World Academy of Science, Engineering and Technology
International Journal of Aerospace and Mechanical Engineering
Vol:15, No:2, 2021

A Robust and Adaptive Unscented Kalman Filter for the Air Fine Alignment of the Strapdown Inertial Navigation System/GPS

Jian Shi, Baoguo Yu, Haonan Jia, Meng Liu, Ping Huang

***Abstract*—**Adapting to the flexibility of war, a large number of guided weapons launch from aircraft. Therefore, the inertial navigation system loaded in the weapon needs to undergo an alignment process in the air. This article proposes the following methods to the problem of inaccurate modeling of the system under large misalignment angles, the accuracy reduction of filtering caused by outliers, and the noise changes in GPS signals: first, considering the large misalignment errors of Strapdown Inertial Navigation System (SINS)/GPS, a more accurate model is made rather than to make a small-angle approximation, and the Unscented Kalman Filter (UKF) algorithms are used to estimate the state; then, taking into account the impact of GPS noise changes on the fine alignment algorithm, the innovation adaptive filtering algorithm is introduced to estimate the GPS's noise in real-time; at the same time, in order to improve the anti-interference ability of the air fine alignment algorithm, a robust filtering algorithm based on outlier detection is combined with the air fine alignment algorithm to improve the robustness of the algorithm. The algorithm can improve the alignment accuracy and robustness under interference conditions, which is verified by simulation.

***Keywords*—**Air alignment, fine alignment, inertial navigation system, integrated navigation system, UKF.

I. INTRODUCTION

AIR alignment is an alignment process under a moving base, which is adapt to the flexibility of modern warfare. The air alignment can be divided into coarse alignment and fine alignment. The coarse alignment algorithm is used to quickly and roughly determine the attitude matrix between the body coordinate system and the navigation coordinate system. And the fine alignment algorithm is used to further improve the accuracy of the matrix.

For the fine alignment algorithm, the accuracy of the model determines the accuracy of the moving base alignment. An accurate model is a prerequisite for correct alignment results. In order to improve the alignment accuracy, Kong et al. [1] used the psi angle to establish a nonlinear error model. This method considers the case where the three misalignment angles are large misalignment angles, and they can be directly aligned precisely. Subsequently, Kong et al. continued to study the psi angle error model, and proposed to use the quaternion for large misalignment angle modeling, but the principle of this method is the same as that of using the psi angle to establish a nonlinear error model, but only use the different filtering methods [2]. Wei et al. assume that the misalignment angle is no longer a small angle, and use the equivalent rotation vector to describe the misalignment angle between two coordinate systems, and derive a nonlinear error equation [3]. Scherzinger et al. [4]-[6] also studied the error model of SINS, and established a more accurate model.

For the nonlinear systems model mentioned above, it is necessary to select robust and accurate nonlinear filtering for state estimation. Extended Kalman Filter (EKF) is a very classic and widely used nonlinear filtering algorithm, which uses the Taylor expansion to convert the nonlinear system into a linear system. But when the nonlinearity of the system is strong, EKF will produce large errors. In order to improve the accuracy of nonlinear filtering, Julier et al. proposed the UKF algorithm. Compared with EKF, The UKF uses sigma points to sample system state variables to avoid truncation errors due to linearization. Therefore, the accuracy of UKF is higher than the EKF [7].

In the process of aligning the moving base, the maneuver of the carrier and the interference of other external environments may affect the noise distribution characteristic of SINS and other external auxiliary sensors. The adaptive filtering algorithm is currently the main method to solve such problems. Zhong et al. [8] used an adaptive filter algorithm with adjustable window to estimate the noise covariance matrix of INS for the problem of INS noise change, which improved the accuracy of air alignment. In view of the uncertainty of measurement noise, Bing et al. proposed a robust adaptive filtering algorithm based on projection statistics. This method can estimate the covariance of measurement noise when the measurement is disturbed, which improves the stability of the alignment algorithm [9]. In view of the instability of the filter caused by the uncertain system noise and the inaccurate modeling of system, Cheng et al. used the H∞ robust filtering algorithm to filter the system, which improved the robustness of the navigation system [10]. When there are outliers in GPS, Wang et al. used robust Kalman filtering based on innovation to deal with the interference of GPS outliers and improved the stability of air alignment [11].

In order to improve the precision and robustness of air fine alignment, the following methods are adopted in this paper. First, the errors of the coarse alignment are modeled more

Jian Shi was with Harbin Engineering University, Harbin, China.
Meng Liu was with Harbin Engineering University, Harbin, China (e-mail: 2872795124@qq.com).
Baoguo Yu and Haonan Jia are with State Key Laboratory of Satelite Navigation System and Equipment Technology (e-mail: jiahaonan1022@163.com)
Ping Huang is with Harbin Engineering University, Harbin, China (corresponding author, e-mail: hppmonkeyking@163.com).

accurately, instead of the approximation of small angle misalignment angles, and the nonlinear UKF filter algorithm is used for filtering estimation. Then, in order to achieve the robustness of the nonlinear filtering algorithm under the conditions of outliers and the variable noise, an innovation adaptive filtering algorithm is introduced to estimate the GPS measurement noise in real time and improve the suppression of changing noise. At the same time, the robust filtering algorithm and the air precision alignment algorithm are combined to improve the ability of suppressing GPS outliers.

II. The Improved Error Model

A. Nonlinear Attitude Error Equation

The SINS system's attitude differential equations are:

$$\dot{\boldsymbol{C}}_b^n = \boldsymbol{C}_b^n \boldsymbol{\omega}_{nb}^b \times \tag{1}$$

As $\boldsymbol{\omega}_{nb}^b = \boldsymbol{\omega}_{ib}^b - \boldsymbol{\omega}_{in}^b$, then:

$$\dot{\boldsymbol{C}}_b^n = \boldsymbol{C}_b^n \left(\boldsymbol{\omega}_{ib}^b \times\right) - \left(\boldsymbol{\omega}_{in}^n \times\right) \boldsymbol{C}_b^n \tag{2}$$

Considering the effect of error, (2) can be written as:

$$\dot{\boldsymbol{C}}_b^{n'} = \boldsymbol{C}_b^{n'} \left(\tilde{\boldsymbol{\omega}}_{ib}^b \times\right) - \left(\tilde{\boldsymbol{\omega}}_{in}^n \times\right) \boldsymbol{C}_b^{n'} \tag{3}$$

According to (2) and (3), the following results can be obtained:

$$\boldsymbol{C}_b^{n'} - \boldsymbol{C}_b^n = \left(\boldsymbol{I} - \boldsymbol{C}_{n'}^n\right) \boldsymbol{C}_b^{n'} \tag{4}$$

Differentiate the both sides of the equal sign, then the result is:

$$\begin{gathered} \boldsymbol{C}_{n'}^n \boldsymbol{C}_b^{n'} \left(\delta\boldsymbol{\omega}_{ib}^b \times\right) - \boldsymbol{C}_{n'}^n \left(\tilde{\boldsymbol{\omega}}_{in}^n \times\right) \boldsymbol{C}_b^{n'} + \dot{\boldsymbol{C}}_{n'}^n \boldsymbol{C}_b^{n'} + \\ \left(\boldsymbol{\omega}_{in}^n \times\right) \boldsymbol{C}_{n'}^n \boldsymbol{C}_b^{n'} = \boldsymbol{0} \end{gathered} \tag{5}$$

By multiplying the matrix on the right of the both sides, the result is:

$$\begin{gathered} \boldsymbol{C}_{n'}^n \boldsymbol{C}_b^{n'} \left(\delta\boldsymbol{\omega}_{ib}^b \times\right) \boldsymbol{C}_{n'}^b - \boldsymbol{C}_{n'}^n \left(\tilde{\boldsymbol{\omega}}_{in}^n \times\right) + \dot{\boldsymbol{C}}_{n'}^n + \\ \left(\boldsymbol{\omega}_{in}^n \times\right) \boldsymbol{C}_{n'}^n = \boldsymbol{0} \end{gathered} \tag{6}$$

as

$$\boldsymbol{C}_{n'}^n \left(\delta\boldsymbol{\omega}_{ib}^{n'} \times\right) = \left(\delta\boldsymbol{\omega}_{ib}^n \times\right) \boldsymbol{C}_{n'}^n \tag{7}$$

Then, (6) can be written as:

$$\left(\delta\boldsymbol{\omega}_{ib}^n \times\right) \boldsymbol{C}_{n'}^n - \boldsymbol{C}_{n'}^n \left(\tilde{\boldsymbol{\omega}}_{in}^n \times\right) + \dot{\boldsymbol{C}}_{n'}^n + \left(\boldsymbol{\omega}_{in}^n \times\right) \boldsymbol{C}_{n'}^n = \boldsymbol{0} \tag{8}$$

as

$$\dot{\boldsymbol{C}}_{n'}^n = \boldsymbol{C}_{n'}^n \left(\boldsymbol{\omega}_{nn'}^{n'} \times\right) \tag{9}$$

We substitute (9) into (8), and multiply the matrix $\boldsymbol{C}_n^{n'}$ on the right side on both sides of the equal sign:

$$\boldsymbol{C}_n^{n'} \left(\delta\boldsymbol{\omega}_{ib}^n \times\right) \boldsymbol{C}_{n'}^n - \left(\tilde{\boldsymbol{\omega}}_{in}^n \times\right) + \left(\boldsymbol{\omega}_{nn'}^{n'} \times\right) + \boldsymbol{C}_n^{n'} \left(\boldsymbol{\omega}_{in}^n \times\right) \boldsymbol{C}_{n'}^n = \boldsymbol{0} \tag{10}$$

By simplifying (10), the result is:

$$\delta\boldsymbol{\omega}_{ib}^{n'} - \tilde{\boldsymbol{\omega}}_{in}^n + \boldsymbol{\omega}_{nn'}^{n'} + \boldsymbol{\omega}_{in}^{n'} = \boldsymbol{0} \tag{11}$$

By changing the coordinates, we can get the relationship between $\boldsymbol{\omega}_{nn'}^{n'}$ and $\boldsymbol{\phi}$.

$$\boldsymbol{\omega}_{nn'}^{n'} = \begin{bmatrix} \cos\phi_y & 0 & -\sin\phi_y \cos\phi_x \\ 0 & 1 & \sin\phi_x \\ \sin\phi_y & 0 & \cos\phi_y \cos\phi_x \end{bmatrix} \dot{\boldsymbol{\phi}} \tag{12}$$

Defining the matrix $\begin{bmatrix} \cos\phi_y & 0 & -\sin\phi_y \cos\phi_x \\ 0 & 1 & \sin\phi_x \\ \sin\phi_y & 0 & \cos\phi_y \cos\phi_x \end{bmatrix}$ as $\boldsymbol{C}_\omega$, then formula can be written as:

$$\dot{\boldsymbol{\phi}} = \boldsymbol{C}_\omega^{-1} \boldsymbol{\omega}_{nn'}^{n'} \tag{13}$$

We substitute (13) into (11), the nonlinear attitude error equation can be written as:

$$\dot{\boldsymbol{\phi}} = \boldsymbol{C}_\omega^{-1} \left[\left(\boldsymbol{I} - \boldsymbol{C}_n^{n'}\right) \boldsymbol{\omega}_{in}^n + \delta\boldsymbol{\omega}_{in}^n - \boldsymbol{C}_b^{n'} \delta\boldsymbol{\omega}_{ib}^b\right] \tag{14}$$

B. Nonlinear Velocity Error Equation

The specific force equation can be written as:

$$\dot{\boldsymbol{v}}^n = \boldsymbol{C}_b^n \boldsymbol{f}^b - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \boldsymbol{v}^n + \boldsymbol{g}^n \tag{15}$$

The specific force equation with errors is written as follows:

$$\dot{\tilde{\boldsymbol{v}}}^n = \boldsymbol{C}_b^{n'} \tilde{\boldsymbol{f}}^b - \left(2\tilde{\boldsymbol{\omega}}_{ie}^n + \tilde{\boldsymbol{\omega}}_{en}^n\right) \times \tilde{\boldsymbol{v}}^n + \tilde{\boldsymbol{g}}^n \tag{16}$$

By subtracting (15) from (16), the nonlinear velocity equation can be written as:

$$\begin{gathered} \delta\dot{\boldsymbol{v}}^n = \left(\boldsymbol{C}_n^{n'} - \boldsymbol{I}\right) \boldsymbol{C}_b^n f^b + \boldsymbol{C}_b^{n'} \boldsymbol{\nabla}^b - \left(\delta\boldsymbol{\omega}_{ie}^n + \delta\boldsymbol{\omega}_{in}^n\right) \times \\ \boldsymbol{v}^n - \left(\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{in}^n\right) \times \delta\boldsymbol{v}^n + \delta\boldsymbol{g}^n \end{gathered} \tag{17}$$

C. Nonlinear Position Error Equation

Under nonlinear conditions, the position error equation is the same as which under linear conditions, and will not be derived here. The equations are written as follows:

$$\begin{cases} \delta\dot{\lambda} = \dfrac{\sec L}{R_N + h}\delta v_E + \dfrac{v_E \sec L \tan L}{R_N + h}\delta L - \dfrac{v_E \sec L}{\left(R_N + h\right)^2}\delta h \\ \delta\dot{L} = \dfrac{1}{R_M + h}\delta v_N - \dfrac{v_N}{\left(R_M + h\right)^2}\delta h \\ \delta\dot{h} = \delta v_U \end{cases} \tag{18}$$

D.System State Equation

The state vector is built as follows:

$$\boldsymbol{X} = \left[\boldsymbol{\phi}^{\mathrm{T}} \quad \left(\delta\boldsymbol{v}^n\right)^{\mathrm{T}} \quad \left(\delta\boldsymbol{p}\right)^{\mathrm{T}} \quad \left(\boldsymbol{\varepsilon}^b\right)^{\mathrm{T}} \quad \left(\boldsymbol{\nabla}^b\right)^{\mathrm{T}}\right]^{\mathrm{T}} \tag{19}$$

Among them, ϕ is the Misalignment angle, $\delta\boldsymbol{v}^n$ is the error of velocity, $\delta\boldsymbol{p}$ is the error of position, $\boldsymbol{\varepsilon}^b$ is the gyroscope's constant drift, and $\boldsymbol{\nabla}^b$ is the constant drift of accelerometer.

As $\boldsymbol{\varepsilon}^b$ and $\boldsymbol{\nabla}^b$ are constant, so the differential equations about them are as:

$$\begin{cases} \dot{\boldsymbol{\varepsilon}}^b = \boldsymbol{0} \\ \dot{\boldsymbol{\nabla}}^b = \boldsymbol{0} \end{cases} \tag{20}$$

From (14), (17), (18) and (21), we can get the nonlinear system equations as:

$$\dot{\boldsymbol{X}}(t) = f\left(\boldsymbol{X}(t)\right) + \boldsymbol{G}(t)\boldsymbol{w}(t) \tag{21}$$

where $\boldsymbol{G}(t)$ is the system noise allocation matrix and $\boldsymbol{w}(t)$ is the system noise.

III. Adaptive UKF Algorithm

During the air fine alignment, the aircraft's maneuver and electromagnetic interference will change the GPS measurement noise. Therefore, the innovation adaptive filtering algorithm is used to estimate the GPS measurement noise.

In the case where the measurement equation is linear and the measurement noise covariance matrix R is unchanged, the theoretical innovation covariance is:

$$\boldsymbol{P}_{ZZ} = E\left[\tilde{\boldsymbol{Z}}_{k/k-1}\tilde{\boldsymbol{Z}}_{k/k-1}^{\mathrm{T}}\right] = \boldsymbol{H}_k\boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^{\mathrm{T}} + \boldsymbol{R} \tag{22}$$

For the innovation covariance matrix $\boldsymbol{P}_{ZZ}$, we can use the following formula to achieve the estimation in time:

$$\hat{\boldsymbol{C}}_Z = \frac{1}{N}\sum_{i=i_0}^{k}\tilde{\boldsymbol{Z}}_{i/i-1}\tilde{\boldsymbol{Z}}_{i/i-1}^{\mathrm{T}} \tag{23}$$

where N is width of sliding window and can be set according to the actual situation, $\hat{\boldsymbol{C}}_Z$ is the estimated value of $\boldsymbol{P}_{ZZ}$.

If the estimated $\hat{\boldsymbol{C}}_Z$ is not consistent with the theoretical $\boldsymbol{P}_{ZZ}$, then the statistical characteristics of the measurement noise have changed. And the difference between $\hat{\boldsymbol{C}}_Z$ and $\boldsymbol{P}_{ZZ}$ can be used to estimate the measurement noise variance matrix R. The calculation formula of the real-time estimated value $\hat{\boldsymbol{R}}_k$ of the measurement noise variance matrix is:

$$\hat{\boldsymbol{R}}_k = \hat{\boldsymbol{C}}_Z - \boldsymbol{H}_k\boldsymbol{P}_{k/k-1}\boldsymbol{H}_k^{\mathrm{T}} \tag{24}$$

Formula (24) can realize the adaptive estimation of the measurement noise variance matrix.

IV. Algorithm to Suppress GPS Outliers

For the nonlinear system, the innovation is:

$$\tilde{\boldsymbol{Z}}_{k/k-1} = \boldsymbol{Z}_k - \boldsymbol{H}_k\hat{\boldsymbol{X}}_{k/k-1} \tag{25}$$

which can be used to detect the outliers of GPS. When the outliers exist, the value of $\boldsymbol{Z}_k$ will have a big difference from the truth which cause the value of $\tilde{\boldsymbol{Z}}_{k/k-1}$ too large. And then it will pollute the measurement update process of the filter.

In theory, the innovation sequence satisfies the normal distribution with mean 0 and variance $\boldsymbol{P}_{ZZ}$ and that is:

$$\begin{cases} E\left[\tilde{\boldsymbol{Z}}_{k/k-1}\right] = \boldsymbol{0}, \quad \forall k \\ E\left[\tilde{\boldsymbol{Z}}_{k/k-1}\left(\tilde{\boldsymbol{Z}}_{k/k-1}\right)^{\mathrm{T}}\right] = \boldsymbol{P}_{ZZ}\delta_{kj} \end{cases} \tag{26}$$

where δ_{kj} is the Kronecker function. If (26) is not valid in the filtering process, there may be abnormal values in the system. In the SINS/GPS integrated system, the method of hypothesis test is used to detect whether there are abnormal values in the system.

In the hypothesis test, assuming that the innovation sequence follows a normal distribution and that is $\tilde{\boldsymbol{Z}}_{k/k-1} \sim N\left(\boldsymbol{0}, \boldsymbol{P}_{ZZ}\right)$. As a judgment condition for GPS outlier detection, the test statistics η_k are constructed as:

$$\eta_k = M_k^2 = \tilde{\boldsymbol{Z}}_{k/k-1}^{\mathrm{T}}\boldsymbol{P}_{ZZ}^{-1}\tilde{\boldsymbol{Z}}_{k/k-1} \tag{27}$$

where $M_k = \sqrt{\tilde{\boldsymbol{Z}}_{k/k-1}^{T}\boldsymbol{P}_{ZZ}^{-1}\tilde{\boldsymbol{Z}}_{k/k-1}}$ is the value of Mahalanobis distance. If the assumption holds, the test statistics η_k will satisfy the distribution of χ^2 with N degrees of freedom. If the calculated η_k is greater than the quantile $\chi_\alpha^2(n)$ corresponding to the significant level α of the chi-square distribution, then there is an abnormal value in the GPS information, which can be described as:

$$\Pr\left\{\eta_k > \chi_\alpha^2(n)\right\} = \alpha \tag{28}$$

where $\Pr\{\bullet\}$ describes the probability of an event, and $\eta_k > \chi_\alpha^2(n)$ is a little probability event. If that happened, there must be an abnormal value in GPS information.

The chi-square test can only detect the existence of outliers. In order to prevent the filtering algorithm from being affected by outliers, we construct scale factor λ_k to enlarge the innovation covariance matrix.

$$\hat{\boldsymbol{P}}_{ZZ} = \lambda_k \boldsymbol{P}_{ZZ} \tag{29}$$

where the scale factor λ_k is constructed as:

$$\lambda_k = \begin{cases} 1 \quad , & \eta_k \le \chi_\alpha^2(n) \\ \dfrac{\eta_k}{\chi_\alpha^2(n)}, & \eta_k > \chi_\alpha^2(n) \end{cases} \tag{30}$$

Substituting the amplified $\hat{\boldsymbol{P}}_{ZZ}$ into the filter gain formula, we can get:

$$\hat{\boldsymbol{K}}_k = \boldsymbol{P}_{XZ}\hat{\boldsymbol{P}}_{ZZ}^{-1} = \frac{1}{\lambda_k}\boldsymbol{P}_{XZ}\boldsymbol{P}_{ZZ}^{-1} < \boldsymbol{K}_k \tag{31}$$

Because of the matrix inversion, the filter gain matrix $\hat{\boldsymbol{K}}_k$ obtained by (31) becomes smaller, and when updating the status, using $\hat{\boldsymbol{K}}_k$ to calculate $\hat{\boldsymbol{X}}_k$ will reduce the proportion of external measurements thereby reducing the influence of outliers and improving the robustness of the filtering algorithm.

V. Simulation and Analysis

A. Simulation Settings

The system simulation parameters are set as follows: SINS/GPS system parameter settings: the constant offset and random drift of gyroscope are 1°/h and 0.1°/h, the constant offset and random drift of accelerometer are 100 ug and 50 ug. The frequencies of the gyroscope and accelerometer are 100 Hz; the speed error of the GPS system is 0.05 m/s, the position error is 5 m, and the frequency of the GPS system is 10 Hz. The initial state of the SINS system is set as follows: the initial position of the SINS system is north latitude 45° and longitude 126°, initial attitude: heading angle 45°, pitch angle 0° and roll angle 0°, initial velocity: east velocity 0 m/s, north velocity 0 m/s and sky velocity 0 m/s. The state of the SINS system changes as follows: acceleration movement at a forward acceleration of 1 m/s^2 for 10 s, and then uniform movement for 500 s.

B. Simulation Results and Analysis

In order to verify the adaptive ability of the improved algorithm when the GPS measurement noise changes, the measurement noise within 200 s to 260 s is amplified in the GPS simulation data. Figs. 1 and 2 show the added noise of speed and position. Figs. 3 and 4 are the speed and position error graphs of the algorithm after adding abnormal noise, and Fig. 5 is the attitude error graphs of the algorithm after adding abnormal noise. The red line is the alignment effect of the improved air fine alignment algorithm, and the blue line is the UKF fine alignment algorithm without adaptive algorithm.

It can be seen Figs. 1 and 2 that when abnormal noise appears in the GPS data, the algorithm in this paper is more stable and will not fluctuate due to the change of measurement noise. And, there is no singular value phenomenon in the improved air precision alignment algorithm during the operation of the algorithm.

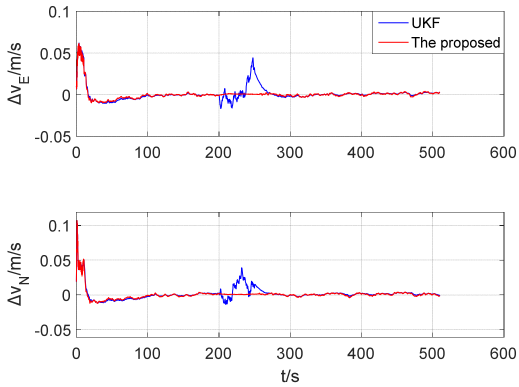

Fig. 1 Speed noise

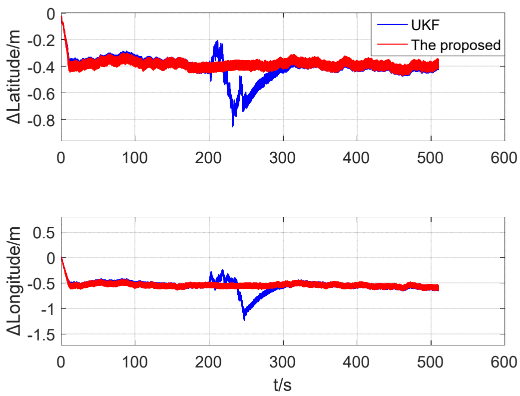

Fig. 2 Position nose

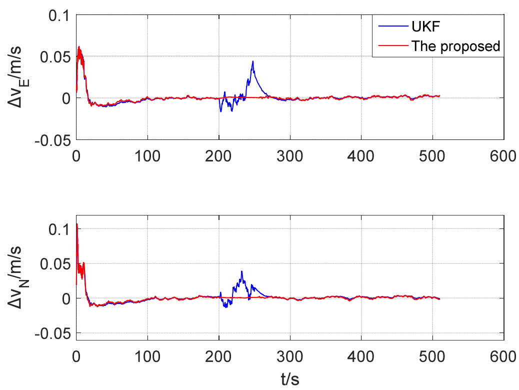

Fig. 3 Comparison of speed error

In order to verify the robustness of the air precision alignment algorithm when there are outliers in the GPS output, on the basis of the above simulation parameter settings, three outliers are added to the GPS speed and position, and then the air precision alignment is performed.

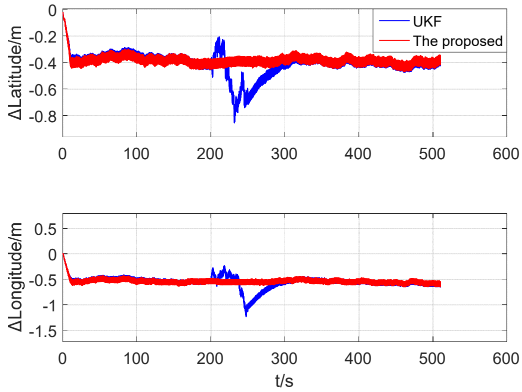

Fig. 4 Comparison of position

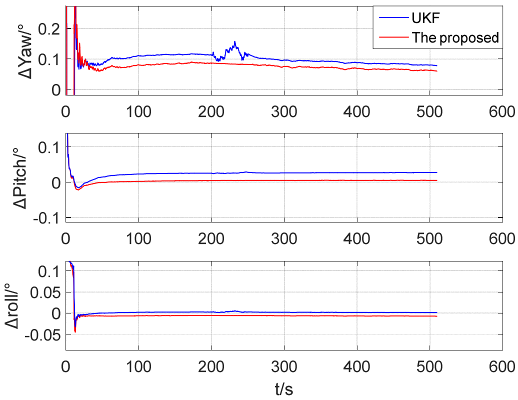

Fig. 5 Comparison of attitude

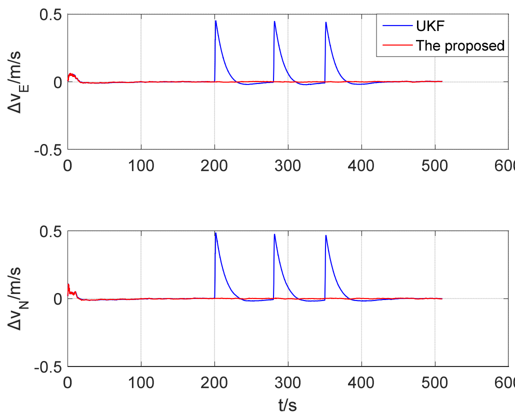

Fig. 6 Comparison of speed

Figs. 6 and 7 are the speed and position error graphs of the algorithm after adding GPS outliers, and Fig. 8 is the attitude error graphs after adding GPS outliers. It can be seen from the results that for the UKF fine alignment algorithm, no matter the speed, position or attitude, it will be affected by GPS outliers and produce large fluctuations, while the algorithm in this paper will not be affected by outliers. Even if there are outliers in the system, it can still converge normally and have good robustness.

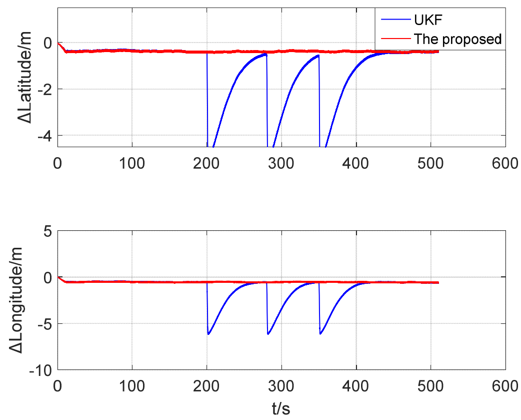

Fig. 7 Comparison of position

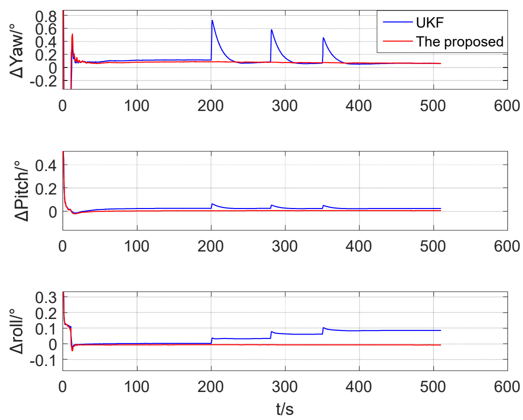

Fig. 8 Comparison of attitude

VI. Conclusion

In order to improve the accuracy and robustness of the SINS/GPS system air precision alignment algorithm, this paper proposes an improved air fine alignment algorithm. Firstly, the system error is accurately modeled to establish a nonlinear error model; then, the UKF is studied to make it adaptive to the changeable measurement noise; and at the same time, considering the influence of GPS outliers on the fine alignment results, the algorithm based on outlier detection improves the robustness of the algorithm. Through simulation experiments, the effectiveness of the proposed algorithm is verified.

Acknowledgment

Jian Shi thanks Meng Liu for his effort at the preliminary stage of the modeling of the algorithm.

References

[1] Kong XY, Nebot E M, Durrant-Whyte H. Development of a nonlinear psi-angle model for large misalignment errors and its application in INS alignment and calibration (C). IEEE International Conference on Robotics & Automation, 1999:1430-1435.

[2] Kong XY. INS algorithm using quaternion model for low cost IMU (J). Robotics and Autonomous Systems, 2004, 46(4):221-246.

[3] Wei Chunling, Zhang Shuyue, Hao Shuguang. SINS Nonlinear Alignment with Large Azimuth Misalignment Angles (J). Aerospace

Control, 2003, 21(4):25-35.
[4] Scherzinger B M. Inertial navigator error models for large heading uncertainty (C). Position Location & Navigation Symposium, 2002:477-484.
[5] Gul F, Fang JC, Gaho A A. GPS/SINS navigation data fusion using quaternion model and unscented Kalman filter (C). IEEE International Conference on Mechatronics & Automation, 2006:1854-1859.
[6] Bai M, Zhao XG, Hou ZG, et al. Application of an adaptive extended Kalman filter in SINS/GPS integrated navigation system (C). World Congress on Intelligent Control & Automation, 2008:2707-2712.
[7] Qin Yongyuan, Zhang Shuyue, Wang Chunhua, Theory of Kalman Filter and Integrated Navigation (M). Northwestern Polytechnical University Press, 2012.
[8] Zhong MY, Guo J, Zhou DH. Adaptive in-flight alignment of INS/GPS systems for aerial mapping (J). IEEE Transactions on Aerospace and Electronic Systems, 2017, 53(3):1184-1196.
[9] Zhu Bing, Xu Jiangning, Wu Miao, Robust adaptive UKF approach for underwater moving base initial alignment (J). Chinese Journal of Scientific Instrument, 2018,39(2):73-80.
[10] Cheng Jiao-jiao, Xiong Zhi, Yu Feng, Wu Xuan, Zhao Hui, Research on Algorithm of Robust Filtering in SINS/GPS/CNS Integrated Navigation System (J). Aeronautical Computing Technique, 2013(6):30-34.
[11] Wang DJ, Dong Y, Li QS, et al. Estimation of small UAV position and attitude with reliable in-flight initial alignment for MEMS inertial sensors (J). Metrology and Measurement Systems,2018,25(3):603-616.

Jian Shi obtained a Bachelor degree in 2018 from the Harbin Engineering University, Harbin, China, where he is currently working toward the Ph.D degree. His research interests conclude multisensor fusion in terms of navigation technologies such as INS/GPS/Camera/Lidar.