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Parametric Approach for Reserve Liability Estimate in Mortgage Insurance

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***Abstract*—**Chain Ladder (CL) method, Expected Loss Ratio (ELR) method and Bornhuetter-Ferguson (BF) method, in addition to more complex transition-rate modeling, are commonly used actuarial reserving methods in general insurance. There is limited published research about their relative performance in the context of Mortgage Insurance (MI). In our experience, these traditional techniques pose unique challenges and do not provide stable claim estimates for medium to longer term liabilities. The relative strengths and weaknesses among various alternative approaches revolve around: stability in the recent loss development pattern, sufficiency and reliability of loss development data, and agreement/disagreement between reported losses to date and ultimate loss estimate. CL method results in volatile reserve estimates, especially for accident periods with little development experience. The ELR method breaks down especially when ultimate loss ratios are not stable and predictable. While the BF method provides a good tradeoff between the loss development approach (CL) and ELR, the approach generates claim development and ultimate reserves that are disconnected from the ever-to-date (ETD) development experience for some accident years that have more development experience. Further, BF is based on subjective a priori assumption. The fundamental shortcoming of these methods is their inability to model exogenous factors, like the economy, which impact various cohorts at the same chronological time but at staggered points along their life-time development. This paper proposes an alternative approach of parametrizing the loss development curve and using logistic regression to generate the ultimate loss estimate for each homogeneous group (accident year or delinquency period). The methodology was tested on an actual MI claim development dataset where various cohorts followed a sigmoidal trend, but levels varied substantially depending upon the economic and operational conditions during the development period spanning over many years. The proposed approach provides the ability to indirectly incorporate such exogenous factors and produce more stable loss forecasts for reserving purposes as compared to the traditional CL and BF methods.

***Keywords*—**Actuarial loss reserving techniques, logistic regression, parametric function, volatility.

I. Introduction

LOSS reserving, a key actuarial function, involves the estimation of the unpaid losses (ultimate losses) that an insurer is liable to pay in the present and future time periods. An insurer must carry enough reserves to fulfill the claim obligations as per the insurance contract, against both reported and unreported claims for all the policies in force as of a given accounting date. Typically, there is a delay between reporting and claim payment, during which the contractual obligation is known as an outstanding claim. The two types of outstanding claims are, Reported But Not Settled (RBNS), and Incurred But Not Reported (IBNR). The focus of this paper is on the RBNS claims, which are the predominant in MI [3]. Claims are estimated on a paid basis as ultimate losses on a delinquency inventory categorized into homogenous groups of delinquencies based on their delinquency development history. This step is critical for estimation of the insurer's underwriting income as well as the valuation of the insurer, which supports various management and strategic decisions. While the insurer might hold and report reserves (case reserve or carried loss reserve) to reflect the potential liability against the known claims, this number might not be the best estimate.

Actuaries apply various reserving techniques on homogeneous groups of data in the reserve estimation process. Estimates generated by such methods are known as indicated loss reserves, and actuarial judgement is used in selecting an indicated loss reserve from one method or a combination of the methods. Once the best estimate is generated through the process, either a reserve deficiency or surplus can result from the difference between the selected (indicated) loss reserve and the carried loss reserve. This paper uses a Private Mortgage Insurance (PMI) example for the loss reserving application. PMI is an insurance policy which protects the mortgage lender or titleholder when a borrower defaults on the mortgage. The PMI provider pays the lender's first loss, up to the covered amount in the event of a loss/default from the borrower. Borrowers pay the PMI premium as a part of their mortgage payments for access to credit with low down payment. After a borrower misses monthly payments on the covered mortgage for 1-3 months, the mortgage servicer reports the delinquency to the PMI provider, which records it as a delinquency. Reserves need to be held against these delinquencies because the loan is impaired and has a higher probability of ending up as a claim and eventual loss to the PMI provider. Some of the reported delinquencies in a development period cure (i.e., the borrower resumes the monthly payment) and become current, while a small fraction eventually go to foreclosure with possibility of a claim and loss. The whole process can take several years. The reserve estimation process considers historical ETD claim experience to generate estimates of ultimate losses on each delinquency group.

II. Methodology

The most common reserving techniques used by General

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Insurance actuaries are: CL method, ELR Method, and BF method. In general insurance, outstanding claim liability is represented in the form of run-off triangles (claim counts and/ or claim amounts) by actuaries for depicting, assessing, and estimating ultimate claim liability. Although the depiction in the loss development triangle can be on a paid basis or incurred basis, this paper uses the paid loss development triangle rather than the incurred or the reported one. For the Paid Loss Triangle, losses are grouped in rows by the delinquency period (referred to as Accident Year in P&C) in which the delinquency is reported by the lender. The claim payments for any delinquency period (row) are summed and organized by development periods in columns. For this paper, the delinquency period and development period are defined at a regular interval. As of a given valuation date, only the upper left triangle information is known, and the lower right triangle and the ultimate cumulative losses are not known and must be estimated through one of the methods described.

A general mathematical framework following the notation as in [1], [4] is developed for exploring the approaches. In this paper, 100% severity on the covered risk is assumed once a reported delinquency is paid out as a claim, meaning the PMI provider will indemnify the lender for 100% of the covered loss amount. Hence the loss development triangle for paid claim count (frequency) is used for the purpose of this paper. So, the estimation problem resolves to delinquency to ultimate claim roll rate estimation and development.

Let Z_{ij} be a random variable representing the paid claims from a delinquency period i with $i \in \{1, \ldots, n\}$ paid in development period j with $j \in \{0, \ldots, n-1\}$. Given this, the claim dataset can be represented by $\{Z_{ij} : i = 1, \ldots, n; j = 0, \ldots, n-1\}$. As shown in Fig. 1, incremental paid claims are tabulated in a Run-off Triangle, where row represents the delinquency period/cohort, and column represents the development period.

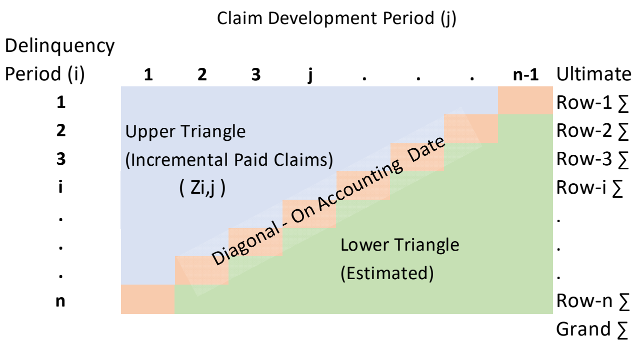

Fig. 1 Incremental Claim Run-off Triangle Data

To facilitate the estimation process, the incremental claim run-off triangle is converted into a cumulative form as shown in Fig. 2, such that Cij is a random variable representing the cumulative claims paid from delinquency period i, and development period j. Such that, $C_{ij} = \sum_{j=1}^{n} Z_{ij}$; Zij, $i = 1, \ldots,$ n. Claims are accumulated until period n for each delinquency period i such that $C_{in} = \sum_{j=1}^{n} Z_{ij}$ represents the Ultimate Claim (Observed or Estimated). Given the construct, the CL method utilizes the loss development method through loss development factors defined by:

$$\tilde{f}_j = \frac{\sum_{i=1}^{n-j} C_{ij}}{\sum_{i=1}^{n-j} C_{i,j-1}}, \text{ for } j = 1, 2, \ldots, n\text{-}1 \tag{1}$$

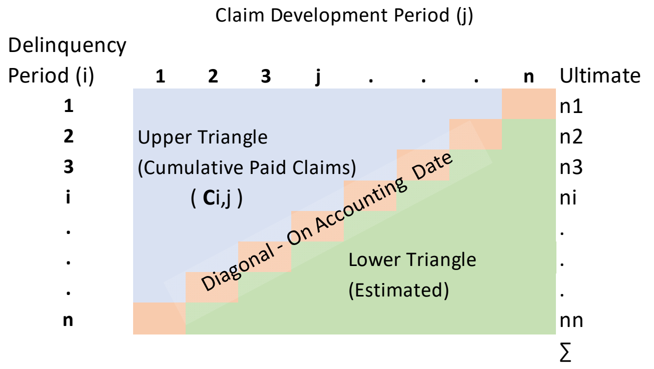

Fig. 2 Cumulative Claim Run-off Triangle Data

Given a set of loss development factor estimates ($\tilde{f}$) for each development period j, and observed cumulative claim development in the upper triangle captured by $Ci1$, $Ci2$, …, $Ci,j{-}1$, for delinquency period i, the Cumulative Claim Estimate for delinquency period i and development period j is given by $\tilde{C}_{i,j} = \tilde{f}_j . C_{i,j-1}$, and the expected outstanding c claims estimate for delinquency period i is given by, $E[C_{i,n-1}] = (\prod_{j=n-i+1}^{n-1} f_j). C_{i,n-1}$ for j = 1, 2, . n-1, and i+j ≤ n. The estimate for incremental claims from delinquency period i, for development period j, is given by $\tilde{Z}_{ij} = \tilde{C}_{ij} - \tilde{C}_{i,j-1}$

ELR approach is another commonly used approach in general insurance when adequate historical claim experience is lacking. In this approach, the assumption is that the ELR is the best available information for estimating the reserves. This approach also requires information on earned premium, which is multiplied by ELR to obtain the estimate of ultimate cumulative losses for any delinquency period, which is given by: $\tilde{C}_{i,n}^{LR} = E(s). \tilde{P}_i$, where $\tilde{P}_i$ is the earned premium for the i'th delinquency period (group/Cohort), and E(s) is the selected (or predicted) loss ratio either from internal data or external benchmarking. Total Reserves = Estimated Ultimate Cumulative Losses – Ever to Date Paid Losses. Case reserves for RBNS, and IBNR make up total reserves. One of the drawbacks of this approach is that it can result in negative reserves when ETD claim (or loss) development experience is less than the estimated ultimate claims (losses) under the ELR. Since a single value is targeted, variability in the ultimate loss rates for reserving is dampened.

BF method serves as a hybrid between the ELR approach and CL technique. In this method, an initial Loss estimate (a priori) is generated as under the ELR approach. A pro-rated remaining percentage of this expectation is added to the ETD Loss Rate, to generate the Ultimate Loss Rate for Reserving, which varies by delinquency cohort due to difference in seasoning. The Pro-Rating can be based on the IBNR factor that has been developed using CL approach.

IBNR Factor = 1 – 1/(LDF_j), where LDF_j is the Loss Development Factor Selected under the CL approach for j'th development period. While the BF approach offers a good compromise between the CL and ELR approaches, it still suffers the subjectivity bias from the a priori selection of Ultimate Loss Rate. While the Loss Development is muted, compared to CL approach, BF approach still does not mean-revert to a single ultimate loss rate. Both the CL and BF approaches address the estimation problem using linear assumptions, while loss development might not be a linear process. More sophisticated approaches use stochastic modeling, including multivariate regression-based approaches that are complex and difficult to understand [5]. The more complex multivariate regression-based techniques also make use Markov Chain transition framework based Generalized Linear Models (GLM) that leverage information on predictor variables like economic conditions [5], which is beyond the scope of this paper.

We propose an alternative to the CL, ELR, and BF approaches by explicitly modeling the non-linearity through parametric approach to generate an asymptote for ultimate loss rate by accident year. The Logistic function or a Sigmoid Curve (S-shaped) was extensively used in population growth modeling [6] as illustrated in Fig. 3. The function addresses the problem at hand elegantly because the function exhibits an initial geometric (exponential) rate of growth, followed by an arithmetic (linear) rate of growth, approaching the asymptote. The function is also governed by few parameters that address the non-linearity in loss development, while generating an asymptote for ultimate loss estimate for reserving.

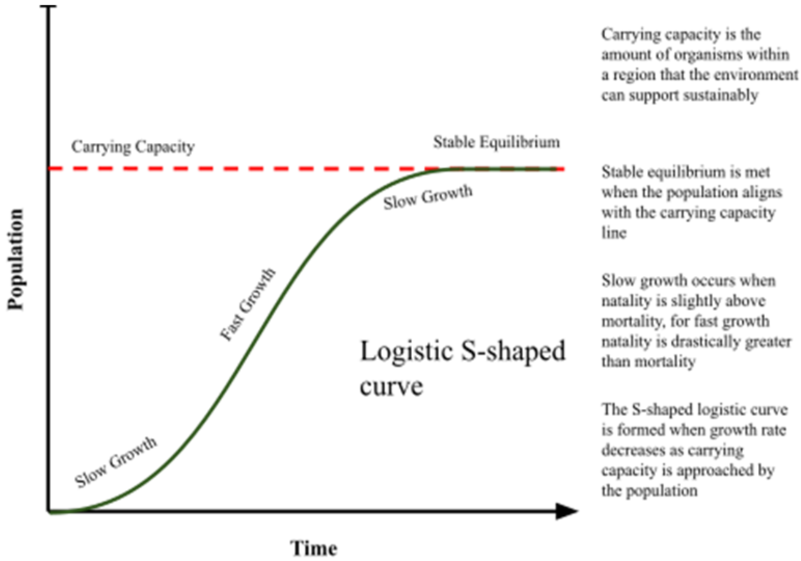

Fig. 3 Logistic Function Describing Population Growth over Time

The parametric form of a Logistic equation can be represented by [7],

$$f(x) = \frac{a}{1+\exp^{-d(x-f)}} \tag{2}$$

where a = Maximum Value of the Function or the upper asymptote, f is the mid-point of the curve, and d is the growth rate or steepness of the curve. However, to have more control over the sigmoid function, a generalized logistic function [4] that better controls the scale and shape of the function is used, as in (3).

$$f(x) = \frac{a}{b+c.\exp^{-d(x-f)}} \tag{3}$$

where, 'c' is a value which depends on the value of the function when f(x) = 0; Parameters b, c, and f govern the scale and shape of the function.

The optimal set of parameter values (a, b, c, d, f) can be generated by non-linear optimization on the historic Loss-Triangle data, which can then be used for generating the loss rate projection to ultimate, at which point the function is an asymptote. Borrowing from the Spline-fitting method, we also force the calculated f(x) to be >= the observed last available roll rate to ensure that there is no inconsistency between the last diagonal of the triangle and the projected part of the sigmoid curve. For our case, the loss triangle data for each of the accident period and development period is used. The optimal set of parameters are fit on the observed cumulative claim roll rate or frequency f(x) based on the observed historic claim rate, expressed as a function of development, or seasoning of the cohort 'x'.

Although a logistic function addresses the non-linearity and asymptote (mean reversion) features, the approach might still generate an asymptote that is either too low or too high, especially for accident years with little seasoning and (or) abnormal ETD loss development. To overcome this problem, we tried two approaches: leveraging the first derivative of the logistic function and forcing four prior cohort average asymptotes as an additional constraint during optimization for cohorts that are seasoned less than eight periods. The first derivative of the logistic is also known as the Hubbert's Curve [7], which was used during the 1950's to model the production and growth rates of oil. Hubbert used the first derivative of the logistic function to model production rate as a function of time. The justification is that curve fitting to model physical phenomena is subjective, unless the resulting functions are useful and meet some first principles based on fundamental dynamics [7].

The first derivative of the logistic function (3) is:

$$f' = \frac{a.c.d.exp^{-d(x-f)}}{(b+c.exp^{-d(x-f)})^2} \tag{4}$$

For the Loss Rate Projection of each development period for each cohort, we first compute the projected loss rate using both the logistic function, as well as the derivative of the logistic function. Taking the maximum of the two values at each projection point ensures that ETD Loss Rate at development period j is always less than the projected loss rate at development period j+1.

After careful comparison and review of the results from the two approaches, we ended up using the second method of forcing a priori asymptote for unseasoned cohorts, as this ensured the best fit for all the available historic data points, while tracking a sigmoid function with an a priori asymptote reflecting recent mature cohort experience. We call this the Hybrid Sigmoid approach (HS), which models the asymptote for the ultimate while capturing the impact of exogenous economic factors that can impact roll rates at different points in time for the various cohorts.

Development Period (Per-x)

	Per-0	Per-1	Per-2	Per-3	Per-4	Per-5	Per-6	Per-7	Per-8	Per-9	Per-10	Per-11	Per-12	Per-13	Per-14	Per-15	Per-16	Per-17	Per-18	Per-19	Per-20	Per-21	Per-22	Per-23
Cohort-1	0%	0%	0%	2%	5%	8%	15%	27%	31%	34%	35%	36%	36%	38%	39%	39%	40%	42%	43%	44%	44%	44%	44%	44%
Cohort-2	0%	0%	2%	5%	7%	12%	22%	27%	29%	30%	32%	35%	37%	38%	39%	39%	41%	43%	44%	44%	45%	45%	45%	45%
Cohort-3	0%	1%	4%	5%	9%	16%	20%	23%	25%	27%	33%	36%	38%	39%	39%	41%	43%	43%	43%	45%	46%	46%	47%	48%
Cohort-4	0%	0%	1%	4%	10%	13%	16%	18%	21%	28%	33%	35%	36%	37%	38%	40%	41%	42%	43%	45%	45%	46%	46%	
Cohort-5	0%	0%	2%	6%	11%	15%	17%	19%	26%	30%	32%	34%	36%	37%	39%	40%	41%	44%	45%	46%	47%	47%		
Cohort-6	0%	0%	3%	7%	10%	12%	14%	17%	21%	24%	26%	29%	30%	33%	36%	39%	41%	43%	43%	44%	46%			
Cohort-7	0%	1%	3%	5%	8%	9%	11%	14%	16%	19%	23%	25%	29%	34%	36%	39%	41%	42%	44%	45%				
Cohort-8	0%	0%	2%	5%	6%	9%	11%	14%	18%	22%	26%	30%	35%	38%	40%	42%	43%	45%	46%					
Cohort-9	0%	0%	2%	4%	6%	8%	12%	17%	23%	27%	31%	36%	40%	41%	42%	44%	46%	47%						
Cohort-10	0%	0%	1%	3%	4%	7%	12%	19%	22%	27%	33%	38%	39%	41%	42%	43%	44%							
Cohort-11	0%	0%	0%	1%	2%	7%	14%	17%	22%	28%	33%	36%	37%	39%	41%	41%								
Cohort-12	0%	0%	1%	2%	7%	16%	18%	24%	30%	34%	36%	37%	40%	42%	43%									
Cohort-13	0%	0%	1%	5%	14%	21%	29%	36%	41%	43%	45%	46%	49%	50%										
Cohort-14	0%	1%	5%	14%	23%	35%	44%	49%	52%	54%	56%	57%	58%											
Cohort-15	0%	1%	5%	14%	25%	41%	47%	50%	52%	55%	57%	57%												
Cohort-16	0%	1%	6%	13%	32%	42%	47%	49%	51%	54%	54%													
Cohort-17	0%	0%	2%	17%	27%	36%	40%	42%	44%	44%														
Cohort-18	0%	1%	7%	13%	20%	26%	29%	32%	33%															
Cohort-19	0%	2%	5%	10%	17%	22%	26%	28%																
Cohort-20	0%	1%	4%	10%	18%	24%	26%																	
Cohort-21	0%	1%	6%	13%	21%	25%																		
Cohort-22	0%	2%	8%	16%	20%																			
Cohort-23	~~0.0%~~	~~1.9%~~	~~8.9%~~	~~14.0%~~																				
Cohort-24	~~0.0%~~	~~2.7%~~	~~6.0%~~																					
Cohort-25	~~0.0%~~	~~0.7%~~																						

Training
Testing
~~Excluded~~

Fig. 4 Cumulative Delinquency-to-Claim Roll Rate Triangle (Training and Testing Data)

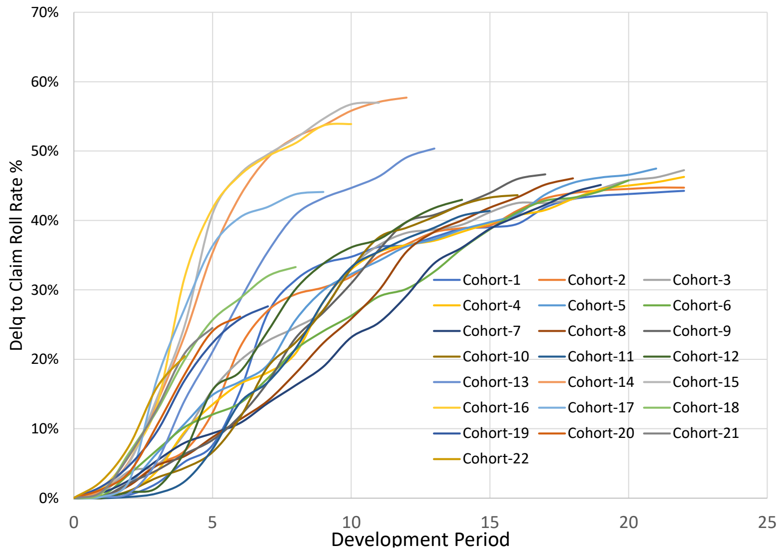

Fig. 5 Cumulative Delinquency-to-Claim Roll Rate Development by Delinquency Cohort

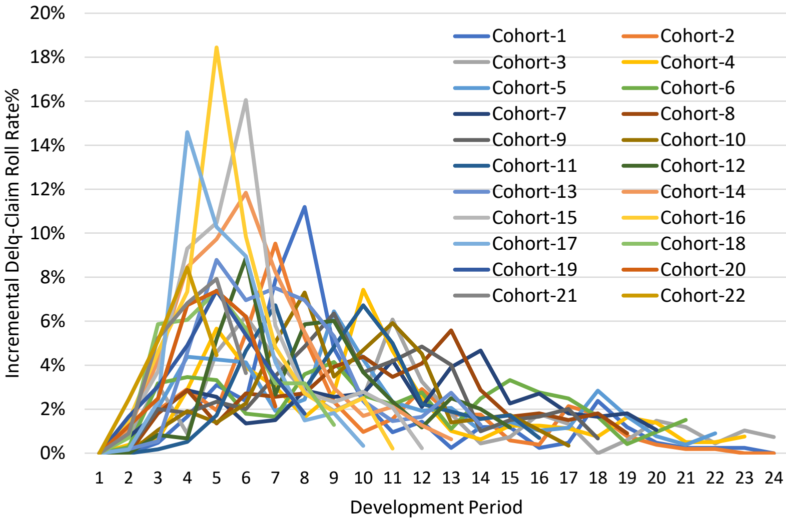

Fig. 6 Incremental Delinquency-Claim Roll Rate Development by Delinquency Cohort

Parameter	a	b	c	d	e
Average	0.52	1.06	17.09	0.18	2.93
Std. Deviation	0.14	0.26	4.51	0.08	0.52
Coefficient of Variation	27%	24%	26%	43%	18%

Fig. 7 GRG Solver Optimal Parameters – Summary Statistics

III. Data And Assumptions

Comparative analysis of the claim frequency projections under the CL, BF, and proposed approach (HS) was performed on the MI delinquency and claim data. The Loss Development Triangle (Cumulative Delinquency-to-Claim Roll Rate), as shown in Fig. 4, comprises of 25 accident periods, and 23 development periods for the oldest accident cohort. While all the data are used for finding the optimal parameters, last three diagonals (3 periods) are removed for back testing the projected cumulative claim rate under the three methods. Out of the 22 delinquency cohorts, about 2-3 cohorts are fully developed to ultimate, exhibiting an asymptote; however, evaluating the claim triangle reveals that the more recent cohorts have adverse early development compared to the earlier cohorts.

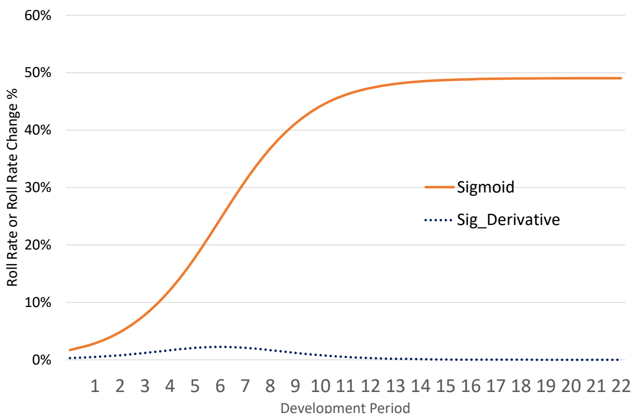

Fig. 8 Optimized Sigmoid Roll Rate Function & Sigmoid Derivative

The last three cohorts (cohort 23, 24, 25) have been excluded from the training purpose as they are extremely low-seasoned. The corresponding Cumulative Delinquency-to-Claim Roll Rates are illustrated in a Vintage Analysis Chart in Fig. 5.

It is evident that the recent cohorts have less development experience but have higher roll rate, and most of the cohorts that are seasoned beyond 10 periods are exhibiting signs of an asymptote. The corresponding incremental Roll Rate is illustrated in Fig. 6.

As noted earlier, the incremental roll rate development also exhibits volatility, especially stemming from the recent delinquency cohorts.

CL and BF projections of the lower triangle are developed as discussed in the methodology section earlier. The optimal HS parameters were obtained by using the Upper Triangle data in MS Excel by applying the Generalized Reduced Gradient (GRG) solver. Since GRG is a smooth nonlinear method [2], which is sensitive to initial conditions, appropriate bounds were placed on the parameter values through iterative testing to ensure a global minimum is reached (Engineer Excel, 2016) rather than the local minimum. The GRG method evaluates the gradient or slope of the objective function as inputs to determine the optimum solution by setting partial derivative(s) = 0. The optimal parameter values are obtained by minimizing the sum of squared errors (SSE) between the observed and calculated delinquency to claim roll rate for all the data points in the upper claim triangle. Also, the parameters were relatively stable, as illustrated by the summary statistics in Fig. 7, especially the coefficient of variation. Of all the parameters, parameter d, which reflects the growth rate or steepness of the sigmoid function, had the highest coefficient of variation. This is precisely the reason that linear approaches like CL and BF fail to accurately model the loss development pattern more accurately.

Method & Seasoning Period (Per)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Act_18 Per Seasoned	0%	0%	2%	5%	6%	9%	11%	14%	18%	22%	26%	30%	35%	38%	40%	42%	43%	45%	46%					
CL_18 Per Seasoned																			46%	47%	48%	49%	50%	51%
BF_18 Per Seasoned																			46%	47%	48%	49%	50%	51%
Logistic_18 Per Seasoned																			46%	47%	47%	47%	48%	48%
Act_14 Per Seasoned	0%	0%	1%	2%	7%	16%	18%	24%	30%	34%	36%	37%	40%	42%	43%									
CL_14 Per Seasoned															43%	45%	46%	48%	49%	51%	52%	52%	53%	54%
BF_14 Per Seasoned															43%	45%	46%	48%	49%	51%	52%	52%	53%	54%
Logistic_14 Per Seasoned															43%	43%	43%	44%	44%	44%	44%	44%	44%	44%
Act_10 Per Seasoned	0%	1%	6%	13%	32%	42%	47%	49%	51%	54%	54%													
CL_10 Per Seasoned											54%	58%	62%	65%	68%	70%	73%	75%	77%	79%	81%	82%	84%	85%
BF_10 Per Seasoned											54%	56%	59%	61%	63%	64%	66%	68%	69%	70%	71%	72%	73%	73%
Logistic_10 Per Seasoned											54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%	54%
Act_6 Per Seasoned	0%	1%	4%	10%	18%	24%	26%																	
CL_6 Per Seasoned							26%	30%	34%	38%	42%	46%	49%	51%	53%	56%	58%	59%	61%	63%	64%	65%	66%	67%
BF_6 Per Seasoned							26%	29%	33%	36%	39%	42%	44%	46%	48%	50%	52%	53%	54%	56%	57%	57%	58%	59%
Logistic_6 Per Seasoned						24%	29%	36%	41%	45%	47%	49%	49%	50%	50%	50%	50%	50%	51%	51%	51%	51%	51%	51%
Act_4 Per Seasoned	0%	2%	8%	16%	20%																			
CL_4 Per Seasoned					20%	28%	33%	38%	43%	48%	54%	58%	61%	64%	67%	70%	73%	75%	77%	79%	81%	82%	83%	85%
BF_4 Per Seasoned					20%	25%	28%	32%	35%	38%	42%	44%	47%	49%	50%	52%	54%	55%	57%	58%	59%	60%	61%	61%
Logistic_4 Per Seasoned				16%	22%	31%	38%	44%	47%	49%	50%	50%	51%	51%	51%	51%	51%	51%	51%	51%	51%	51%	51%	51%

Fig. 9 Comparison of Model Projections (CL, BF, HS) & ACT

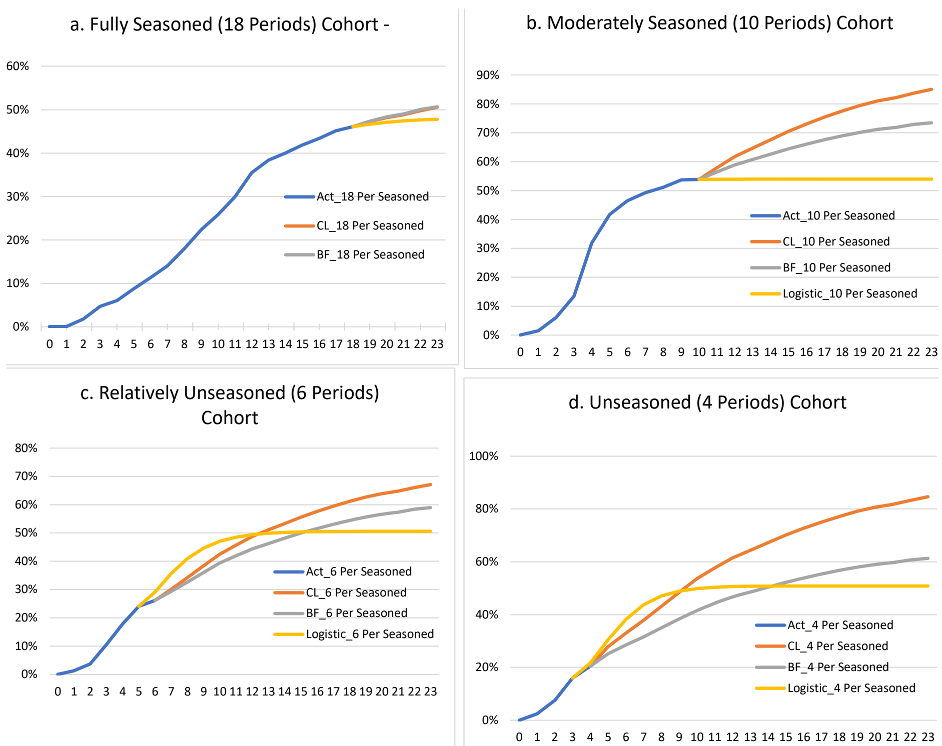

Fig. 10 Comparative of Projections from Various Techniques (CL, BF, HS) by Varying Seasoning

Although parameters were developed and used at a cohort-level, for the illustration of generalizability, the average parameter set is used for sigmoid and sigmoid-derivative functions. The average parameters for the logistic function result in an asymptote for ultimate roll rate at about 48% (38th development period), and peak roll rate change of 2.26% at 18th development period. The asymptote of ~ 48% is the direct result of older delinquency cohorts (more data points) with relatively benign experience. The resulting logistic function and derivative function are shown in Fig. 8.

IV. Results and Discussion

Results from selected delinquency cohorts seasoned by development periods, 4, 6, 10, 14, 18, are used to provide a comparison of projections to ultimate, among CL, BF, and HS approaches, along with actuals (ACT). The results of the back test are shown in Fig. 9.

As shown in Fig. 9, both CL and BF techniques overshoot, compared to the actuals or HS projections, more so for cohorts seasoned less than 10 periods. As noted earlier, this data set has the ETD performance of cohorts covering 4 periods, way above that of other cohorts observed at similar development times. This results in CL and BF techniques projecting very high Ultimate Roll Rates. The HS technique alleviates the problem of ever-increasing roll rate, even past 23 development periods that both CL and BF techniques exhibit by reaching a reasonable asymptote. However, the HS technique tracked more closely with the slope of the actuals, compare Fig. 10.

V. Summary and Conclusions

Although CL, ELR, and BF methods are some of the most used actuarial reserving techniques in general insurance, they make a linearity assumption and/or a priori assumptions. At times, this results in very high ultimate loss rate (roll rate in our example) that will be applied for reserve estimation. The proposed approach of combining a Logistic function with asymptote assumptions for nascent cohorts simultaneously addresses the issues of non-linearity in loss development, avoiding the a priori assumption (as in BF), while still generating an asymptote that is expected due to the inherent parametric sigmoidal behavior of the loss development. The proposed HS approach incorporates the influence of the exogenous effects like economic conditions during the development to ultimate parametrically, unlike the traditional reserving methods. This is technically more meaningful because some development periods might experience an economic stress, but relying on that as a trend for ultimate projections is unreasonable. By using a sample MI claim development dataset, Ultimate Claim Frequency (for Reserving) estimated through the proposed approach is shown to be more stable (mean reverting roll rate change) while fully considering the cumulative loss development experience for each homogeneous delinquency group/cohort, compared to the CL and BF techniques. Results from a back-test performed for four cohorts that differ by seasoning has shown that, while the proposed technique performed reasonably well in generating an asymptote for loss rate, the projections in the back-test period came slightly under the actuals. In summary, the proposed technique offers a promising alternative to CL and BF techniques by modeling non-linearity in an explicit manner.

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