], and also, a data mining technique was added to PSO which extracts the knowledge from the solution sets to find the near optimal solution and avoid trapping in a local minimum [11]. To enhance the PSO local search capability a Pareto approach was added [12]. Simulated annealing (SA) [13] is used to reduce the chance of getting stuck in local optima to minimize lateness [14] and to obtain best makespan [15].

Recent researches in AI have employed different hybridization techniques instead of a unique one to solve complex large-scale optimization problems like JSP [16]. To do so, a hybrid breeding model combines the standard velocity and position update rules of PSO with the ideas of selection, crossover and mutation of GA using additional parameters; the Breeding ratio determines the proportion of the population which undergoes breeding using Breeding Swarm (BS) [17]. However, this hybrid breeding model does not suit JSP which is a discrete optimization problem; so, we present DBS and apply it in the cost minimization of the CSOA problem which is a practical example of parallel JSP [18], [19].

The remainder of this paper is organized as follows: The mathematical optimization problem equations of CSOA are formulated in Section II and Section III reviews concepts of PSO, GA, and SA. Section IV introduces the proposed DBS and the experimental study is presented in Section V, and finally the conclusion and future work are presented.

II. CSOA Problem Formulation

CSOA is a real practical application of JSP and we present both its characteristic and mathematical model.

A. Problem Characteristics

The problem has the following characteristics [18], [19]:

1 Operations must be executed in a consecutive time span without breaks during execution.
2 Operation starting of each job should be after completion of the previous operation of the same job.
3 Parallel operations are not allowed in any job at any time.
4 Any operator should perform only one operation at any time.
5 No two jobs have similar operations throughout its repair time.
6 Operators are assigned to jobs based on their availability.
7 Each operator must be given scheduled time off in a day, which are not consecutive, and is unique for different operators.
8 The job starts after their arrival and no preemption is

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allowed for any job.

9 Tasks should be completed with the available time span.

If there are $n$jobs,$p$operations, and$m$operators, the number of possible schedules is $(np!)^p$ and the objective is to find the schedule among all of them that realizes the best possible cost minimization [19].

B. Mathematical Model

It is essential to consider all the above-mentioned characteristics as the constraints to be strictly followed while realizing the objective function of cost minimization of CSAO. The objective function can be written as:

$$Min \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{z_i} X_{ijk} \delta_{ijk} C_{jk} \quad (1)$$

where i Index of jobs, $i = 1, 2, \ldots, n$, j Index of operators, $j = 1, 2, \ldots, m$, k Index of operations, $k = 1, 2, \ldots, z_i$.

$$p = \sum_{i=1}^{n} z_i$$

X_{ijk} Operational time of operator j to perform operation k of job. δ_{ijk}Decision variable equals 1 if operator j is assigned to perform operation k of job i and 0 otherwise. C_{jk} Cost of operator j to perform operation k, subject to these constraints,

$$\sum_{i=1}^{n} \sum_{k=1}^{z_i} X_{ijk} \delta_{ijk} \leq T_j \ \ \forall\, k \in i \quad (2)$$

where T_j is the scheduling time duration of operator j.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ijk} \delta_{ijk} - \sum_{i=1}^{n} \sum_{l=1}^{P_j} V_j t_l^j = 0 \quad (3)$$

V_j Decision variable equals 1 if operator j is free to perform any operation of any job and 0 otherwise. t_l^j Time during which operator j is free and $l = 1, 2, .., P_j$ and P_j is the number of times during which operator j is free.

$$Ar_i \leq St_{ijk} \quad (4)$$

where $Ar_i$is the arrival time of the job i and St_{ijk} is the starting time of operator j to perform operation k of job i.

$$Ft_{ijk} = St_{ijk} + X_{ijk} \delta_{ijk} \quad (5)$$

$$St_{ijk+1} = Ft_{ijk} \quad (6)$$

where Ft_{ijk} is the starting time of operator j to perform operation k of job i.

$$X_{ijk} > 0, Ft_{ijk} > 0$$
$$St_{ijk} \geq 0, T_j \geq 0, t_j \geq 0, Ar_i \geq 0 \quad (7)$$

III. Discrete Breeding Swarm (DBS)

The proposed DBS in this study will be added to the standard BS of PSO/and GA as:

- First: According to the PSO concept [9], the construct swarm of size N particles and each particle P_i represents a solution of the optimization problem where i = 1, 2,...N
- Second: We calculate particles' new cost function $F(P_i^{new})$ for all particles, the particle cost function $F(P_{ibest})$ and the global best route $F(P_{best})$ are adjusted according to:

$$F(P_i^{new}) = \begin{Bmatrix} F(P_{ibest}) & F(C_{ibest}) \geq F(P_i^{new}) \\ F(P_{best}) & F(C_{best}) \geq F(P_i^{new}) \end{Bmatrix} \quad (8)$$

where $F(P_{best})$ = min($F(P_{ibest})$) ∀ i = 1,2,... N.

- Third: For all particles in the swarm, we apply the inverse mutation concept [4], [20] of GA by randomly selects two chromosomes in the old particle P_i^{old} Invert the chromosomes in the substring between these two chromosomes to get the new particle P_i^{new}.

Example: Consider the following old particle $P_i^{old} P_i^{old}$ = (2 3 4 5 6 1 7 9 8).

If a substring (4 5 6) within $P_i^{old}$is selected to be inversely mutated, thus the new particle will be P_i^{new} = (2 3 6 5 4 1 7 9 8).

- Fourth: Apply the DBS to the swarm particles using the following steps:

Step1. The swarm will be divided into two portions, the first is the discarded portion *(N**Ψ) containing the worst particles cost function where Ψ is the arbitrary selected breeding ratio and the other is the breeding portion which is the remaining (N*(1-Ψ)) particles [17].

Step2. Select randomly two particles from the breeding portion as parent particles $(P_{i1}^{old}, P_{i2}^{old})$ where, $i_1, i_2 = 1, 2, \ldots, \mathrm{N} * (1 - \Psi), i_1 \neq i_2$

Step3. For the parent particles $(P_{i1}^{old}, P_{i2}^{old})$, apply modified partially-mapped crossover [20] to get two modified parent particles (P_{i1}^{m}, P_{i2}^{m})in a new searching direction away from the discarded portion direction.

Step4. For new modified parent particles (P_{i1}^{m}, P_{i2}^{m}), apply displacement mutation [21] to get two new particles $(P_{i1}^{new}, P_{i2}^{new})$ to ensure the diversity while keeping the obtained reinforcement direction in the search space.

Step5. These new particles $(P_{i1}^{new}, P_{i2}^{new})$ will replace two randomly selected particles $(P_{j1}^{old}, P_{j2}^{old})$ from the discarded portion where, $j_1, j_2 = 1, 2, \ldots, \mathrm{N} * (\Psi), j_1 \neq ji_2$.

Step6. Apply SA concept [13] for acceptance of the new solution as [8]:

$$\Delta F(P_i) = F(P_i^{new}) - F\left(P_j^{old}\right) \quad (9)$$

$$\rho = \begin{Bmatrix} 1 & \Delta F \leq 0 \\ exp\left(-\frac{\Delta F}{\tau}\right) & \Delta F > 0 \end{Bmatrix} \quad (10)$$

where, ρ is the acceptance probability of the new solution and τ is the temperature control parameter which decreases during each iteration reduced according to the cooling equation as:

$$\tau_i = \alpha\tau_0 + \tau_\theta \tag{11}$$

where, α is arbitrary selected cooling coefficient in the range between 0 and 1, T_0 is the initial temperature and T_θ is the lowest temperature value.

IV. Results and Discussion

We considered that the CSOA is a real practical application of JSP [18], [19], while repair works for all cars may be distinct, each comprises of non-identical service time and cost characteristics. The operators are assumed to be paid on an hourly basis and there are no previous days pending works to be done. The parameters such as car arrival time and tasks to be executed in each car are given in Table I. The operator costs and their associated repair time are presented in Table II. Table III shows the availability time span of the operator in the repair shop and Table IV shows the operation procedures constraints.

TABLE I
CAR ARRIVAL AND REPAIR DATA

Car No.	Arrival Time	Type of Tasks						
		Brakes	Gasket	Fender	Muffler	Transmission	Oil Change	Tune Up
1	0	-	-	-	-	1	-	2
2	0	3	4	-	-	5		-
3	4	6		7	-	-	8	-
4	5	9	10	-	-	-	-	-
5	5	-	-	-	11	12	-	-

TABLE II
OPERATOR COST AND REPAIR DURATIONS

Operator	Cost/Hr	Brakes	Gasket	Fender	Muffler	Transmission	Oil Change	Tune Up
Al	4	1	2	2	1	2	1	3
Bert	1	3	3	4	2	5	2	5
Chip	2	3	3	4	2	3	2	3
Joe	3	1	2	2	1	3	1	3
Charles	2	3	3	2	1	3	1	3

TABLE III
OPERATOR AVAILABILITY

Operator	Available Time Span (hours)
Al	(4-8), (9-12)
Bert	(1-9), (11-16)
Chip	(2-7), (9-16)
Joe	(2-5), (7-12)
Charles	(2-8), (10-14)

Our experiment is conducted in three different cases; the first case is when all the constraints in Table III should be strictly met, and its associated time scheduling and cost are shown in Table V. The second case is when the availability time is changed to start from 0 to 16 while keeping the rest period for all operators unchanged, and a comparison between the results of the two cases is shown in Table VI. The third case is when cancelling the rest period, and its associated result is shown in Table VII.

TABLE IV
OPERATION PRECEDENCE CONSTRAINTS

Car No.	Precedence Order
Car 1	Fix Transmission, Tune Up
Car 2	Fix Brake, Change Gasket, Fix Transmission
Car 3	Fix Brake, Fix Fender, Oil Change
Car 4	Fix Brake, Change Gasket
Car 5	Change Muffler, Fix Transmission

Case1. The operators to job allocations for cost minimization is 3-3-2-2-5-1-5-5-4-2-5-3 leading to a minimum possible cost of $48, as shown shown in Table V, with using all the five operators.

TABLE V
OPERATORS TO JOBS ALLOCATION CASE 1 WITH A MINIMUM COST OF $48

Car No.	Tasks	Starting Time (hours)	Finishing Time (hours)	Operator	Job Costs ($)
Car 1	Fix Transmission 1	2	5	Chip	6
	Tune Up 2	9	12	Chip	6
Car 2	Fix brakes 3	1	4	Bert	3
	Change Gasket 4	4	7	Bert	3
	Fix Transmission 5	10	13	Charles	6
Car 3	Fix brakes 6	4	5	Al	4
	Fix fender 7	6	8	Charles	4
	Oil Change 8	13	14	Charles	2
Car 4	Fix brakes 9	7	8	Joe	3
	Change Gasket 10	11	14	Bert	3
Car 5	Muffler 11	5	6	Charles	2
	Fix Transmission 12	12	15	Chip	6

Case2. The best possible operators to job allocations for cost minimization is 3-3-2-2-5-4-5-2-1-2-5-3 leading to a minimum possible cost of $48 with using all the five operators. It is obvious that the proposed DBS outperforms the SA technique [19] which achieves a minimum cost of $55 in Case 1 and $54 in Case 2. Also, it is obvious that the increasing of operators' time availability did not change the minimum cost value. Moreover, among all jobs, only three jobs are differently allocated to different operators as shown in Table VI.

Case3. The best possible operators to jobs allocations is 2-5-4-2-2-4-5-5-4-2-5-5 leading to the minimum possible cost of $45 with only three operators allocated to all jobs as shown in Table VII. The proposed DBS in this study is more efficient than the SA technique [19] which allocated four operators to get the minimum possible cost of $45.

TABLE VI
DIFFERENCE IN JOBS TO OPERATORS' ALLOCATION BETWEEN CASE 1 AND CASE 2

Car No.	Task	Operator	
		Case 1	Case 2
Car 3	Fix brakes 6	1	4
Car 3	Oil Change 8	5	2
Car 4	Fix brakes 9	4	1

TABLE VII
CASE 3 AND A MINIMUM COST OF $45

Car	Tasks	Starting Time (hours)	Finishing Time (hours)	Operator	Job Costs ($)
Car 1	Fix Transmission 1	0	5	Bert	5
	Tune Up 2	6	9	Charles	6
Car 2	Fix brakes 3	0	1	Joe	3
	Change Gasket 4	5	8	Bert	3
	Fix Transmission 5	11	16	Bert	5
Car 3	Fix brakes 6	4	5	Joe	3
	Fix fender 7	9	11	Charles	6
	Oil Change 8	14	15	Charles	2
Car 4	Fix brakes 9	5	6	Joe	3
	Change Gasket 10	8	11	Bert	3
Car 5	Muffler 11	5	6	Charles	2
	Fix Transmission 12	11	14	Charles	6

IV. CONCLUSION AND FUTURE WORK

In this paper, we introduce DBS by dividing the swarm into discarded and breeding portions. It successfully achieved the cost minimization of CSOA problem due to enabling the diversity in the search space and improving the searching capability proving that it outperforms the SA technique. Also, it could be applied to other large scale practical real life JSP.

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