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Model of Optimal Centroids Approach for Multivariate Data Classification

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Abstract—Particle swarm optimization (PSO) is a population-based stochastic optimization algorithm. PSO was inspired by the natural behavior of birds and fish in migration and foraging for food. PSO is considered as a multidisciplinary optimization model that can be applied in various optimization problems. PSO's ideas are simple and easy to understand but PSO is only applied in simple model problems. We think that in order to expand the applicability of PSO in complex problems, PSO should be described more explicitly in the form of a mathematical model. In this paper, we represent PSO in a mathematical model and apply in the multivariate data classification. First, PSOs general mathematical model (MPSO) is analyzed as a universal optimization model. Then, Model of Optimal Centroids (MOC) is proposed for the multivariate data classification. Experiments were conducted on some benchmark data sets to prove the effectiveness of MOC compared with several proposed schemes.

Keywords—Analysis of optimization, artificial intelligence-based optimization, optimization for learning and data analysis, global optimization.

I. Introduction

PARTICLE Swarm Optimization [3] is one of the most popular swarm intelligence techniques that mimic the navigation mechanism of a swarm of birds or a school of fish in nature. PSO algorithm is considered as a multidisciplinary optimization model that can be applied in many different application models such as financial data analysis [4], [5], medical data analysis [6], image data analysis [7], [8], text data analysis [9], biological and environmental data analysis [10], many-feature data analysis [11], cluster analysis [12]. However, most applications of PSO algorithm are simple or the models of applications are very close to PSO algorithm. We reason that:

i) All phenomena occurring in nature can be represented by mathematical models.

ii) Applying a natural phenomenon to a particular problem is a simulation process.

iii) To simulate a natural phenomena we must know their mathematical model.

For example, to apply PSO algorithm to any problem we need to know the mathematical model of PSO algorithm. Kennedy and later works have only described the mechanisms of animal behavior, based on their natural behavior, but they have not yet modeled those behaviors mathematically. This motivates us to recommend this paper.

In this paper, we propose to re-represent PSO algorithm as a general mathematical model and a multivariate data classification model. First, PSOs general mathematical model is analyzed in detail to each specific component so that can be applied into complex application models. Then, Model of Optimal Centroids - MOC is proposed for the multivariate data classification.

The remaining part of this paper is organized as follows: In Section II, we re-present the general mathematical model of PSO algorithm. In Section III, the proposed MOC model and some constraint aspects of MOC model are analyzed and clarified. In Section IV, the application model of MOC in the multivariate data classification is proposed and analyzed for clarification. Section V shows the experimental results on some benchmark data sets to prove the effectiveness of MOC model compared with several proposed models. A conclusion of this paper is given in Section VI.

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II. The General Mathematical Model of MPSO

The use of mathematical models in the phenomena analysis of behavior has increased over the years, and they offer some advantages [13]. Mathematical models help us to see the phenomena that it reflects more easily. As such we will easily apply them to specific applications. The animal swarm of PSO is such a natural phenomenon.

PSO's general mathematical model consists of five main components: Ω: Space of the model; I: Input data; J: Objective function; S: Swarm; O: Output. We have taken the main characters of the components' names in order of their appearance to create the mathematical model combination MPSO in (1).

$$[\Sigma = \{\Omega, I, J, S, O\} \tag{1}$$

where,

A. Ω Space of MPSO

$\Omega = \{\Omega, D\}$ is the space of the problem to be applied. This component represents the versatility of PSO algorithm. For example, data mining Ω is a real space; automatic control Ω can be a complex numeric space. D is the dimensional number of Ω, and D is the dimensional number of the input data, too.

B. Input Data X of MPSO

$I = \{X, D\}$ is the input data in D dimensional space. $X = \{x_1, x_2, \dots, x_N\}, x_i \in \Omega^D, i = \overline{1, N}$.

C. Application Model of MPSO

$J = J(X)$ is the mathematical model of the applied problem (the optimal objective function). That is, the J problem uses PSO to find the optimal solution.

D. Swarm of MPSO

$S = \{P, f, \alpha\}$ is the swarm that is the basic component of MPSO.Where,

1) Particles of MPSO: $P = \{P_1, P_2, \ldots, P_M, P_{PB}, P_{GB}\}$ is the swarm of particles. The particles move in the search space, the best personal position and the best global position. M is the size of the swarm.

$P = \{A, C\}$, where

$A = \{A_1, A_2, \ldots, A_M, A_{PB}, A_{GB}\}$ is the velocity vector of the particles $P_i, i = \overline{1, M}$ and the personal best position P_{PB} and the global best position P_{GB} in the space Ω^D.

$C = \{C_1, C_2, \ldots, C_M, C_{PB}, C_{GB}\}$ is the position vector of the particles $P_i, i = \overline{1, M}$ and P_{PB} and P_{GB} in the space Ω^D.

2) The Fitness Function of MPSO: $f = f(J, P)$ is a relationship function between J and PSO. f is used to identify potential positions in the swarm to guide the swarm during movement. $f = \{f_1, f_2, \ldots, f_M, f_{PB}, f_{GB}\}$ called the fitness function of the swarm that measures the optimal index available on the particles.

3) The Rule Set of MPSO: $\alpha = \alpha(P, P_{PB} \cup P_{GB})$ is a relation function between particles $P_i, i = \overline{1, M}$ with the swarm's mechanism of movement.$\alpha = \{\alpha_1, \alpha_2, ..., \alpha_M, \alpha_{PB}, \alpha_{GB}\}$ called the set of rules that identify potential solutions P_{PB} v P_{GB}. The structure of α is decided by the fitness function where,

$\alpha_i = \alpha_i(P_i, P_{PB}) \Leftrightarrow \alpha_i(f_i, f_{PB}), i = \overline{1, M}$ is the rule that specifies the relationship between the ith particle and the P_{PB} personal optimal particles which is through their fitness function.

$\alpha_{PB} = \alpha_{PB}(P_{PB}, P_{GB}) \Leftrightarrow \alpha_{PB}(f_{PB}, f_{GB})$ is a rule that regulates the relationship between P_{PB} and P_{GB} which is also passed to their fitness function.

$\alpha_{GB} = \alpha_{GB}(P_{GB}, \beta) \Leftrightarrow \alpha_{GB}(f_{GB}, \beta)$ is the rule that specifies the relationship between the P_{GB} particle and the search stop condition.

$P_{GB} = \{C_{GB}, A_{GB}\}$, $P_{GB} \in \{P_1, P_2, \ldots, P_M\}$ is the best of the M particles in the swarm. P_{GB} corresponds to an optimal solution for the J application model and is the expected result of the PSO algorithm. P_{GB} is determined among the particles based on the optimal index available on these particles. The particle with the highest optimal index is chosen as P_{GB}.

E. The Optimal Solution of MPSO

$O = O(J, P_{GB})$ is the result set of the swarm's output. The structure of O is decided by J. The amount of O is determined by P_{GB}. Usually O is the best global position.

F. The Operation of MPSO

The swarm operates according to the following rule. Particles in the swarm move sequentially, with the same number of steps, the velocity calculated by (2) and position according to (3).

$$A_i^{(t+1)} = \omega A_i^{(t)} + c_1 r_1^{(t)}(P_{lb}^{(t)} - P_i^{(t)}) + c_2 r_2^{(t)}(P_{gb}^{(t)} - P_i^{(t)}) \quad (2)$$

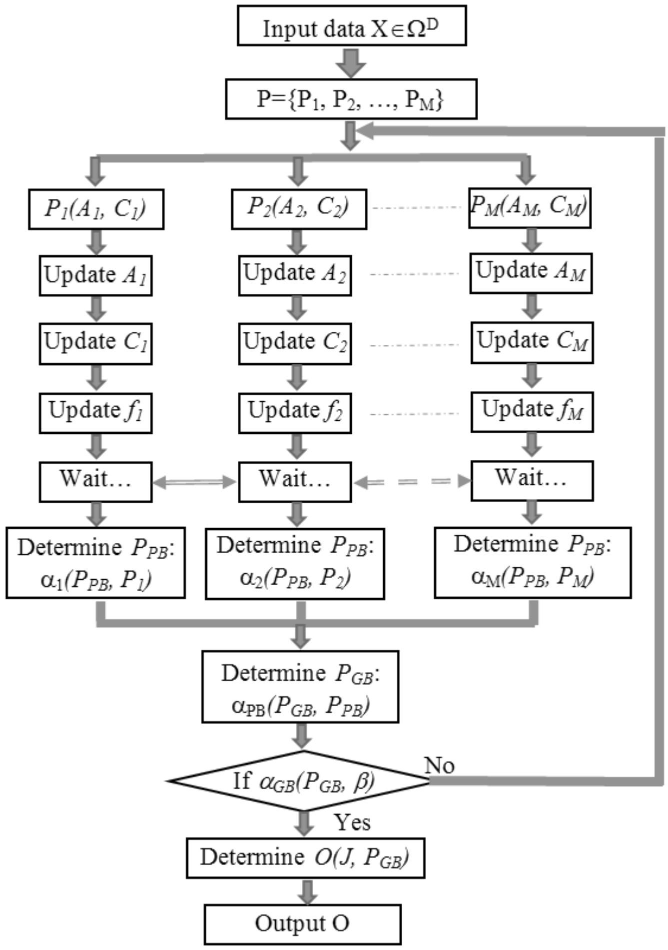

Fig. 1 The general mathematical model of Particle Swarm Optimization algorithm - MPSO

$$C_i^{(t+1)} = C_i^{(t)} + A_i^{(t)} \quad (3)$$

where, the user-defined behavioral parameter w is called the inertia weight and controls the amount of recurrence in the particles velocity. Stochastic variables r_1 and r_2 are random numbers between 0 and 1. Positive constant c_1 and c_2 are the learning factors of the stochastic acceleration terms and determine the impact of the personal best and the global best, respectively.

Completing each move, the particles calculate velocity, position and the fitness function independently. Afterwards, particles must remain in the waiting state in turn to guide the local optimum position. After all the particles have completed the guiding of local optimal position P_{lb}, the swarm continues to guide the next global optimal position P_{gb}. After the swarm has determined the global optimization location that satisfies the condition $alpha$, the swarm will end the searching. With the mechanism of operation of such swarm, we can show MPSO model in Fig. 1.

III. The Model of Optimal Centroids

PSO has been widely exploited to solve complex clustering tasks, where simpler clustering algorithms, such as K-means, Fuzzy c-means, Fuzzy co-clustering are likely to get stuck into a local optimum possibly far from a satisfactory result.

In this paper, the problem of applying MPSO model to find the initial cluster center for FCoC algorithm: Let data set $X = \{x_1, x_2, \ldots, x_N\}, x_i \in R^D, i = \overline{1,N}$ Conduct cluster X into G different clusters. The problem of "Determining the optimal initialization cluster center solution for FCoC algorithm using MPSO model" is called The Model of Optimization Centroids (MOC). The MOC model is shown in (4).

$$\Sigma_M = \{\Omega_M, I_M, J_M, S_M, O_M\} \tag{4}$$

where,

A. The Space of MOC

$\Omega_M = \{R, D\} \in R^D$ with R is the real number field. D is the dimension number of the space and is also the dimension number of the data.

B. Input Data of MOC

$I_M = \{X, D\}$ where, $X = \{x_1, x_2, \ldots, x_N\}, x_i \in R^D, i = \overline{1,N}$ is a data set in D-dimensional space.

C. The Objective Function J_M

In the MOC model, the PSO algorithm is used to find the optimal initialization cluster center solution for the FCoC algorithm. Therefore, J_M is the optimal objective function of FCoC, whose form is $J_M = J_{FCoC}(X, U, V, C)$ where,

$$\begin{aligned} J_{FCoC}(X,U,V,C) &= \sum_{k=1}^{G}\sum_{i=1}^{N}\sum_{j=1}^{D} u_{ki} v_{kj} d_{kij} \\ &+ T_u \sum_{k=1}^{G}\sum_{i=1}^{N} u_{ki} \log u_{ki} + T_v \sum_{k=1}^{G}\sum_{j=1}^{D} v_{kj} \log v_{kj} \end{aligned} \tag{5}$$

u_{ki} stands for object membership degree of the x_i to cluster with centroid c_k, $U = \{u_{ki}\}$ is the GxN object membership function matrix; v_{kj} stands for the feature membership degree defined as the membership grade of feature j to the cluster k and $V = \{v_{kj}\}$ be the feature membership matrix with size GxD.

$C_{FCoC} = \{c_1, c_2, ..., c_G\}$, $c_k \in R^D, k = \overline{1,G}$ are centroids of X which will be used by FCoC algorithm to group X into G clusters.

D. The Swarm MOC

$S_M = \{P, f, \alpha\}$ where,

1) The Particles of MOC: $P = \{P_1, P_2, \ldots, P_M, P_{PB}, P_{GB}\}$ is the particle set of the swarm. P_{PB} is the position corresponding to the personal best particle at each move. P_{GB} is the position corresponding to the global best particle of the swarm up to the present migration time. Each particle $P_i = \{A_i, C_i\}, i = \overline{1,M}$ where, $A = \{A_1, A_2, \ldots, A_M, A_{PB}, A_{GB}\}$, $A_i, A_{PB}, A_{GB} \in R^D, i = \overline{1,M}$ is the velocity vector and $C = \{C_1, C_2, \ldots, C_M, C_{PB}, C_{GB}\}$, $C_i, C_{PB}, C_{GB} \in R^D, i = \overline{1,M}$ is the position vector of the particles $P_i, i = \overline{1,M}$ and P_{PB} and P_{GB} positions in R^D space.

Each velocity vector $A_i = \{a_{i1}, a_{i2}, \ldots, a_{iG}\}$ with $a_{ij} \in R^D, i = \overline{1,M}; j = \overline{1,G}$ is the jth velocity component of the ith element. $a_{ij} = \{a_{ij1}, a_{ij2}, ..., a_{ijG}\}$ with $a_{ijk} \in R, i = \overline{1,M}; j = \overline{1,G}; k = \overline{1,D}$ is the velocity component in the direction k of the jth velocity component of the ith element, the a_{ijk} of the next move is calculated by (6).

$$\begin{aligned} &a_{ijk}^{(t+1)} = \omega a_{ijk}^{(t)} + q_1 r_1^{(t)} (c_{PB-jk}^{(t)} - c_{ijk}^{(t)}) + \\ &q_2 r_2^{(t)} (c_{GB-jk}^{(t)} - c_{ijk}^{(t)}), i = \overline{1,M}; j = \overline{1,G}; k = \overline{1,D} \end{aligned} \tag{6}$$

Each position vector $C_i = \{c_{i1}, c_{i2}, \ldots, c_{iG}\}$ with $c_{ij} \in R^D, i = \overline{1,M}; j = \overline{1,G}$ is the jth position of the ith particle. $c_{ij} = \{c_{ij1}, c_{ij2}, ..., c_{ijG}\}$ with $c_{ijk} \in R, i = \overline{1,M}; j = \overline{1,G}; k = \overline{1,D}$ is the position component of the kth dimension of the jth element of the ith element, c_{ijk} of the next move is calculated by:

$$c_{ijk}^{(t+1)} = c_{ijk}^{(t)} + a_{ijk}^{(t)}, i = \overline{1,M}; j = \overline{1,G}; k = \overline{1,D} \tag{7}$$

2) The Fitness Function of MOC: $f = f(J, P)$ is a relationship function between J and PSO algorithm. f is used to identify potential positions in the swarm to guide the swarm during movement. $f = \{f_1, f_2, \ldots, f_M, f_{PB}, f_{GB}\}$ is called the fitness function of the swarm.

Often the fitness function is tied to the optimal objective function J. For example, [14] used the PCM objective function to calculate the fitness function for image segmentation; [15] used the KFECSB objective function to calculate the fitness function for MRI brain image segmentation; [16] used the FCM objective function to calculate fitness function for medical diagnosis. In this paper, we use MOC model to find the optimal initialization centroids solution for FCoC algorithm, so the fitness function is determined by (8).

$$f = 1/J_{FCoC} \tag{8}$$

3) The Rule Set of MOC: $\alpha = \alpha(P, P_{PB} \cup P_{GB})$, $\alpha = \{\alpha_1, \alpha_2, ..., \alpha_M, \alpha_{PB}, \alpha_{GB}\}$ is called a set of rules defining P_{PB} and P_{GB} potential solutions. In the MOC model, the fitness function f uses the J_{FCoC} objective function according to (8) with J_{FCoC} as the minimum optimal objective function. That is, the format of f is an array of real numbers and the well-positioned particle has the fitness function f increasing. Therefore, the rule on f is the comparison rule on real numbers. That is:

For $\alpha_1, \alpha_2, ..., \alpha_M$: Under the MPSO model, the rules $\alpha_i, i = \overline{1,M}$ is used to determine the best personal particle P_{PB} at every move. Therefore, $\alpha_i, i = \overline{1,M}$ are rules compare the fitness function of the particles with the P_{PB}. Mean,

$$\alpha_i(P_i, P_{PB}) : If\,(f_i > f_{PB}) \quad \text{then} \quad P_{PB} = P_i, i = \overline{1,M} \tag{9}$$

For α_{PB}: According to the MPSO model, the rule α_{PB} is used to determine the best particle of the swarm at each P_{GB} move. Therefore, α_{PB} is the rule to compare the fitness function of P_{GB} with P_{GB}. Mean,

$$\alpha_{PB}(P_{PB}, P_{GB}) : If\,(f_{PB} > f_{GB}) \quad \text{then} \quad P_{GB} = P_{PB} \tag{10}$$

For α_{GB}: According to the MPSO model, the rule α_{GB} is used to determine the best particle of the swarm and finish the search process. Therefore, α_{GB} is the rule comparing the fitness function of the P_{GB} with a given stop condition β.That is,

$$\alpha_{GB}\left(P_{GB}, \beta\right) : If\left(f_{GB} \geq \beta\right) \quad \text{then} \quad \text{Stop} \quad \text{searching} \tag{11}$$

4) The Optimal Initialization Centroids Solution of MOC: $O = O\left(J_{FCoC}, P_{GB}\right)$ is a set of positions extracted from P_{GB} to serve as the optimal initialization cluster for J_{FCoC}. Therefore, O is $C_{GB} \in P_{GB}$. Mean,

$$O = C_{GB} = \{C_{GB-1}, C_{GB-2}, , C_{GB-G}\} \tag{12}$$

E. A Sample Model MOC

To better understand the MPSO model and MOC model, we represent an overview of the PSO swarm where the particles move in a 2-dimensional data clustering space in Fig. 2.

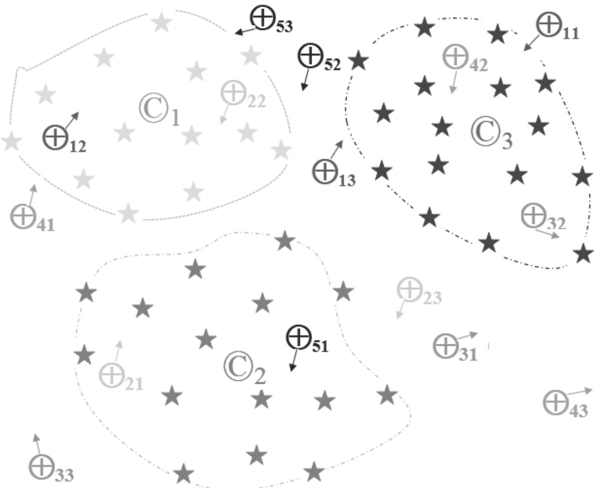

Fig. 2 The model of PSO swarm in 2-dimensional space which can be grouped into 3 classes

Data set $X = \{x_i\}, i = \overline{1, 44}$ includes 44 data objects which can be grouped into 3 classes: First class: 13 particles ★. Second layer: 15 elements ★. Third class: 16 elements ★.

There are 3 optimal centroids: $c_1{}^* = ©_1 = (©_{11}, ©_{12})$, $c_2{}^* = ©_2 = (©_{21}, ©_{22})$, $c_3{}^* = ©_3 = (©_{31}, ©_{32})$ is displayed in 2-dimensional space.

The PSO swarm is initialized five particles in random positions and directions:

The first particle: $P_1 = \{\bigoplus_{11}, \bigoplus_{12}, \bigoplus_{13}\}$.

The second particle: $P_2 = \{\bigoplus_{21}, \bigoplus_{22}, \bigoplus_{23}\}$.

The third particle: $P_3 = \{\bigoplus_{31}, \bigoplus_{32}, \bigoplus_{33}\}$.

The fourth particle: $P_4 = \{\bigoplus_{41}, \bigoplus_{42}, \bigoplus_{43}\}$.

The fifth particle: $P_5 = \{\bigoplus_{51}, \bigoplus_{52}, \bigoplus_{53}\}$.

IV. Application of MOC for Data Clustering

In this paper, we propose a data clustering model, we call MOC-FCoC algorithm. This model consists of two main learning loops corresponding to two core models. The first loop is the loop of the MOC model. MOC model is based on PSO algorithm to find the optimal initialization centroids solution. The input of MOC is X data set in D-dimensional space. The output of MOC model is the position of the best particle $P_{gb} = (C_{gb}, A_{gb})$ corresponding to the optimal initialization centroids solution $C_{MOC} = C_{gb}$. The MOC algorithm will end when the fitness function does not improved after a number of loop τ. The second loop is the loop of the FCoC algorithm which is the next after the MOC model is completed. The FCoC algorithm uses C_{MOC} as the initialization centroids which replaces the random centroids in traditional clustering algorithms. The clustering algorithm is described in Algorithm 1

Algorithm 1 MOC-FCoC algorithm

Input:
Data set $X = (x_1, x_2, ..., x_N)$, $x_i \in R^D, i = \overline{1, N}$.
Swarm of particles $\Sigma_M = \{\Omega, I, J, S, O\}$; Number of clusters G, ε, τ_{max}.
Output: Clustering results.
Initiation: Initialize $\Omega = \Omega(X) \in R^D$ and randomly swarm M particles $P = \{P_1, P_2, \ldots, P_M, P_{PB}, P_{GB}\}$, $P_i = \{C_i, A_i\}, i = \overline{1, M}$
τ=1;
While $\left(\left|f_{GB}^{(\tau)} - f_{GB}^{(\tau-1)}\right| \leq \varepsilon\right)$ or $(\tau \geq \tau_{\max})$ do
Calculate fitness function f using Eq.(8);
If $(f_i > f_{PB})$ **then**
Save local best solution: P_{PB}=P_i ($C_{PB} = C_i; A_{PB} = A_i$); f_{PB}=f_i;
End if
If $(f_{PB} > f_{GB})$ **then**
Save best solution: P_{GB}=P_{PB} by $C_{GB} = C_{PB}; A_{GB} = A_{PB}$); f_{GB}=f_{PB};
End if
Update velocity components $a_i = \{a_{i1}, a_{i2}, \ldots, a_{iG}\}$ using Eq.(6).
Update position components $c_i = \{c_{i1}, c_{i2}, \ldots, c_{iG}\}$ using Eq.(7).
End For
τ=τ+1
End While
$C_{MOC} = C_{GB}$
Initialize Centroids $C = C_{MOC}$, Membership function matrix U using X and G.
τ=1;
Do
Update centroids c_{kj};
Update feature membership function v_{kj};
Update object membership function u_{ki};
τ=τ+1
While$(max\left(\left|u_{ki}(\tau) - u_{ki}(\tau - 1)\right|\right) \leq \varepsilon \quad \text{or} \quad \tau \geq \tau_{max})$

V. Experimental Results

In this section we present results of clustering experiments to demonstrate the advantages and effectiveness of the proposed models. We ran experiments utilizing three labeled data sets from the UC Irvine Machine Learning Repository [1] (see Table I). Then, the performance of the proposed method is compared with some well known methods in literature for unsupervised clustering: the Interval Type-2 Fuzzy C-means (IT2FCM, M_L=2, M_R=3.5) [17], the Fuzzy Co-Clustering (FCCI, T_u=9, $T_v = 10^6$) [1], the Interval-Valued Fuzzy Co-Clustering (IVFCoC, $T_u = 9$, $T_v = 10^6$, $M_L = 1.5$, $M_R = 3.5$) [2] and the Interval Type-2 Fuzzy C-Means Clustering Combining Neighborhood Information (nr-IT2FCM, M_L=2, M_R=3) [18]. We use four validity indices to compare the performance of clustering algorithms: Recall index (Rec.) and Precision index (Pre.) [19], Accuracy (Acc.) and F1 score (F1) [20].

TABLE I
THREE REAL DATA SETS FROM THE UCI

Datasets	Clusters	Objects	Features
CNAE-9	9	1080	857
Spambase	2	4601	57
Landsat	7	4435	36

Clustering results are summarized in Table II. In Table II, the most of values of indexes Precision, Recall, AR and RI are obtained from MOC-FCoC algorithm which are greater than the values obtained from algorithms IT2FCM, FCoC, IVFCoC and nr-IT2FCM. That is, the classification accuracy of MOC-FCoC algorithm is better than the compared clustering algorithms.

TABLE II
CLUSTERING RESULTS ON LABELED DATA SETS USING ALGORITHMS IT2FCM, FCOC, IVFCOC, NR-IT2FCM AND MOC-FCOC

Data sets	**Algorithms**	**Pre.**	**Rec.**	**F1**	**Acc.**	τ
CNAE-9	IT2FCM	0.905	0.904	0.904	0.914	17
	FCCI	0.924	0.923	0.923	0.930	15
	IVFCoC	0.959	0.958	0.958	0.961	13
	nr-IT2FCM	0.914	0.909	0.909	0.920	12
	MOC-FCoC	**0.985**	**0.984**	**0.985**	**0.985**	**7**
Spambase	IT2FCM	0.896	0.892	0.894	0.903	19
	FCCI	0.912	0.904	0.907	0.914	15
	IVFCoC	0.953	0.951	0.952	0.953	13
	nr-IT2FCM	0.922	0.911	0.916	0.920	10
	MOC-FCoC	**0.979**	**0.979**	**0.979**	**0.980**	**8**
Landsat	IT2FCM	0.881	0.887	0.882	0.899	18
	FCCI	0.926	0.930	0.927	0.935	15
	IVFCoC	0.949	0.955	0.951	0.957	13
	nr-IT2FCM	0.922	0.935	0.928	0.940	10
	MOC-FCoC	**0.973**	**0.977**	**0.975**	**0.978**	**8**

From the results in Table II, we can also observe the number of loops τ so that the classification algorithms converge to the optimal result. The MOC-FCoC algorithm almost achieves convergence with fewer iterations than the compared algorithms.

We can learn from Table II that the MOC-FCoC algorithm has obvious advantages over the other four methods in multi-dimensional data classification. The average correct classification rate of the MOC-FCoC algorithm is higher than that of other methods. It shows that MOC model can search the optimal centroids solution for fuzzy co-clustering better than other models, and it is more capable of data classification.

[1] https://archive.ics.uci.edu/ml/index.php

VI. CONCLUSION

In this paper, we present a general mathematical model of PSO algorithm and apply in the data classification. We look forward to the attention and development to the contributions of this paper for several reasons follows: 1) General mathematical model of PSO algorithm: The PSO algorithm has a simple idea but can be applied in different fields. Therefore, the mathematical model of PSO algorithm, is an essential tool to apply PSO algorithm in complex application problems. The MPSO model is expected to fully reflect the issues of concern so that we can fully exploit the advantages of PSO algorithm. 2) Model of Optimal Centroids: MOC model is a mathematical model of the problem using MPSO to find the optimal centroids solution for fuzzy co-clustering. PSO is a versatile algorithm, PSO algorithm has recently been applied in many different fields which have different mathematical models. PSO algorithm has been applied to find the optimal centroids solution for some clustering algorithms such as K-means, FCM. MOC model is first proposed and analyzed in detail in this paper. 3) MOC-FCoC algorithm: Fuzzy co-clustering is an important unsupervised learning technique that is suitable for multi-dimensional data clustering (such as remote sensing images) compared to traditional clustering techniques. One of the main limitations of FCoC algorithm is sensitivity to initialization. Data has more dimensions, the sensitivity level is higher. Furthermore, FCoC's mathematical model is more complex than FCM and K-means. Therefore, MOC model is very meaningful to form MOC-FCoC algorithm for multi-dimensional data classification. In the future, we will continue developing and expanding MOC-FCoC algorithm to classify the multi-spectral images; Classify and detect targets in the hyper-spectral images and the medical images.

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