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Crystalline Model Approach for Studying the Nuclear Properties of Light Nuclei

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***Abstract*—**A study of the structure of the nucleus with the analogy by solid-state physics has been developed. We have used binding energy to calculate R (a parameter that is proportional to the radius of the nucleus) for deuteron, alpha, and ^{8}Be. The calculated parameter r calculated from solid state physics produces a probe for calculation the nuclear radii. ^{8}Be has special attention as it is radioactive nucleus and the latest nucleus to be calculated from crystalline model approach. The distribution of nucleons inside the nucleus is taken to be tetrahedral for ^{16}O. The model has failed to expect the radius of ^{9}Be which is an impression about the modification should be done on the model at near future. A comparison between our calculations and those from literature has been made, and a good agreement has been obtained.

***Keywords*—**The structure of the nucleus, binding energy, crystalline model approach, nuclear radii, tetrahedral for ^{16}O.

I. Introduction

THERE are two classes of models used to describe the nucleus structure. The first group deals with strong interaction of nucleons inside the nucleus and known as collective models, liquid drop model is one of these groups, nucleons have small mean free path. The second group of models is independent particle model where Pauli Exclusion Principle is applied. The mean free path is larger here, shell model belongs to this set of models. It is known today that the description of the nucleus needs two sets of models [1]. It is important to make some approaches to be close to the whole picture in nuclear physics. The regularity of the universe gives us an impression to choose another shape of the nucleus. The hypothesis for cluster and crystal states are widely discussed in [2] when the authors state that the possibility of describing the cluster states as hybrid states between quantum liquid and crystal states. The crystal model for nuclei has been discussed in details [3] and the model succeeded only for light nuclei such as deuteron, triton and alpha particles. The dependence of density on temperature is well-known, when the temperature increases the density decreases. This idea has been used for long time to shift from one state of matter to another state. The motivation of this work arises from understanding the structure of nuclei under regular structure of universe. The nucleus should has a regularity as a whole our universe.

We will focus here on alpha nucleus and discuss our various achievements on the calculation of the nuclear factors and parameters. The lattice model is nearly geometrical approach for the arrangement of nucleons inside the nucleus.

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II. Results and Discussions

The nucleons are arranged into the sides or in corners of the crystal as shown in Fig. 1. The general equation of potential between two nucleons to describe energy states has the form [4]:

$$U = (4\varepsilon)\left(\frac{a}{r^{12}} - \frac{b}{r^{7}}\right) \tag{1}$$

where a is the number of protons, b is the mass number of the nucleus and ε is an arbitrary constant. The first term of (1) represents the repulsion force where second term is the attractive force (short range force). From the binding energy of the system, we could calculate r for different nuclei. For example, the binding energy of deuteron is 2.21 MeV is used instead of U and by analytical method; (1) has been solved as:

$$\frac{1}{r^{12}} - \frac{2}{r^{7}} = 2.21 \tag{2}$$

Thus for deuterons, r = 0.8270181702 and r = -1.036306604. For alpha particles, r has been calculated using (1):

$$\frac{2}{r^{12}} - \frac{4}{r^{7}} = 28MeV \tag{3}$$

and r has the values: r = 0.7571552829 and r = -0.8443682319.

We supposed that the crystal model of nucleus (CMN) is applicable in light nuclei as deuteron, alpha, and ^{8}Be which agree with the previous result in literature [2]. ^{8}Be is unstable nuclei, so, it will separate into two alphas. The distance at which ^{8}Be will separate into two similar parts has been calculated from (1) to be larger than the value of r in case of alpha particles. The calculated value of r = 0.8697992638 at where ^{8}Be will be broken into two alphas. As seen from previous assumptions, we have found that calculated values of r for deuteron, alpha and ^{8}Be is a good parameter (probe) to expect the radius of nucleus as shown in Table I compared with calculated radii from literature [2].

For ^{8}Be, using (1) where U = Q-value, we could calculate the distance that ^{8}Be will separate into two alphas. The transformation parameter α (R1/R2) for ^{8}Be has been calculated as shown in Table I to be 2.6 which is bigger than that calculated for alpha particle by nearly the factor 1.53. ^{8}Be has special attention in our discussion as it gives us a nice state to discuss our description of the nucleus. Our results agree with above assumptions as for ^{9}Be the calculated radius is less than alpha nucleus which reflect disorder in this state.

TABLE I
CALCULATED PARAMETERS FROM CMN

Z	Nucleus	R1 from Literature [2] fm	Extracted	R2 Calculated from (CMN)	α = R1/R2 transformation parameter	a lattice spacing
1	2H	2.1421	---	0.827	1.769	*2.52fm*
2	4He	1.6755	---	0.7515	1.259	*1.970fm*
4	8Be	---	2.249	0.869	2.6	*2.646fm*
4	*9Be*	*2.519*		*0.7501*	*3.356*	*2.141fm*

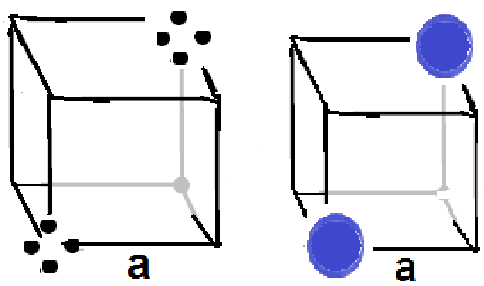
We can imagine the distribution of ^{8}Be nucleus as two alpha each one in two opposite corners of a cubic lattice with lattice parameter which is given by:

$$R = \frac{a\sqrt{3}}{2} \quad (4)$$

where R is the radius of the nucleus calculated from (1) given in Table I. The condition of validity of (4) is that $R \leq a$. The extracted radius value of ^{8}Be nucleus has been obtained from comparing radius of deuteron with ^{8}Be as given in Table I. The calculated value of a is 2.646fm from (4).

Our model (CMN) produces an approach for some states of nuclei in nuclear physics like ^{16}O. The model is nice till alpha particles (b = 4) in (1), which is supported by discussions in [4].

A lot of models have been developed to overcome the ambiguities arise inside the nucleus and everywhere there is still a hope to find out the standard model to describe the nucleus [5]-[8]. Optical model is still a nice approximation for studying nuclear reactions. However, we are trying to overcome such obstacles by introducing our model for describing the arrangement of nucleons inside the nucleus. By applying (1), we could calculate the parameter r for light nuclei A ≤ 4. This is parameter r is a perfect probe for the size of nuclei as it succeeded to introduce the relative sizes of deuteron to alpha as discussed in the first part of the text. From our calculations, the relative size of alpha to deuteron is about 0.9 which close to that from literate 0.8 [2].



(a) (b)

Fig. 1 Lattice model configurations: (a) ^{8}Be and (b) for deuteron

Transformation parameter in Table I for ^{9}Be did not succeed to interpret the radius of nucleus as ^{9}Be in ground state as two alphas and neutron in between. The value of transformation parameter is 3.3, twice the value of deuteron and nearly three times this value for alpha, which has been discussed in details in [1] and [2]. The classification of nuclei depending on atomic number (A) and transformation parameter for light nuclei are presented in Table I.

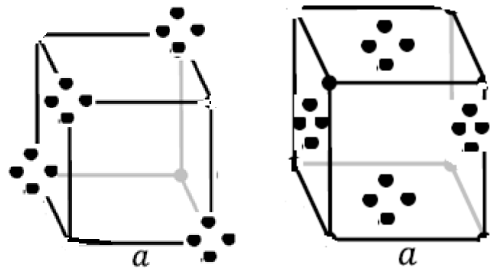
For typical lattice spacing a = 2fm, we could calculate the radius of nucleus for any nuclear system under consideration. The lattice spacing (a) for deuteron and alpha has been calculated as given in Table I. The lattice model of both ^{8}Be and deuteron which is nearly geometrical approach for the arrangement of nucleons inside the nucleus is shown in Fig. 1.

The model is inapplicable for ^{9}Be as the calculated lattice spacing is smaller than deuteron where it is not supported by literature. The importance of our approach arises from the simplicity of calculations and the dependence of radius on the binding energy of nuclei. For heavier nuclei ^{9}Be and ^{10}B cluster is more suitable for study in spite of our attempt to modify (1) to describe intermediate nuclei (A > 8). The importance of approaches like ours here arises when the spin and parity are studied for some intermediate nuclei like ^{16}O and ^{12}C. The distribution of nucleons on crystal lattice is taken as tetrahedral for the symmetry in ^{16}O [9]. *From this point of view, we have introduced an approach for distribution* of nucleons inside ^{16}O nucleus. The isotopes are the perfect choice for studying by our model as we can add more nucleons anywhere on the crystal lattice to satisfy the known properties of the nucleus under considerations. We expect to arrange alpha in ^{8}Be, ^{12}C and ^{16}O nuclei as shown in Fig. 1.

Calculations depending on lattice spacing could be used to determine the radius of nucleus and hence the binding energy for ground and excited states. Reversible process is available for studying any nuclear system from (1) where we have calculated the parameter proportional to radius of nuclei. If we have the radius of nucleus one can calculate a parameter proportional to binding energy of the nucleus or directly the binding energy from transformation parameter α.

Transformation parameter α has been calculated for deuteron, alpha and ^{8}Be. The transformation parameter establishes a relation between electrostatic force and nuclear force (sh. More modifications should be achieved on (1) to be suitable for A > 4). The transformation parameter α is a relation between nuclear force and electrostatic force in spite of the mechanism of two forces are completely different. We have produced here a transformation parameter to connect between classical view of force and quantum view.

For tetrahedral configuration, Fig. 2 (a), the nucleons are arranged in the center of the edges with co-orientation 0,1/2,0 or 1/2, 0,0 and 0,0,1/2, the nucleons have been arranged in the corners with absence in some corners which is suitable for ^{16}O. In each corner there will be an alpha particle with bond length between alphas $R = \frac{a\sqrt{3}}{2}$. The tetrahedral of ^{16}O has high symmetry parameters through the diagonal lengths. For square lattice, the lattice spacing is calculated from $R = \frac{a}{\sqrt{2}}$ which gives higher value of lattice spacing.



(a) (b)

Fig. 2 (a) Tetrahedral and (b) Square lattice configurations of ^{16}O

How to imagine the nucleus with our model for light nuclei is obvious and nearly explained from deuteron to ^{8}Be nucleus. The crystal lattice now is about twice the radius of nucleon which differs from one nucleus to the others. As shown in Table I, the last column contains the values of lattice spacing for different nuclei. An inverse calculation has been done to calculate the binding energy U as r is known. The condition r ≤ 3.5 fm , the separation distance two alphas, has been applied to calculate the binding energy of ^{8}Be nucleus:

$$U = (3.5 * 4 * 10^3)\left(\frac{4}{(3.35)^{12}} - \frac{8}{(3.35)^{7}}\right) = 52MeV \quad (5)$$

where U is binding energy of ^{8}Be.

Another approach could be applied if we use transformation parameter (α) for ^{8}Be equal 3.5 obtained on our calculations as shown in Table I. The radius of ^{8}Be on the separation moment is 3.05fm. The radius of ^{8}Be on the moment of separation into two alphas is nearly equal the radius of ^{20}Ne which contains more than the twice number of nucleons of ^{8}Be nucleus and equal ^{24}Mg which three times of nucleons number of ^{8}Be. The solution of (1) succeeded to give us the transformation parameter which is proportional to the radius of nuclei from deuteron up to ^{8}Be as shown in Table I. Also, the distribution of nucleon inside the nucleus is known and has a structure as shown in Fig. 3.

The loss of energy resulting from nuclear reaction known as Q-value will produce stretching. This stretching will change the shape of cubic and its original distribution of nucleons to hexagonal close packed (HCP). The two alphas will occupy the faces up and down of the hexagonal in special arrangement known as HCP as shown in Fig. 3. We have chosen the place of nucleons as shown in Fig. 4, where three corners have been removed and four nucleons occupy the upper face and four nucleons on lower one of HCP.

The density distribution of ^{6}Li, ^{7}Li, and ^{7}Be nuclei could be taken as a summation of two Gaussians distribution as one for the first cluster and another one for the second cluster. For example, ^{6}Li is taken as d+alpha or t+^{3}He (two configurations with the same probability) and the density distribution of ^{6}Li now is a summation of two Gaussians one for deuteron and the other for alpha. The possibility of applying our imagination of density distribution will be discussed in the near future as we will study double folding of deuteron elastically scattering by ^{9}Be using the new form of density distribution from CMN.

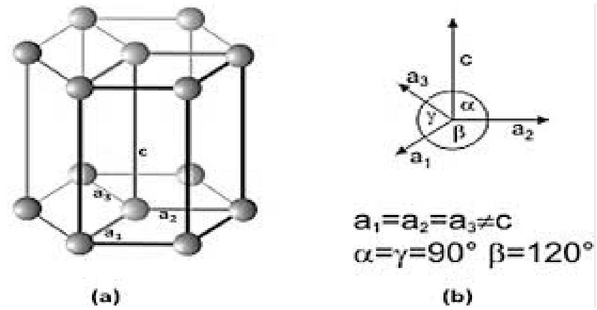


Fig. 3 HCP

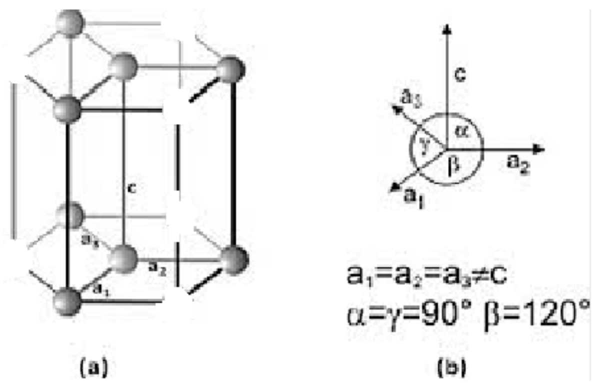


Fig. 4 An arrangement for two alphas each one on a face of HCP

An approximation has been applied on the nuclei using crystal model approach that the density distributions of ^{6}Li, ^{7}Li, and ^{7}Be nuclei are taken as a summation of two Gaussians distribution as one for the first cluster is alpha and the other part of nucleus is the residual cluster of nucleus. For example, ^{6}Li is taken as d+alpha or t+^{3}He (two configurations with the same probability) and the density distribution of ^{6}Li now is a summation of two Gaussians one for deuteron and the other for alpha.

Matter nuclear distribution for ^{6}Li, ^{6}He and ^{9}Be has been discussed in [10] theoretically in the three body model. Binding energy and nuclear density distribution have been calculated for light nuclei depending on alpha as a core of cluster. The authors in [10] choose ^{6}Li, ^{6}He as (α+n+n) and ^{9}Be as α+α+n where in case of ^{6}Li it was an approximation to take neutron instead of proton in the configuration. The authors [11] have summarized the form of nuclear density distribution of light nuclei in the form of summation of two Gaussians.

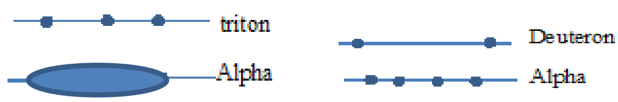


Fig. 5 Cluster model distribution for ^{7}Li and ^{6}Li

The density distribution of $^{7}Li \equiv \alpha$+t configuration is described as in cluster structure and hence alpha will be the core of nucleus and triton as valence (see Fig. 5).

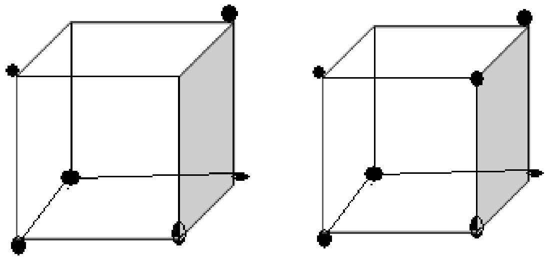


Fig. 6 Density distribution of nucleons on crystal model in case ^{6,7}Li where (a) the distribution of nucleons of ^{6}Li as alpha on the base and deuteron on the upper face, (b) ^{7}Li case with alpha on the base and triton on the upper face

In another approach from the principles of nuclear structure the overlapping between two constituents of nucleus is forbidden [10], [11]. The density distribution of many body systems may be written:

$$\rho(R) = \sum_{cluster=i,j,k} \rho_{cluster}(R) \quad (6)$$

^{6}Li density distribution, for example, will be written as:

$$\rho(R) = \rho_{\alpha}(R) + \rho_{d}(R) \quad (7)$$

Also, the calculations of alpha nucleus have been done using:

$$\rho(r) = 0.17\{(1 + e^{\frac{(r-1.97)}{0.65}})^{-1}\} \quad (8)$$

where R = 1.97fm, a = 0.65fm, and ρ_0 = 0.17 (1/fm^3).

Our imagination of ^{6}Li agrees with that from literature [12]:

$$\rho = \frac{6}{8\pi^{\frac{3}{2}}}\left(\frac{1}{a^3}\exp(-\frac{r^2}{4a^2}) - \frac{c^2(6b^2 - r^2)}{4b^7}\exp(-\frac{r^2}{4b^2})\right) \quad (9)$$

where a, b, and c are constants and a^2 = 0.78fm, b^2 = 1.7fm and c^2 = 0.205fm. Also, density distribution of ^{7}Li nucleus was taken in the form which is two summations of Gaussians [13]:

$$\rho(R) = \rho_0\left(1 + \alpha(\frac{R}{a})^2\right)\exp\left(-(\frac{R}{a})^2\right), \quad (10)$$

where for harmonic oscillator a = 1.77fm and α = 0.327fm. ^{6}Li is taken here in case of crystal model approach to be alpha on the base of the lattice and deuteron on the upper face of crystal which is shown in Fig. 6.

The calculations have been done for ^{6}Li ($^{6}Li \equiv \alpha$+d):

$$\rho(r) = 0.17\{(1 + e^{\frac{(r-2.5)}{1.2}})^{-1} + (1 + e^{\frac{(-r-2.5)}{1.2}})^{-1} - 1\} \quad (11)$$

where R = 2.5fm, a = 1.2fm (as very smooth nucleus) and ρ_0 = 0.17(1/fm^3). Also, the calculations of alpha nucleus have been done here using the relation:

$$\rho(r) = 0.17\{(1 + e^{\frac{(r-1.97)}{0.65}})^{-1}\} \quad (12)$$

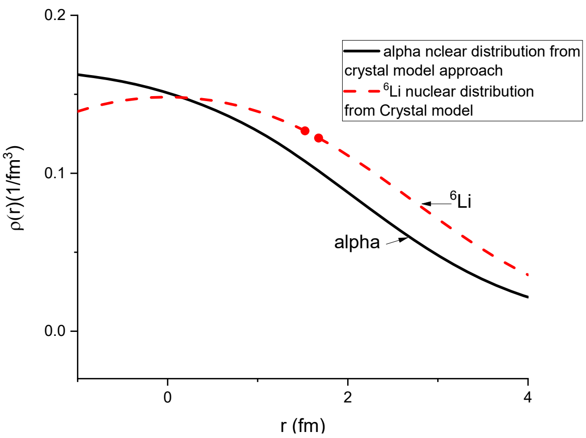


Fig. 7 Matter density distribution ^{6}Li nucleus from crystal model approach

III. Conclusion

An approach for nucleus structure has been developed by analogy of crystal lattice. A calculation for deuteron, alpha, and ^{8}Be have been done by crystalline model and a transformation parameter α has been extracted. Transformation parameter for different nuclei has been calculated. The model succeeded to compare between radius of deuteron and alpha particles. The relation used to study electrostatic force between atoms in crystal has been applied inside the nucleus. The relation between electrostatic force and nuclear force has been detected through transformation parameter. Configuration of deuteron, ^{8}Be, and ^{16}O has been expected under lattice model.

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