World Academy of Science, Engineering and Technology
International Journal of Nuclear and Quantum Engineering
Vol:14, No:12, 2020

Optical and Double Folding Analysis for ^{6}Li+^{16}O Elastic Scattering

Abd Elrahman Elgamala, N. Darwish, I. Bondouk, Sh. Hamada

Abstract—Available experimental angular distributions for ^{6}Li elastically scattered from ^{16}O nucleus in the energy range 13.0–50.0 MeV are investigated and reanalyzed using optical model of the conventional phenomenological potential and also using double folding optical model of different interaction models: DDM3Y1, CDM3Y1, CDM3Y2, and CDM3Y3. All the involved models of interaction are of M3Y Paris except DDM3Y1 which is of M3Y Reid and the main difference between them lies in the different values for the parameters of the incorporated density distribution function $F(\rho)$. We have extracted the renormalization factor N_R for ^{6}Li+^{16}O nuclear system in the energy range 13.0–50.0 MeV using the aforementioned interaction models.

Keywords—Elastic scattering, optical model, folding potential, density distribution.

I. INTRODUCTION

NUCLEAR processes induced by lithium nuclei had attracted nuclear physicists for decades due to the interesting features which could be derived from these studies. Cluster structure of ^{6}Li as consisting of a core (α-alpha) and a valence particle (d- deuteron) orbiting the core is one of these features. During the past decades, several experimental studies and theoretical calculations were performed for ^{6}Li+X nuclear systems (where, X is any target nuclei) and consequently lithium scattering angular distributions have become available at different beam energies for different nuclear systems among them: a) ^{6}Li+^{12}C [1]-[14], b) ^{6}Li+^{16}O [15]-[22], and c) ^{6}Li+^{28}Si [9], [13], [16], [19], [23]-[27]. There are various studies performed to reproduce the experimental angular distribution measurements for ^{6}Li+X nuclear systems using either phenomenological optical [28]-[30] as well as double folding (DF) potentials. The essential information deduced from previous lithium elastic scattering of different targets and especially ^{16}O nucleus are: a) Optical model is a successful tool to reproduce the experimental data over a wide range of bombarding energies. b) For a given reaction, potential parameters are either constant or change smoothly with bombarding energy. c) Spin orbit effects do not play a significant role. d) Remarkable deviations between experimentally and calculated cross sections at large angles are observed. In other words "the potentials are not unique". e) The observed ambiguities of potential parameters families

cannot experimentally be reduced unless data at higher bombarding energies become available.

Various studies showed the effectiveness of optical model (OM) to reproduce the experimental angular distributions data for ^{6}Li+^{12}C and ^{7}Li+^{12}C elastic scattering at beam energies ranging from 4.5 to 13.0 MeV [28], ^{6}Li+^{12}C elastic scattering at energies > 20.0 MeV [29], ^{6}Li and ^{7}Li elastically scattered from ^{10}B, ^{12}C, ^{13}C, ^{16}O, and ^{28}Si at energies between 4.0 and 13.0 MeV [30], ^{6}Li+^{16}O system at energies 25.7 and 50.0 MeV [17], [22]. Vineyard et al. [17] and Trcka et al. [22] have measured and analyzed the angular distributions for ^{6}Li+^{16}O system at energies 25.7 MeV and 50.0 MeV. The data were analyzed using double folded potentials with renormalization factors 0.61 and 0.65 at the two studied energies 25.7 MeV and 50.0 MeV respectively, in other words the potentials needed to be reduced in strength by about 37% in order to reproduce the data.

The current work aims to reanalyze the available angular distribution data for ^{6}Li+^{16}O in the energy range 13.0–50.0 MeV using both OM and DFOM. In the DFOM, the real part of potential is created on the basis of DF using different models of interactions DDM3Y1, CDM3Y1, CDM3Y2, and CDM3Y3 in order to examine the sensitivity of ^{6}Li+^{16}O nuclear system to the different used interaction models.

The paper is organized as follows. In Section II the theoretical calculations of the experimental data is presented. Section III is devoted to the results of the theoretical analysis and discussion. The summary is given in Section IV.

II. THEORETICAL ANALYSIS

The angular distributions for ^{6}Li+^{16}O elastic scattering in energy range 13.0–50.0 MeV are reanalyzed using both phenomenological and semi-microscopic potentials: OM potential and DF potential of different models of interaction: DDM3Y1, CDM3Y1, CDM3Y2, and CDM3Y3. Firstly, the available experimental angular distributions data for ^{6}Li+16O elastic scattering at energies 13.0 MeV [15], 20.0 MeV [16], 25.7 MeV [17], 30.0 MeV [18], 32.0 MeV [19], 36.0 MeV [20], 48.0 MeV [21], and 50.0 MeV [22] are investigated within the framework of OM. We have started with the same parameters of. [22] as starting parameters. The analyses employed real and imaginary volume central potentials together with a Coulomb potential. Spin orbit interaction for ^{6}Li has not been included due to the well-known its little influence on the differential cross section. The used potential has the following form:

Abd Elrahman Elgamala, N. Darwish, I. Bondouk, and Sh. Hamada are with Faculty of Science, Tanta University, Tanta, Egypt (e-mail: elgmalhabdo@gmail.com, nema.darwish@science.tanta.edu.eg, ibrahim.bondok@science.tanta.edu.eg, sh.m.hamada@science.tanta.edu.eg).

World Academy of Science, Engineering and Technology
International Journal of Nuclear and Quantum Engineering
Vol:14, No:12, 2020

$$U(R) = V_C(r) - \frac{V_0}{\left[1+\exp(r-r_V A^{1/3})/a_V\right]} - i\frac{W_0}{\left[1+\exp(r-r_W A^{1/3})/a_W\right]} \quad (1)$$

where $V_C(r)$ is the Coulomb potential due to a uniform sphere with charge equal to that of the target nucleus and radius $r_C A_t^{1/3}$. It is worth to mention that the current work completes our ongoing studies for ^{6}Li+X [31], [32] nuclear systems.

Secondly, the available ^{6}Li+^{16}O elastic scattering angular distributions are analyzed in the framework of double folding optical model (DFOM). In this model, the real part of the interaction potential was derived on the basis of DF, where ^{6}Li and ^{16}O densities in their ground states are folded to nucleon-nucleon potential using code DFMSPH [33]. The real part of potential is calculated from:

$$V_{DF}(R) = \iint \rho_P(r_1)\rho_T(r_2)\upsilon_{NN}(s)d\vec{r}_1 d\vec{r}_2 \quad (2)$$

where $\upsilon_{NN}(s)$ is the effective nucleon-nucleon interaction potential which is taken to be of the CDM3Y1, CDM3Y2, and CDM3Y3 [34] forms based on the M3Y-Paris potential:

$$v_D(s) = 11061.625\frac{\exp(-4s)}{4s} - 2537.5\frac{\exp(-2.5s)}{2.5s}, \quad (3)$$

$$v_{EX}(s) = -1524.25\frac{\exp(-4s)}{4s} - 518.75\frac{\exp(-2.5s)}{2.5s} - 7.8474\frac{\exp(-0.7072s)}{0.7072s},$$

and also is taken to be of the DDM3Y1 [35] form based on the M3Y- Reid potential:

$$v_D(s) = 7999.0\frac{\exp(-4s)}{4s} - 2134.25\frac{\exp(-2.5s)}{2.5s}, \quad (4)$$

$$v_{EX}(s) = -4631.38\frac{\exp(-4s)}{4s} - 1787.13\frac{\exp(-2.5s)}{2.5s} - 7.8474\frac{\exp(-0.7072s)}{0.7072s},$$

The M3Y-Paris and M3Y- Reid interactions are scaled by an explicit density-dependent function $F(\rho)$:

$$v_{D(EX)}(\rho,s) = F(\rho)v_{D(EX)}(s), \quad (5)$$

where $v_{D(EX)}$ are the direct and exchange components of the M3Y-Paris and M3Y- Reid, ρ is the nuclear matter (NM) density, s is the distance between the two interacting nucleons. Density-dependent function $F(\rho)$, is taken to be of exponential dependence and can be written as:

$$F(\rho) = C[1 + \alpha\exp(-\beta\rho) - \gamma\rho^n], \quad (6)$$

The parameters C, α, β, γ for the different concerned interaction models are listed in Table I. The DF potential in the present work is the sum of direct folded potential and exchange folded potential, and can be written as

$$V^{DF}(R) = V_D^{DF}(R) + V_{EX}^{DF}(R) \quad (7)$$

TABLE I
PARAMETERS OF DENSITY-DEPENDENCE FUNCTION $F(P)$

Interaction Model	c	α	β (fm³)	γ (fm³ⁿ)	n	K (MeV)
DDM3Y1	0.2845	3.6391	2.9605	0.0	0	171
CDM3Y1	0.3429	3.0232	3.5512	0.5	1	188
CDM3Y2	0.3346	3.0357	3.0685	1.0	1	204
CDM3Y3	0.2985	3.4528	2.6388	1.5	1	217

The density distribution of ^{16}O is expressed in a modified form of the Gaussian shape as: $\rho(r) = \rho_0(1 + wr^2)\exp(-\beta r^2)$, where ρ_0=0.1317, w=0.6457 and β=0.3228 [36]. The density distribution of ^{6}Li nucleus is taken from [37] as:

$$\rho(r) = 0.203 \exp(-0.3306\,r^2) + \left[-0.0131 + 0.001378\,r^2\right]\exp(-0.1584\,r^2) \quad (8)$$

The imaginary part of potential is taken in the Woods–Saxon shape, the optimal imaginary potential parameters obtained from OM calculations are taken the same in DFOM calculations, and consequently only one variable parameter N_R "renormalization factor for the real part of potential" was used in the calculations using this approach. The nuclear potential in this case has the following shape:

$$U(R) = V_C(r) - N_R\,V_{DF}(r) - iWf(r,r_W,a_W) \quad (9)$$

III. RESULTS AND DISCUSSION

We have performed an OM analysis for the ^{6}Li+^{16}O elastic scattering angular distributions in the energy range 13.0–50.0 MeV using pure phenomenological Woods-Saxon potentials for both real and imaginary volume parts. The comparisons between the experimental data at energies (13.0, 20.0, 25.7, 30.0, 32.0, 36.0, 48.0, and 50.0 MeV) and the theoretical calculation using OM are shown in Figs. 1 and 2. The optimal extracted potential parameters are listed in Table II. Two parameters "r_V and r_W" were fixed during search to the values 1.37 and 1.88 fm respectively, and the rest four parameters V_0, a_V, W_0, and a_W were allowed to be changed freely in order to obtain the least χ^2 value defined by:

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma(\theta_i)^{\text{cal}} - \sigma(\theta_i)^{\text{exp}}}{\Delta\sigma(\theta_i)}\right)^2, \quad (10)$$

where N is the number of experimental data points. $\sigma(\theta_i)^{\text{cal}}$ and $\sigma(\theta_i)^{\text{exp}}$ are the calculated and experimental cross sections, $\Delta\sigma(\theta_i)$ is the uncertainty of the data. Using this technique, we could obtain an energy dependence on both real and imaginary potential depths. Theoretical calculations conducted in the present study are performed using the code FRESCO [38], and SFRESCO search code was used in searching for the optimal potential parameters. The Coulomb

World Academy of Science, Engineering and Technology
International Journal of Nuclear and Quantum Engineering
Vol:14, No:12, 2020

radius parameter was fixed at 2.3 fm during all the calculations performed in this work.

Available experimental data for ^{6}Li+^{16}O are then analyzed within the framework of DFOM using different models of interaction: DDM3Y1, CDM3Y1, CDM3Y2, and CDM3Y3. The data were fitted using only one parameter "N_R", while the imaginary phenomenological potential parameters are taken the same as those obtained from OM calculations. The comparisons between the experimental angular distributions data for ^{6}Li+^{16}O nuclear system at different concerned energies and theoretical calculations using DDM3Y1, CDM3Y1, CDM3Y2, and CDM3Y3 models are shown in Figs. 3-10. It is clearly shown that using such hard constraints, the quality of fitting got worse "see the values of χ^2/N extracted from the OM and DFOM calculations". Data analysis using DFOM with the different concerned interactions showed and emphasized the necessity for reduction N_R by about 40%. The optimal potential parameters used in the DFOM calculations using DDM3Y1, CDM3Y1, CDM3Y2, and CDM3Y3 interaction models are listed in Table III.
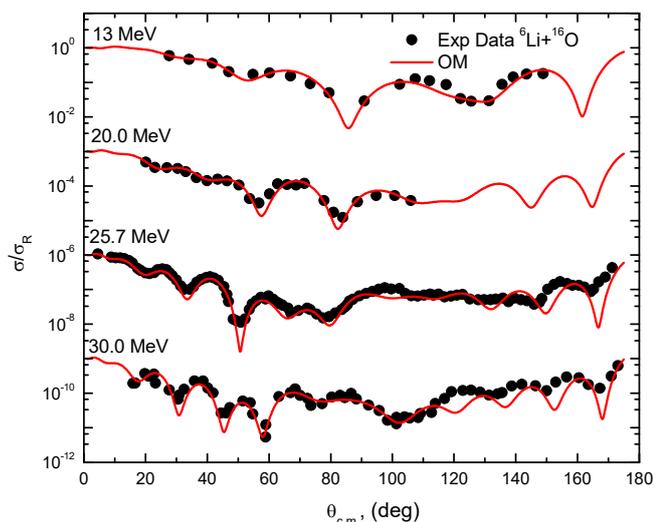

Fig. 1 Comparison between experimental angular distributions for ^{6}Li+^{16}O elastic scattering and the theoretical calculations using OM at E_{lab}= 13.0, 20.0, 25.7, and 30.0 MeV. The data sets at different energies have been displaced by successive factors of 10^{-3} for the sake of clarity
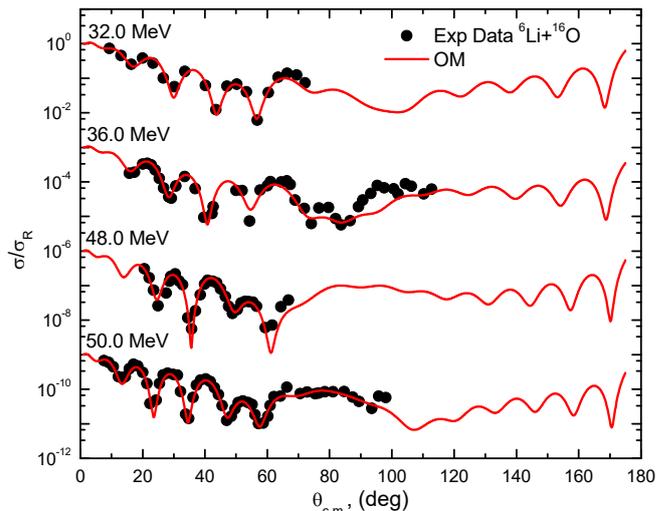

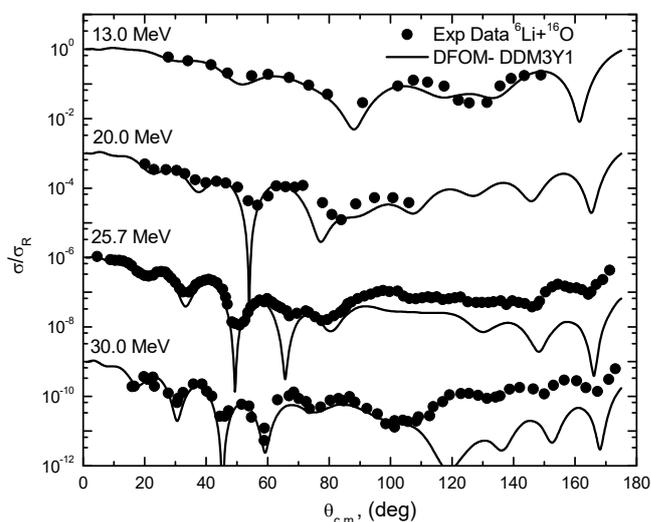Fig. 2 Same as Fig. 1 but at E_{lab}= 32.0, 36.0, 48.0, and 50.0 MeV

Fig. 3 Comparison between experimental angular distributions for ^{6}Li+^{16}O elastic scattering and the theoretical calculations using DFOM – DDM3Y1 at E_{lab}= 13.0, 20.0, 25.7, and 30.0 MeV

TABLE II
OPTIMAL OPTICAL POTENTIAL PARAMETERS FOR ^{6}Li+^{16}O NUCLEAR SYSTEM EXTRACTED FROM THE OM ANALYSIS,

E (MeV)	V_0 (MeV)	r_V (fm)	a_V (fm)	W_0 (MeV)	r_W (fm)	a_W (fm)	χ^2/N	σ_R (mb)	J_V (MeV.fm^3)	J_W (MeV.fm^3)
13.0	186.01	1.37	0.719	7.14	1.88	0.949	7.4	1183	477.0	46.3
20.0	185.87	1.37	0.666	9.11	1.88	0.93	8.1	1288	456.3	58.3
25.7	183.13	1.37	0.716	11.23	1.88	0.93	11.5	1438	468.5	71.9
30.0	180.06	1.37	0.734	11.65	1.88	0.93	16.6	1477	467.6	74.6
32.0	182.15	1.37	0.721	12.59	1.88	0.93	7.5	1496	467.9	80.6
36.0	182.46	1.37	0.701	13.64	1.88	0.93	17.9	1513	460.9	87.4
48.0	188.0	1.37	0.664	14.09	1.88	0.95	16.8	1530	460.8	91.3
50.0	185.6	1.37	0.708	16.25	1.88	0.862	5.2	1439	471.6	100.0

The values of reaction cross sections σ_r as well as real J_V and imaginary J_W volume integrals are also listed.

World Academy of Science, Engineering and Technology
International Journal of Nuclear and Quantum Engineering
Vol:14, No:12, 2020

TABLE III
POTENTIAL PARAMETERS FOR ^{6}Li+^{16}O SYSTEM EXTRACTED FROM THE DFOM ANALYSIS

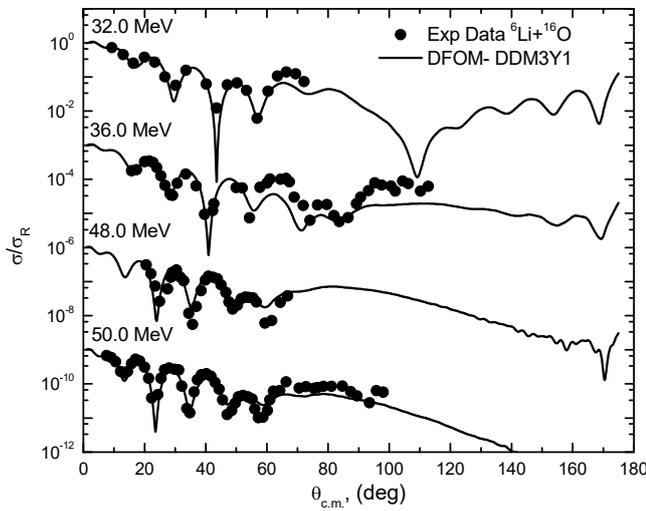| E (MeV)	Interaction model	N_R	W_0 (MeV)	r_W (fm)	a_W (fm)	χ^2/N	σ_R (mb)	J_V (MeV.fm^3)	J_W (MeV.fm^3)
13.0	DDM3Y1	0.795	7.14	1.88	0.949	13.5	1211	403.9	46.3
	CDM3Y1	0.774				14.5	1217	393.9	
	CDM3Y2	0.795				16.3	1215	403.9	
	CDM3Y3	0.795				14.7	1214	403.9	
20.0	DDM3Y1	0.806	9.11	1.88	0.93	19.9	1415	403.9	58.3
	CDM3Y1	0.79				20.2	1427	395.8	
	CDM3Y2	0.794				20.1	1414	397.8	
	CDM3Y3	0.586				23.9	1423	293.7	
25.7	DDM3Y1	0.59	11.23	1.88	0.93	37.4	1453	294.3	71.9
	CDM3Y1	0.58				38.3	1454	289.4	
	CDM3Y2	0.58				38.2	1453	289.4	
	CDM3Y3	0.58				38.1	1452	289.4	
30.0	DDM3Y1	0.597	11.65	1.88	0.93	41.8	1478	295.9	74.6
	CDM3Y1	0.594				42.9	1479	294.5	
	CDM3Y2	0.593				42.7	1478	294.0	
	CDM3Y3	0.837				35.3	1477	414.9	
32.0	DDM3Y1	0.593	12.59	1.88	0.93	15.0	1498	293.2	80.6
	CDM3Y1	0.581				14.4	1563	287.2	
	CDM3Y2	0.582				15.6	1561	287.7	
	CDM3Y3	0.584				15.7	1560	288.7	
36.0	DDM3Y1	0.521	13.64	1.88	0.93	28.1	1513	256.1	87.4
	CDM3Y1	0.522				29.6	1515	256.6	
	CDM3Y2	0.507				29.1	1514	249.2	
	CDM3Y3	0.521				29.7	1513	256.1	
48.0	DDM3Y1	0.554	14.09	1.88	0.95	82.1	1545	267.6	91.3
	CDM3Y1	0.54				76.5	1544	260.8	
	CDM3Y2	0.54				72.1	1526	260.8	
	CDM3Y3	0.54				69.2	1543	260.8	
50.0	DDM3Y1	0.559	16.25	1.88	0.862	23.2	1473	269.4	100.0
	CDM3Y1	0.543				35.4	1478	261.7	
	CDM3Y2	0.545				35.5	1458	262.7	
	CDM3Y3	0.547				35.6	1477	263.7	

The values (σ_R, J_V and J_W) are also listed.

Fig. 4 Same as Fig. 3 but at E_{lab}= 32.0, 36.0, 48.0, and 50.0 MeV

As shown in Figs. 3-10, the DFOM calculations using DDM3Y1, CDM3Y1, CDM3Y2, and CDM3Y3 interaction models gives equally fitting for the data except at E_{lab}= 25.7 MeV where there is a noticeable deviation especially at large angle > 90°. It was expected that DF calculations using these different interaction models will be close to each other, in other words "the renormalization factors for the different used models are close". The main reason for this is that we used four families of interaction models which has nearly the same incompressibility constant K = 171-217. In case of using different interaction models which have a higher incompressibility constant, we could be able to fit the data with a higher N_R value "more close to one" and consequently the strength of N_R would be reduced by a factor < 40%.
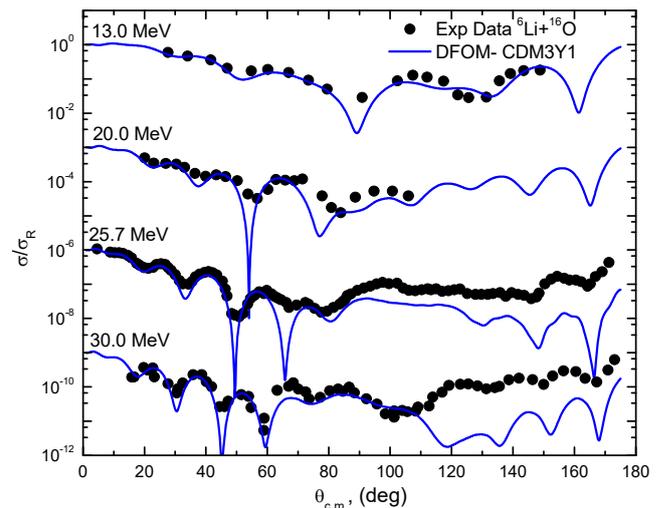

Fig. 5 Comparison between experimental angular distributions for ^{6}Li+^{16}O elastic scattering and the theoretical calculations using DFOM – CDM3Y1 at E_{lab}= 13.0, 20.0, 25.7, and 30.0 MeV

World Academy of Science, Engineering and Technology
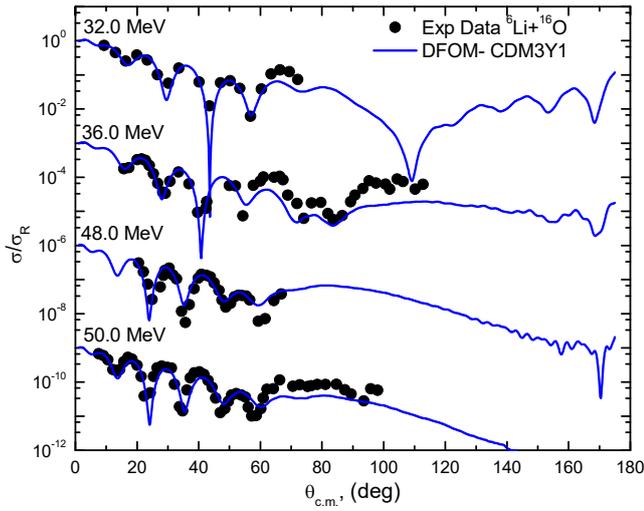International Journal of Nuclear and Quantum Engineering
Vol:14, No:12, 2020

Fig. 6 Same as Fig. 5 but at E_{lab}= 32.0, 36.0, 48.0, and 50.0 MeV

Fig. 7 Comparison between experimental angular distributions for ^{6}Li+^{16}O elastic scattering and the theoretical calculations using DFOM – CDM3Y2 at E_{lab}= 13.0, 20.0, 25.7, and 30.0 MeV

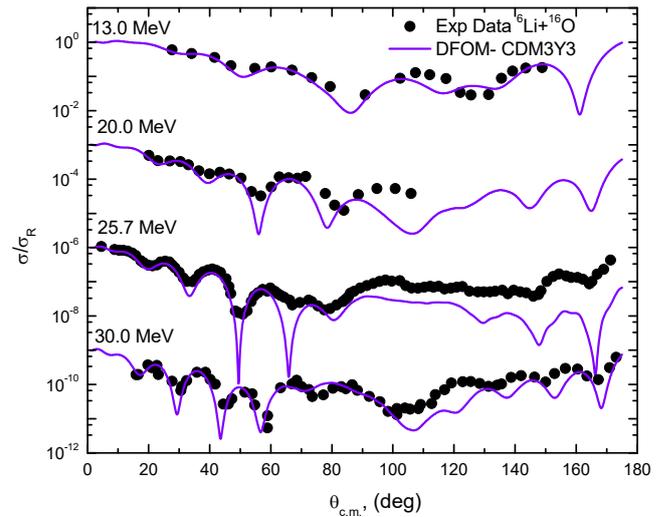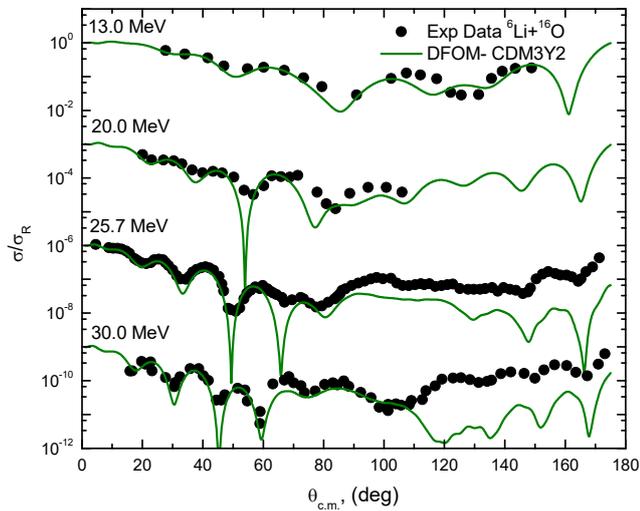Fig. 8 Same as Fig. 5 but at E_{lab}= 32.0, 36.0, 48.0, and 50.0 MeV

Fig. 9 Comparison between experimental angular distributions for ^{6}Li+^{16}O elastic scattering and the theoretical calculations using DFOM – CDM3Y3 at E_{lab}= 13.0, 20.0, 25.7, and 30.0 MeV
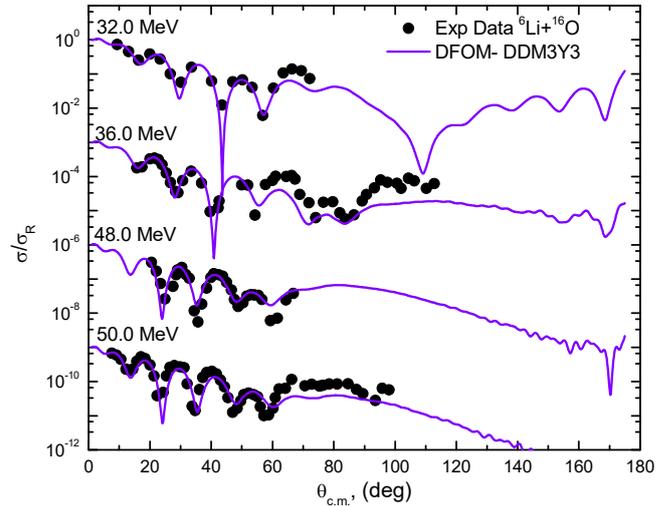

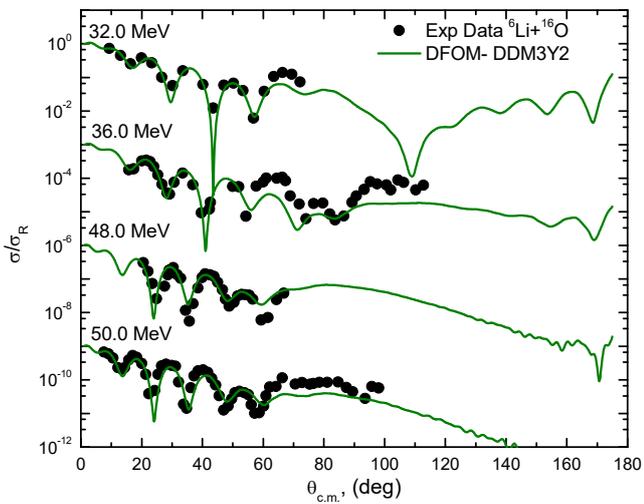Fig. 10 Same as Fig. 9 but at E_{lab}= 32.0, 36.0, 48.0, and 50.0 MeV

IV. SUMMARY

Experimental angular distributions for ^{6}Li+^{16}O elastic scattering in the energy range 13.0–50.0 MeV are investigated and analyzed using OM and DFOM. OM potentials consisting of real volume term and an imaginary volume term both has the phenomenological Woods-Saxon form could fairly reproduce the experimental data in the whole angular range. The radius parameters for real and imaginary parts of potential were fixed in order to obtain the energy dependence on real and imaginary potential depths and consequently reduce the ambiguities which could be associated to OM calculations. Then the optimal imaginary potential parameters extracted from OM are used in DFOM. In DFOM, the real part of potential was calculated on the basis of DF with different interaction models DDM3Y1, CDM3Y1, CDM3Y2, and CDM3Y3. The comparisons between experimental data and the theoretical calculations using the different concerned

World Academy of Science, Engineering and Technology
International Journal of Nuclear and Quantum Engineering
Vol:14, No:12, 2020

models are fairly good at forward hemisphere (angles $< 90^{\circ}$) with worse fitting at larger angles. Data analysis using DFOM of different interaction models showed that potentials needed to be reduced in strength by about 40%; this fact was also previously reported [7], [22] for various ^{6}Li+X nuclear systems. Choosing an appropriate interaction model with a high incompressibility constant could increase the value of N_R to be closer to unity.

References

[1] M. F. Vineyard, J. Cook, K. W. Kemper, and M. N. Stephens, Phys. Rev. C30 (1984) 916

[2] D. L. Gay, E. E. Bartosz, P. D. Cathers, D. D. Caussyn, T. L. Drummer, K. W. Kemper, P. L. Kerr, and D. Robson, Phys. Rev. C54 (1996) 3273

[3] D. E. Trcka, A. D. Frawley, K. W. Kemper, D. Robson, J. D. Fox, and E. G. Myers, Phys. Rev. C41 (1990) 2134

[4] C. L. Woods, B. A. Brown, and N. A. Jelley, J. Phys. G: Nucl. Phys 8 (1982) 1699

[5] A. S. Dem'yanova, J. M. Bang, F. A. Gareev, S. A. Goncharov, S. N. Ershov, A. A. Ogloblin and P. P. Korovin, Nucl. Phys. A501(1989) 336.

[6] L. T. Chua, F. D. Becchetti, J. Janecke and F. L. Milder, Nucl. Phys. A273 (1976) 243

[7] F. Carstoiu, L. Trache, R.E. Tribble, C.A. Gagliardi, Phys. Rev. C70 (2004) 054610

[8] H.G. Bingham, M.L. Halbert, D.C. Hensley, E. Newman, K.W. Kemper, L.A. Charlton, Phys. Rev. C11 (1975) 1913

[9] P. Schwandt, W. W. Jacobs, M. D. Kaitchuck, P. P. Singh, W. O. Ploughe, F. D. Becchetti, J. Janecke, Phys. Rev. C24 (1981) 1522

[10] K. Katori, T. Shimoda, T. Fukuda, S. Shimoura, A. Sakaguchi, M. Tanaka, T. Yamagata, N. Takahash, H. Ogata, M. Kamimura and Y. Sakuragi, Nucl. Phys. A480 (1988) 232

[11] J. Cook, H. J. Gils, H. Rebel, Z. Majka, H. Klewe-Nebenius, Nucl. Phys. A388 (1982) 173.

[12] A. Nadasen, M. McMaster, G. Gunderson, A. Judd, S. Villanueva, P. Schwandt J. van der Plicht, R. E. Warner, F. D. Becchetti and J. W. Janecke, Phys. Rev. C37 (1988) 132

[13] A. Nadasen, T. Stevens, J. Farhat, J. Brusoe, P. Schwandt, J.S. Winfield, G. Yoo N. Anantaraman, F. D. Becchetti, J. Brown, B. Hotz, J. W. Janecke, D.Roberts and R. E. Warner, Phys. Rev. C47 (1993) 647

[14] K. Schwarz, C. Samanta, M. Fujiwara, H. Rebel, R. De Leo, N. Matsuoka, H. Utsunomiya, H. Akimune, I. Daito, H. Fujimura, F. Ihara, K. Ishibashi, Y. Maeda, T. Yamanaka, H. Yoshida, A. Okihana, T. Yoshimura, P. K. J. van Aarle, W. A. T. Uijen, M. Ito, Y. Sakuragi, EPJA 7 (2000) 367

[15] J. E. Poling, E. Norbeck, and R. R. Carlson, Phys. Rev. C13, 648 (1976)

[16] K. Bethge, C. M. Fou, and R. W. Zurmuhle, Nucl. Phys. A123, 521 (1969)

[17] M. F. Vineyard, J. Cook, K. W. Kemper, and M. N. Stephens, Phys. Rev. C30, 916 (1984)

[18] V. V. Davydov, B. G. Novashkii, A. A. Ogloblin, S. B. Sakuta, D. N. Stepanov, V. I. Chuev, J, IZV 35 (11), 2399 (1971)

[19] N. Anantaraman, H. W. Fulbright, P. M. Stwertka, Phys. Rev. C22, 501 (1980)

[20] P. Schumacher, N. Ueta, H. H. Duhm, K.-I. Kubo, and W. J. Klages, Nucl. Phys. A212, 573 (1973)

[21] J. Cook, L. C. Dennis, K. W. Kemper, T. R. Ophel, A. F. Zeller, C. F. Maguire, and Z. Kui, Nucl. Phys. A415, 114 (1984)

[22] D. E. Trcka, A. D. Frawley, K. W. Kemper, D. Robson, J. D. Fox, and E. G. Myers, Phys. Rev. C41, 2134 (1990)

[23] R. M. DeVries, D. A. Goldberg, J. W. Watson, M. S. Zisman, J. G. Gramer, Phys. Rev. Lett. 39, 450 (1977)

[24] Mandira Sinha, Subinit Roy, P. Basu, H. Majumdar, S. Santra, V.V. Park, EPJ Web of Conf. 17, 03004 (2011)

[25] M. Hugi, J. Lang, R. Miller, E. Ungricht, K. Bodek, L. Jarczyk. B. Kam, Nucl. Phys. A368, 173 (1981)

[26] A. Nadasen, M. McMaster, M. Fingal, J. Tavormina, P. Schwandt, Phys. Rev. C39, 536 (1989)

[27] A. Pakou, N. Alamanos, A. Lagoyannis, A. Gillibert, E. C. Pollacco, Phys. Lett. B556, 21 (2003)

[28] J. E. Poling, E. Norbeck, and R. R. Carlson, Phys. Rev. C5, 1819 (1972)

[29] P. K. Bindal, K. Nagatani, M. J. Schneider, and P. D. Bond, Phys. Rev. C9, 2154 (1974)

[30] J. E. Poling, E. Norbeck, and R. R. Carlson, Phys. Rev. C13, 648 (1976)

[31] Sh. Hamada, B. Alshahrani, Abd Elrahman Elgamala, N. Darwish, I. Bondouk and Awad A. Ibraheem, Revista Mexicana de Fisica 66(3) (2020) 322

[32] Sh. Hamada and AwadA.Ibraheem, Int. J. Mod. Phys. E28 (2019) 1950108

[33] I. I. Gontchar and M. V. Chushnyakova, Comput. Phys. Commun. 181, 168 (2010)

[34] D. T. Khoa, G. R. Satchler and W. von Oertzen, Phys. Rev.C 56954 (1997)

[35] D. T. Khoa and W. von Oertzen, Phys. Lett.B 304 8 (1993)

[36] S. Qing-biao, F. Da-chun, Z. Yi-Zhong, Phys. Rev. C43, 2773 (1991)

[37] K.H. Bray et al., Nucl. Phys. A 189(1972) 35.

[38] I. J. Thompson, Comput. Phys. Rep.7, 167 (1988)