Philosophy, Geometry, and Purpose in Islamic and Gothic Architecture as Two Religious-Based Styles

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Abstract-Religion and divinity have always held important meaning to humans, and therefore it affects different aspects of life including art and architecture. Numerous works of art are related to religion whether supporting or denying it. Religion and religious scholars have influenced and changed art throughout history. This paper focuses on Islam and Christianity because these two religions have been the most discussed and most popular of all time, starting from the birth of Jesus to the arrival of Mohammad. Based on this popularity, these religions have influenced the arts and especially architecture. Islam on one hand changed Iranian and Arabian architecture and they applied it in different places around the world. From the appearance of Islam at 622 AD to this day, Islamic architecture has been evolving; however, one of the most important periods for this style was between 1501 AD and 1736 AD in Iran. Christianity, on the other hand, changed European architecture especially between 1150 AD and 1450 AD or the so-called "Gothic" era, which begins at medieval time and reaches its peak at International Gothic ages. At both of these periods, designing buildings based on spiritual concepts and divine statements reached its peak, and architects were considering God and religion as their center of attention. This article studies the focus on the religions of Islam and Christianity in terms of architecture and presents a general philosophy of both styles to comprehend the idea behind each one, followed by an analysis of their geometry and architectural aspects derived from the best examples, all to understand the purpose of each style and to realize, which one was more successful in reaching their purpose. Subsequently, a comprehensive review of each building is provided including 3D visualizations to help achieve the goal of the article. These studies can support diverse inquiries about both Islamic and Gothic architecture and can be used as a resource to support studies and research towards designing based on religion or for divine purposes.

Keywords-Architecture, gothic, Islamic, religion.

I. INTRODUCTION

THERE is no exact definition of what religion is, some like James Martineau (1805-1900) says that "religion is the belief in an ever-living God" but the most well-known definition of religion is as Fredrich Schleiermacher (1768-1834) defined it: "the essence of religion consists in the feeling of absolute dependence". Scholars say that what Schleiermacher meant by the word "feeling" is a method to experience reality [1].

It is no wonder that religion plays a very important role in people's lives and has considerable effects on their lifestyle even to the smallest aspects like their food, dress, houses, or places in which they tend to live [2]. Religion also drastically affects the way in which people relate to a particular place [3].

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That is why, religion is such an important matter to people from all walks of life including ordinary people, politicians, artists, architects, and so on.

Leading artists like Kandinsky and Tolstoy believe that spirituality and creativity are intrinsically linked, in as much as creativity is considered an aspect of spirituality. They saw art as a tool for emotional expression and a way for communicating, whereby the emotions are attached to the soul [4], [5].

From Plato's point of view, man has two dimensions: the body and the soul. The soul existed before entering the body in the divine world but has forgotten everything the moment it entered the body, so when a human observes nature and its beauties, he remembers the true beauty he has seen in the divine world [6]. The more the beauty of something is closer to the beauty he has seen on the divine world, the more he likes it and connects to it [7]. For example, we have different preferences in musical notes or smells, while some images are ugly in our eyes or some voices are considered unpleasing. None of these feelings are predetermined and do not belong to a specific place or time. All of these preferences and feelings are based on the things that we have seen in the divine world [6].

Architects, musicians, and mathematicians have all paid respect to that divine world [8]. This respect can be seen through the buildings all around the globe. In Chapter III and Chapter IV, usage of geometry and its relation to the divine beauty is explained.

Premised upon what mentioned above, man has tried to create artworks and buildings that simulate the beauty of the divine world [8], which in most religions is referred to as "heaven" [9], [10], and holy places are the most important targets of this goal since they are the place in which people go to connect to the other world and more generally, God [11]. In addition to that, religious places guide people to learn their identity and lead them to themselves.

Sacred structures are places of prayer, worship, meditation, or even education. Their design and aesthetics bring the person closer to religious ideas and spirituality. The architectural design and physical elements of the built structure are able to transport the one who truly believes, to a different place and a different reality and make it possible for him to experience heaven on earth. Moreover, these divine places call the believer through symbols. These symbols are not only visual but can be sensed in all five senses of a human being. The believer "observes" the scared lights, structures, and monuments, "hears" the sound of the church and bible or in Islam "Adhan and Quran", "Touches" the divine objects like the bible, the Quran, shrines or the statue of the Christ, "eats or drinks" specific foods like dates or the sacred wine and "smells" special odors like the smell of holy water, the smell of bitter orange trees in mosques. Based on that, it is pleasant to be in a place where divinity is, as a true believer believes [12].

Based on the theories and facts mentioned above, the architect's goal of building sacred places was to build heaven on earth; in the following sections of this paper, we want to see how each religion's architects tried to achieve this goal.

II. GOTHIC CHURCH: A GEOMETRICAL HEAVEN

Reviewing the history of Gothic, it can be said that original Gothic can be found in Horace Walpole's, The Castle of Otranto (1764), Ann Redcliffe's, The Mystery of Adolpho (1794), and Gregory Lewis's, The Monk (1794) [13]. Derived from these different sources, Gothic is "a genre that entertains and attracts its audiences by fear, a fear which brings them a sense of joy, they often feel ashamed to confess" [14].

The starting point for Gothic art was the 12th century in a region in France called Île de France. The first application of the Gothic style into a building was Saint-Denis cathedral under the protection of Abbot Suger the close friend of Louis VI [15]. But Gothic is not just an architectural style, it is an analytical category which has proved to be interesting and beneficial. It can be considered true that shock, anxiety, terror, and oddness have been more attractive topics both for scholars and people for a certain amount of time, rather than serenity, normalcy, or any concept that had spoken of optimism and positivism.

The other idea is that Gothic architecture was to mirror the 18th century's values and morals, some say it was a remaking of the past as the inverted, mirror image of the present. Back then, aesthetics judgments of architecture tend to use classical notions of beauty which was taking credit for simplicity, regularity, and predetermined proportions. Based on these thoughts, Gothic architecture was viewed in a negative way [14]. In the writings of Evelyn's architect and architecture in 1697, we see how vandals or so-called "Goths" at that time ruined beautiful and classic Roman and Greek buildings and introduced their own method of building that was thought as dark, disproportional, heavy and melancholy [16]. But this attitude towards Gothic architecture is not entirely true. The Gothic era came after Romanesque and a simple comparison between these two leads to this conclusion. As Kilde writes in Sacred Power, Sacred Space, "one of the most significant transformations that occurred between the early Christian basilica and these new great churches had to do with light. Whereas the high walls and sparse windows of the Romanesque and later Norman basilicas of earlier periods resulted in dark interiors, efforts in the eleventh century to bring light into the church transformed Christian architecture" [17]. The main reason that Romanesque churches were dark compared to Gothic is the rounded arch, as shown in Fig. 1.

Structurally speaking, if the sides of the arch take more distance from each other than normal, it makes the whole construction collapse and as the doors and windows of the Romanesque churches were made of the rounded arch, these arches could not provide large windows, and they became dark.

The "significant transformation" that Gothic made was the replacement of rounded arches with pointed vaults, see Fig. 1. This vault is originally an Iranian concept. To build this kind of vault, the keystone of the rounded arch will be eliminated and instead, the length of the wedge pieces will be longer so that they meet at the point where the keystone used to be. This vault that is one of the main characteristics of Gothic architecture (similar to Islamic architecture), allowed architects to provide larger windows and they filled them with a stained glass [17].

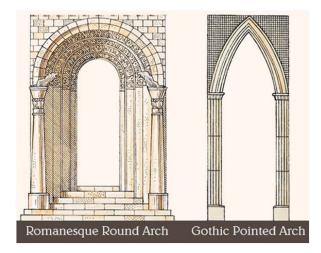


Fig. 1 Romanesque round arch and Gothic pointed arch comparison [37]

In addition to that mentioned above, pointed arches had higher resistance compared to the other kinds of vaults at that period. A study on these arches shows that they can resist a 146 km/h wind pressure which is a considerable number for the 18th century [18].

Separate from darkness, the other criticism of Gothic architecture was that, this style is not proportional in comparison to ancient Greek and Roman architecture. The Gothic style came to existence due to the desperate need of an architectural style, particularly attuned to religion and religious experience, and the designers, following the steps of their predecessors, wanted to create heaven on earth with a different attitude. They achieved this goal by using one main element, geometry [11].

A. Gothic Geometry

Gothic architects have been well aware of the symbolic significance of their projects, but the most important matter they focused on was geometry, and this was the basis of their art. In Gothic architecture, everything is based on proportion and geometry, even a keystone or width of a wall. They developed the magnitude of their building with but a single dimension given all based on strictly geometrical means, see Fig. 2 [11].

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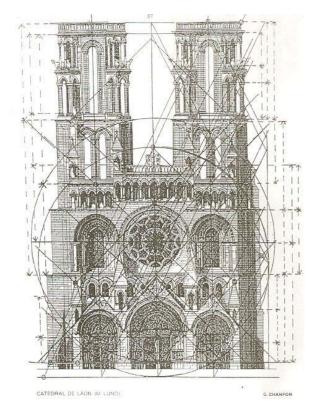


Fig. 2 Strict use of geometry in Laon cathedral in France [37]

In Dudley's writings, it is mentioned that: "It is in the light of the ancient cosmology that one needs to envisage the culture that created the great medieval churches, all of which incorporate a geometry that is purposefully created in order to provide, though it's supposed supernatural power, divine protection from the destructive powers of the earthly world and the Devil, and to attract the presence of the Almighty, creator of all the geometry in the universe" [19].

In the idea of the architects in that period, building a house of God without his geometry would be futile. As an example, the circle and the sphere are forms that are believed to belong to eternity and heavens while the square belongs to the earthy world. Based on these beliefs, in Gothic geometry there are two critical rules:

The circle has to be the beginning of all constructions, and
Maintaining symmetry is a necessity.

These beliefs have roots in Augustinian thoughts in which this universe is stable because of the perfect balance of its elements, created by the Creator. Premised upon this, the most common geometrical patterns are based on Euclidean postulates and are those which are developed from a square within a circle (*ad quadratum*) and a triangle which is again within a circle (*ad triangulum*) and in total, their geometry begins with a circle and from that, the patterns start to unfold, as shown in Figs. 3 and 4 [19].

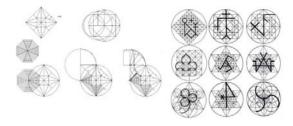


Fig. 3 Application of *ad quadratum* and *ad triangulum* in Gothic architecture [19]

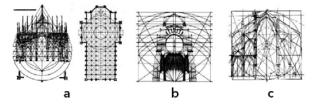


Fig. 4 *Ad quadratum* and *ad triangulum* in Gothic architecture as proportional, imaginary, or working (invisible) lines in the design of gothic churches: (a) Milan Cathedral plan and section, (b) partial section at Chartres Cathedral, and (c) Viollet-le-Duc's cross-section of Notre-Dame in Paris [19]

Architects at that time used to think that these shapes and proportions helped to protect the building, since they are the geometry of God. But Euclidian geometry was not the only method they used in Gothic. Based on the fact that these shapes are a sort of protection, they came up with the theory of "Geometrical progression". This theory exists when a series of a particular shape are continuously repeating itself like a square within a square within a square. This loop never ends and can go to infinite numbers or in their thoughts to the dome of heaven and beyond. This is the reason at that time, the theoretician believed that this geometry could protect the building from distinctive forces since on one hand it has the power of unifying the building into the general geometry of God, and on the other hand, it includes the "Divine Protection" of continuity and it is safe from demolishing forces. Today we know this geometrical progression as "Fractal Geometry" and Gothic cathedrals are one of the best architectural representation of this theory, see Fig. 5.

The next geometrical tool that genius Gothic architects used in order to bring the heaven in a mathematical way into the earth was the golden ratio or golden section, see Fig. 7. The philosophy behind this is that man is the core of God's creation and has the most perfect proportions which represent the divine harmony of being. This thought was one of the basics for designing a Gothic church. The golden ratio is a conclusion of the Fibonacci Series which was first published in a book called *Liber Abaci* in 1202 and happens when two segments of a shape – nonequal to each other – are divided into each other and the conclusion is equal to 1.61803 [19]. Since this is an approximate number, architects used the geometrical method to reach this proportion, as shown in Fig. 6.

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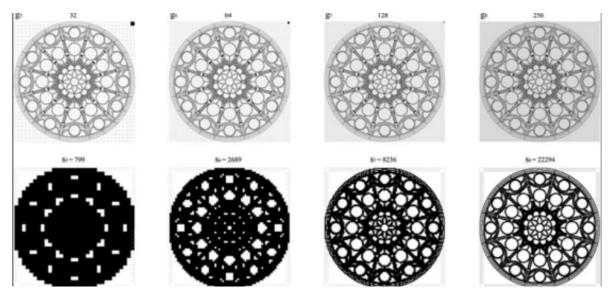


Fig. 5 Geometrical progression or Fractal geometry in a Gothic rose window design by repeating circles within the circle [15]

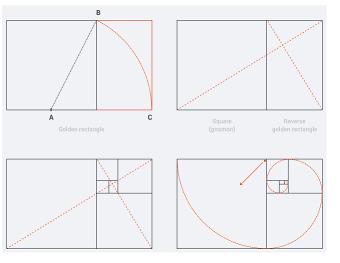
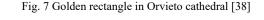


Fig. 6 Steps for drawing the golden rectangle [39]





Considering all of these geometrical laws that gothic

architects used in design, the first thing that catches the eye in a Gothic church is the exaggerated erection.

Gothic architects wanted to create buildings that instilled fear to the local gentry [17]. Extreme height is one of the most important features of gothic style and the effect of this exaggerated height is the very thing that masters of gothic took advantage from [11]. This technique was to spread fear and terror all over the place, the fear of God [13].

Christians throughout Europe used Gothic architecture for buildings in their churches. But in the east, the story was different. Saudi Arabia faced the appearance of a new religion called Islam that had its special method in design and construction.

III. ISLAMIC ARCHITECTURE: THE JOY OF PRAY

Islam is the religion that was introduced by the Prophet Muhammad in Saudi Arabia in the 7th century. The meaning of Islam in the Arabic language is "Surrender" which means to obey the one true god called Allah. Like the holy bible, Islam also has a specific religious book called Quran, which is said to be revealed to Muhamad by Allah himself [20].

In Islam, mathematics and art are praised equally together and in the Islamic idea, each matter in this world is created by calculations and art which represents God himself [7]. Based on this idea, nature was always the most important inspiration for Islamic artists and architects, since nature was the life companion of man from the very first moment of his birth and for satisfying his needs he always referred to nature as the main source; so, it is no wonder that nature is one of the most considered elements to be inspired from. Artists mimicked nature in three ways: Morphologic, Contextually, and Mimicking organisms. The most obvious elements inspired by nature in Islamic architecture are paintings and patterns, which are based on mathematical and geometrical laws [21]. These patterns can be divided into three groups [22]:

- 1. Patterns inspired by plants,
- 2. Patterns inspired by animals, and

3. Geometrical patterns or "Ties".

Animal patterns are the least used among these three, since using the face of a human or animal is not allowed in Islamic art, using ties is the most popular [23].

In the Islamic notion, patterns and painting are a way to express feelings and Islamic patterns do not have any limitation and can be expanded until infinity, this is because these patterns are a symbol of the human's inner dimension which is eternal and knows no limitation, see Fig. 8 [7], [24].



Fig. 8 Islamic geometrical pattern [40]

Mathematics and geometry have always had a huge impact in the Islamic religion since Islam praised mathematics and geometry in various texts and quotes, for instance, in Quran, the world "Ghadr" means dimension or in Imam Reza's quotes, he mentions: "Ghadr is the calculation, as the duration of life and the time of death", and therefore these patterns are based on a complicated geometry [25].

In the past, geometry was considered so sacred that only highly skilled architects and scientists had access to it and the most important duty of an architect was to know and understand the spaces and the structure of a building in three dimensions which he used to design the filled or unfilled spaces based on these facts and proportions. This usage of geometry is called structural use. Architects tend to use this knowledge for creating reasonable proportions, and to make sure all the buildings that are made follow the same true logic; they used a module [26]. The module was also used in gothic architecture.

Islamic architects used geometry in six ways:

- 1. Using geometry in plan,
- 2. Using proportions,
- 3. Using square and cube,
- 4. Using circular geometry,
- 5. Using octagons, and
- 6. Using sacred numbers.

Using Geometry in Plan

Following mathematical and geometrical laws in designing

a building plan help organize the spaces and would lead to the harmony of small details compared to the whole building. Based on the studies on different cases, using the square and rectangle was more popular in Islamic architecture, see Fig. 9.

Using Proportions

Complex but organized geometrical shapes that are used are a symbol of the creation, and not only are they appealing, they help the structure in different ways. Architects used geometrical patterns to influence the believer in a symbolic way, using the philosophy behind these patterns and to empower the sense of unity in a mosque.

As Gothic architects did not limit themselves to Euclidian geometry and used fractal theory in their design, Islamic artists also used this theory with a different meaning. In their idea, fractal geometry is a level of praying towards God [27], see Fig. 10.

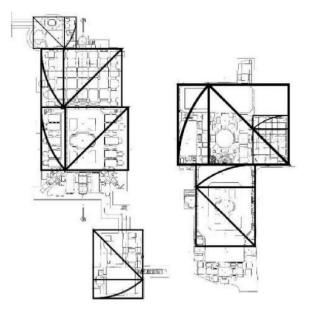


Fig. 9 Golden rectangle in the plan of Agha-Bozorg Mosque in Kashan, Iran [36]



Fig. 10 Fractal geometry in Fatima Masuma Mosque, Qom [41]

Using Square and Cube

The square is the most organized shape of all, the shape which is able to accept the most variations and is assigned to earth and earthy objects [28]. Also, in Islamic cosmology, the square is the symbol of completeness which consists of four lines and four in numerology is the number of wholeness, and the cube, which consists of six surfaces, and six is again a complete number; both show the most stable form of creation. In general, cubic shapes belong to the earth and curvy or spherical shapes belong to the skies and beyond [29].

Using Circular Geometry

As mentioned previously, circle and circular or spherical shapes are from the sky and do not belong to the earth, and despite that the cube is able to transform into different shapes, the sphere on the other side is a perfect shape and denies variation of form and keeps its character [30].

Proportions that are based on the circle are regular shapes that are tangent to circle or drawn inside of it, so it creates a perfect unity between shapes and that is why all of the building's dimensions are derived from a circle. In addition to that, the circle is the symbol of the universe, and design based on the circle would include the whole universe in symbolic terms [27].

The square is earth and the circle is the sky, so the perfect balance between them is a shape which is in the middle of these two. If we try to transform a square into a circle, in the middle of the process we reach a shape known as, an octagon.

Using Octagon

The number eight in Islamic art is the symbol of truth [31]. And also, in numerology, eight is the number of resurrection and regeneration [32]. Islamic architects tend to use numbers in geometrical forms in many of their works and eight is not an exception [31].

There are two ways to show eight in geometry: the first one is to draw a regular octagon and the second one is to rotate a square 90 degrees along with its core and draw a shape named "Shamseh", as shown in Fig. 11, which is a critical shape in Islamic art in general. This shape is used in decorations, tiling, paintings, and patterns, etc. The most important use of this number is in architecture to create an octagonal base for the dome which is called "Jariv", like in the Soltanieh Dome as shown in Fig. 12.

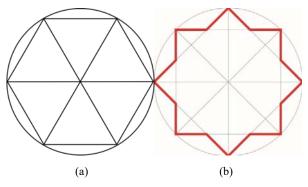


Fig. 11 (a) Regular octagon [42], (b) Shamseh [43]



Fig. 12 Placement of the dome over the regular octagonal structure "Jariv" in Soltanieh Dome, Zanjan, Iran [44]

Using Sacred Numbers

In Islamic numerology, sacred numbers through the years were one, two, three, four, five, six, seven, and eight and each number has its own meaning and philosophy. "One" is the number of perfect unity, so it is the number of God, "Two" is the number of duality or two points, and "Three" is the number of trinity or three points that create the triangle.

Triangle is an important shape in Christian or Gothic architecture as mentioned before (*ad triangulum*), but this shape is not considered so much in Islamic architecture because Christianity believes in the trinity while Islam does not.

The number "four" is the symbol of the earth and material world and wholeness which results in the square. "Five" is the symbol of birth and life, for example, plants which have eatable fruits have five petals, and the pentagon is the symbol of life [33]. "Six" in Islamic culture is the number of the days in which the universe was created based on the Quran and it is the number of nature. Professor Donato admits that in Islamic patterns, the use of the hexagon is unavoidable [34].

And finally, the number "Eight", which is the most important number in Islamic art and culture. One of the reasons this number is important is that it is mentioned many times in the Quran; for instance, heaven has eight levels and the eighth one is the highest and most important level [26]. Besides the geometry, Islamic architecture is based on five principles:

- 1. People orientation,
- 2. Avoiding waste,
- 3. Structure,
- 4. Self-sufficiency, and
- 5. Introversion.

People Orientation

This theory makes architects design buildings based on the human scale and considering his needs. Following this principle, a room which is made for two people has the dimension which is enough for two people, neither bigger nor smaller, and also the design is in a way that it completely responds to their needs.

Avoiding Waste

Waste in Islam is prohibited and is considered as a sin (Quran), so it is no wonder this action is avoided in Islamic architecture. When facing an Islamic building, there are lots of details and decorations which at the first sight may seem a waste of time and material, however, all of those decorations have a structural or architectural function as well. For example, the shades which are used in front of the windows with Islamic patterns are meant to block the sunlight and guide it inside the building in a managed way.

Structure

Not just in Islamic architecture, but in all other styles, structure is one of the main challenges for architects. The similarity between Islamic and Gothic architecture at this point is that both of them used pointed vaults, and not only is it functioning as a structural element, it is also an element of beauty and is shown in elevation.

Self Sufficiency

Islamic architects were trying to gather materials from the nearest area and design the building in a way that they will not need materials or elements that cannot be found nearby. This action meant that the building costs less, and in the end, the building was in harmony with its surroundings.

Introversion

Islam emphasized a lot on the privacy and limits of people, and therefore the architecture had to respect this principle. Islamic buildings have open, semi-private, and private spaces, and only those who are allowed can access the private spaces [35].



Fig. 13 Nasir-al-Mulk mosque, Human scale dimensions and use of color and local materials [37]

IV. COMPARISON

Both of the styles that were analyzed in this paper tried to create heaven on earth but in different ways.

In the Gothic, the focal point is on geometry and proportions since they believe that proportions are sacred and heaven is a well proportional space, while in Islamic architecture proportions are not the only element. Islamic architects used the material and space evaluation as tools to create a pleasant and comfortable space in addition to paying attention to proportions.

One of the key elements in Gothic cathedrals are the circular windows with colorful glass that result in a dramatic light entering into the space, as well as the use of pointed vaults that allowed Gothic architects to bring more light inside the building. On the other side, in Islamic buildings, we have the same colorful windows but in more areas. This element is used to spread joy through colors inside the building, and while Gothic buildings are dark with dramatic light, Islamic buildings are light and colorful.

Both of styles used modules in their design process to create the perfect proportions for the building.

V.CONCLUSION

In the end, considering all of the principles and elements that both architectures used in their buildings, by the 19th century Gothic architecture was no longer used as an architectural style, while the Islamic architecture that was used from the first day continues to be used to this day, and even now, we see mosques being constructed using the same principles of Islamic architecture. In conclusion, creating heaven is more than just proportions, its more about making people feel good in the space, and that place where they feel their best, is heaven.

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