Research on Morning Commuting Behavior under Autonomous Vehicle Environment Based on Activity Method

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Abstract—Based on activity method, this paper focuses on morning commuting behavior when commuters travel with autonomous vehicles (AVs). Firstly, a net utility function of commuters is constructed by the activity utility of commuters at home, in car and at workplace, and the disutility of travel time cost and that of schedule delay cost. Then, this net utility function is applied to build an equilibrium model. Finally, under the assumption of constant marginal activity utility, the properties of equilibrium are analyzed. The results show that, in autonomous driving, the starting and ending time of morning peak and the number of commuters who arrive early and late at workplace are the same as those in manual driving. In automatic driving, however, the departure rate of arriving early at workplace is higher than that of manual driving, while the departure rate of arriving late is just the opposite. In addition, compared with manual driving, the departure time of arriving workplace on time is earlier and the number of people queuing at the bottleneck is larger in automatic driving. However, the net utility of commuters and the total net utility of system in automatic driving are greater than those in manual driving.

Keywords—Autonomous cars, bottleneck model, activity utility, user equilibrium.

I. INTRODUCTION

THE research of morning commute is a long-standing challenge in transportation research. In recent years, due to the rapid development of artificial intelligence and novel automobile technology, autonomous driving technology is becoming increasingly more mature. In urban traffic, AVs - a new element - are beginning to appear, which makes the situation of urban traffic more complicated. AVs are becoming to have an essential role in urban traffic. In the research of morning commute, therefore, AVs have received increasing attention. Compared with normal vehicles (NVs), AVs do not require drivers to drive in travel, which means drivers no longer need to take time for driving tasks, that is, during this time, drivers can do other activities such as rest, work or entertainment. Because of this advantage of AVs, individuals tend to choose it as a commuting tool instead of NVs, which may change the departure pattern of commuters during morning peak hours, and then affect the traffic congestion management strategies. Therefore, it is necessary to analyze the departure pattern and equilibrium nature for commuters in the autonomous driving environment during morning peak hours.

The analysis of morning commute can be traced back to the bottleneck model proposed by Vickrey (1969) [1]. This model assumed that a fixed number of individuals must travel from home to the workplace along a single road in morning peak hours. There is a bottleneck on the road with a fixed capacity. Due to the limited traffic capacity of this road, it cannot satisfy that all commuters can arrive at workplace on time; that is, some commuters will arrive early while others will arrive late. Therefore, the corresponding schedule delay costs will be incurred. Each commuter chooses his departure time to minimize his travel costs which are linear in travel time and schedule delay. Equilibrium is obtained when no commuter has an incentive to alter his departure time. After that, the problem of morning peak commuting was widely concerned, and a considerable number of extended studies of Vickrey’s model have been proposed. These studies mainly include user heterogeneity [2]-[4], randomness of bottlenecks [5]-[7] and mixed travel model [8]-[10]. Most of these studies were based on the assumption that travelers choosing the departure time only depend on travel time and scheduling delay. However, they did not consider the motivations and reasons for travel behavior. Thus, they cannot reflect the connection between travel and activities. These models, if used in traffic management, may make traffic managers to formulate inaccurate strategies and lead to more serious traffic problems [11], such as traffic congestion, unreasonable use of traffic infrastructure and air pollution. Therefore, activity utility must be introduced into the bottleneck model to research the problem of commuting in morning peak hours. Based on the traditional bottleneck model, Zhang et al. [12] introduced commuters’ activities of home and that of workplace to build an extended bottleneck model for characterizing commuters’ choice of departure time. A trade-off between the negative utility of travel costs and the positive utility of activities obtained at home and workplace by commuters was made to maximize their net utility. Li et al. [13] constructed an all-day activity-travel scheduling model based on activity method, and analyzed the equilibrium state of this model. The results showed that the traditional travel-based bottleneck model had a large deviation in estimation of commuters’ departure time decisions. Kim and Kwan [14] analyzed individuals’ activity travel data and real-time traffic congestion data and found that the traffic congestion risk was significantly underestimated if individuals’ activity utility was not considered. From the above...
research, it can be found that activity utility has a significant impact on commuters’ travel departure time choice.

There are a lot of relevant literatures about the commuting problem. However, the research is quite limited about the impact of AVs on the commuting behavior. Among them, Liu [15] compared the parking modes of AVs and that of NVs, and analyzed an equilibrium of commuter parking under the fully AV environment. Vincent et al. [16] considered the effect of AVs on capacity, value of time and preference, and analyzed the equilibrium under the coexistence of AVs and that of NVs. Yu et al. [17] considered the effect of in-vehicle utility of AVs on travel patterns, and analyzed the equilibrium under three AVs supply strategies. However, the above studies only unilaterally considered travel costs or activity utility, that is, they did not consider the impact of both on travel patterns at the same time.

Based on the bottleneck model and activity method, this paper analyzes morning commute under the fully AV environment. Firstly, an activity-based bottleneck model is proposed to model the departure time choices of commuters under the fully AV environment. Then the equilibrium properties are analyzed under the assumption of constant marginal activity utility. Finally, numerical illustrations are presented to verify the properties of the proposed model.

The remainder of this paper is organized as follows. Section II presents the departure time choice behavior of an activity-based bottleneck model under AV environment. In Section III, the properties of the proposed model are analyzed. A numerical experiment is applied to verify the propositions of the proposed model in Section IV. The conclusions and discussions are presented in Section V.

II. MODEL SET-UP

The network is schematically depicted in Fig. 1. As shown in Fig. 1, a bottleneck with a maximum service rate or capacity s is located between home and workplace. It is assumed that, every morning, N commuters drive AVs from home to workplace through the bottleneck. For simplicity, an autonomous vehicle environment (AVE) is that all individuals use AVs, while a normal vehicle environment (NVE) is that all individuals use NVs to commute. Under the AVE, activity utility of commuters is obtained at home, in the car and at the workplace, respectively. Under the NVE, however, the activity utility of commuters is obtained only at home and the workplace, respectively. Therefore, the commuting pattern in NVE can be regarded as a special case in AVE, that is, the commuting pattern in AVE and NVE is the same when the in-vehicle activity utility is zero. The starting and the ending time of commuters’ morning activities are at home and the arrival time of commuter at workplace, respectively. The period in the vehicle includes free-flow travel time and queuing time at the bottleneck, that is, the time commuters spend on the road.

![Fig. 1 Single bottleneck commuter network](image)

![Fig. 2 Activity duration diagram](image)

Let \( u_h(t) \), \( u_p(t) \) and \( u_w(t) \) be the marginal utility of commuters’ activities at home, in the car and at the workplace at time \( t \). Then, the total activity utility of commuters is obtained in the period \([0, \bar{T}]\) as:

\[
U(t) = \int_0^t u_h(t) \, dt + \int_t^{t+T(t)} u_p(t) \, dt + \int_{t+T(t)}^{\bar{T}} u_w(t) \, dt, \tag{1}
\]

where \( T(t) \) is the travel time from home to the workplace at time \( t \) which stands for the departure time from home. \( t + T(t) \) is the arrival time at workplace. The first item of (1) stands for the utility of commuters at home during the period \([0, t]\). The second item of (1) stands for the utility of commuters in vehicle during the period \([t, t + T(t)]\). The third item of (1) stands for the utility of commuters at work during the period \([t + T(t), \bar{T}]\). Travel time \( T(t) \) is defined as:

\[
T(t) = T^f + T^p(t), \tag{2}
\]

where \( T^f \) represents a constant for free-flow travel time from home to the workplace. \( T^p(t) \) represents the queuing time at bottleneck.

Let \( D(t) \) be the length of queue at bottleneck at departure time \( t \) from home, which is defined as:

\[
D(t) = \int_0^t r(u) \, du - s(t - \hat{t}), \tag{3}
\]

where \( \hat{t} \) stands for the moment when the queuing starts, and \( r(t) \) stands for the departure rate. From (3), the derivative of \( D(t) \) is obtained as:

\[
\dot{D}(t) = r(t) - s, D(t) > 0. \tag{4}
\]

where \( \dot{D}(t) \) is the derivative of \( D(t) \). From (4), the queuing time \( T^p(t) \) at bottleneck is obtained as:

\[
T^p(t) = \frac{D(t)}{s}. \tag{5}
\]

According to Vickrey’s model, the negative utility of commuters’ travel can be calculated as:

\[
C(t) = \alpha T(t) + \beta \max\{0,(t^* - t - T(t))\} + \gamma \max\{0,(t + T(t) - t^*)\}, \tag{6}
\]

where \( t^* \) represents the preferred work arrival time. Indeed, (6)
encapsulates the commuter travel time and the schedule delays for all commuters. Let \( \alpha \) be the shadow cost of travel time, \( \beta \) and \( \gamma \) are the schedule penalty for a unit time of early arrival and that of late arrival, respectively. According to the empirical results [18], it assumes that \( \gamma > \alpha > \beta > 0 \).

Following (1) and (6), the net utility of commuters can be represented as:
\[
\psi(t) = U(t) - C(t) = \int_0^t u_\beta(t) \, dt + \int_{t_T}^T u_\gamma(t) \, dt + \int_{t_T}^T u_\alpha(t) \, dt - (\alpha T(t) + \beta \max(0, t - T(t)) + \gamma \max(0, (t + T(t) - t)) \right)
\]

where \( t \) stands for the decision variable, and every commuter needs to trade-off his activity utility and travel negativity to choose the morning departure time for maximizing his net utility. Equilibrium is obtained when no commuter has an incentive to change his departure time.

Similar to the traditional bottleneck model, there are two cases at equilibrium: (i) commuter arrives at workplace earlier than \( t^* \); (ii) commuter arrives at workplace later than \( t^* \). Let \( t_q \) and \( t_q' \) be the departure time of the first commuter and that of the last one, respectively. \( t \) stands for the departure time from home to workplace on time, that is, \( t = t^* - T(t) \). Then we can deduce the departure rate in two cases, respectively.

In case 1, commuters arrive early at workplace. From (7), the net utility of commuters \( \psi(t) \) of early arrival is obtained as:
\[
\psi(t) = \int_0^t u_\beta(t) \, dt + \int_{t_T}^T u_\gamma(t) \, dt + \int_{t_T}^T u_\alpha(t) \, dt - \alpha T(t) - \beta (t^* - T(t)). \tag{8}
\]

At equilibrium, every commuter has the same net utility, that is, \( \frac{d\psi(t)}{dt} = 0 \). From (3), (5) and (8), the departure rate \( r_e(t) \) of early arrival is written as:
\[
r_e(t) = \frac{\alpha + u_\beta(t) - u_\gamma(t)}{\alpha + \beta - u_\beta(t + T(t)) + u_\alpha(t + T(t))} \cdot s \cdot t_q \leq t < \tilde{t}. \tag{9}
\]

In case 2, commuters arrive late at workplace. From (7), the net utility of commuters \( \psi(t) \) of late arrival is obtained as:
\[
\psi(t) = \int_0^t u_\beta(t) \, dt + \int_{t_T}^T u_\gamma(t) \, dt + \int_{t_T}^T u_\alpha(t) \, dt - \alpha T(t) - \gamma (t + T(t)) - \gamma'. \tag{10}
\]

Then the departure rate \( r_l(t) \) of late arrival is written as:
\[
r_l(t) = \frac{\alpha + u_\beta(t) - u_\gamma(t)}{\alpha + \gamma - u_\beta(t + T(t)) + u_\alpha(t + T(t))} \cdot s \cdot \tilde{t} \leq t \leq t_q'. \tag{11}
\]

According to the relationship between the bottleneck capacity and the departure rate, the following propositions can be obtained.

**Proposition 1.** At equilibrium, when \( \beta > u_\beta(t) - u_\gamma(t) - u_\alpha(t + T(t)) + u_\gamma(t + T(t)) \) is satisfied, a queue must exist at bottleneck, otherwise the queue will not exist at bottleneck.

**Proof.** According to (10) and (12) and \( \gamma > \alpha > \beta > 0 \), we have \( r_e > r_l \). When \( \beta > u_\beta(t) - u_\gamma(t) - u_\alpha(t + T(t)) + u_\gamma(t + T(t)) \), we have \( r_e > s \). So a queue exists at the bottleneck. However, when \( \beta \leq u_\beta(t) - u_\gamma(t) - u_\alpha(t + T(t)) + u_\gamma(t + T(t)) \), we have \( r_e < s \). Therefore, the departure rate in both cases is less than \( s \), the queue will not exist at the bottleneck.

In the following, if no special declaration is made, it is assumed that the condition \( \beta > u_\beta(t) - u_\gamma(t) - u_\alpha(t + T(t)) + u_\gamma(t + T(t)) \) is satisfied.

**III. EQUILIBRIUM PROPERTIES ANALYSIS**

This paper analyzes the equilibrium properties under the assumption of constant marginal-activity utility. Let \( u_h(x) = u_h, u_p(x) = u_p, u_w(x) = u_w \) and \( u_w > u_h > u_p \), where \( u_w > u_h \) means the activity utility at work is higher than that at home to ensure that commuters have the motivation to travel. \( u_w > u_p \) means when commuters arrive at the workplace, they are driven to work instead of staying in the vehicle. Then, the net utility of commuters is given by:
\[
\psi(t) = \begin{cases} 
(u_h - u_w) t + (u_p - u_w) \alpha t + u_w (\beta t^* - t) & t < \tilde{t} \\
(u_h - u_p) \gamma t + (u_p - u_w) \gamma T(t) + u_w T(t) + \gamma t^* & \tilde{t} \leq t \leq t_q'.
\end{cases}
\tag{12}
\]

and the departure rate of early arrive and that of late arrive are given by:
\[
r_e(t) = \frac{\alpha + u_h - u_w}{\alpha + \beta + u_w} s \cdot t_q \leq t < \tilde{t}, \tag{13}
\]

\[
r_l(t) = \frac{\alpha + u_h - u_p}{\alpha + \gamma + u_w} s \cdot \tilde{t} \leq t \leq t_q'. \tag{14}
\]

Based on Proposition 1, we have \( \beta > u_w - u_h \). Together with \( \psi(t_q) = \psi(t_q') \) (i.e. equal the net utility for the first commuter and last one) and \( (t_q - t_q') \cdot s = N \), we have:
\[
t_q = \frac{u_h - u_w - \gamma}{\beta + \gamma} s + t^* - T f, \tag{15}
\]

\[
t_q' = \frac{u_h - u_p - \beta}{\beta + \gamma} s + t^* - T f, \tag{16}
\]

\[
\tilde{t} = \frac{(u_h - u_p + \beta)(u_h - u_w - \gamma)}{\beta + \gamma} s + t^* - T f. \tag{17}
\]

Based on \( r_e \) and \( r_l \), the length of queue is determined at time \( t \). Particularly, the length of queue \( D(t) \) can be written as:
\[
D(t) = \frac{u_h - u_w - \beta}{\alpha + \beta + u_w} s (t - t_q) \leq t < \tilde{t}
\]

\[
\frac{u_h - u_w - \gamma}{\beta + \gamma} s + \frac{u_h - u_p - \gamma}{\beta + \gamma} s (t - \tilde{t}) \leq t \leq t_q'. \tag{18}
\]

The user net utility \( \psi(t) \) and system total net utility \( T\psi \) are determined based on the starting and ending time, which are:
\[
\psi(t) = \frac{(u_h - u_w + \beta)(u_h - u_w - \gamma)}{\beta + \gamma} s + (u_h - u_p) t^* - (u_h - u_p + \alpha) T f + u_w \tag{19}
\]
\[
T\phi = \frac{(u_h-u_v+\beta)(u_h-u_v-\gamma)}{(\beta+\gamma)} N s + \left[ (u_h-u_v)T^* - (u_h-u_v+a)T' + u_v T^2 \right] N.
\]

Besides, the number of early arrival commuters and that of late arrival commuters can be determined based on (13)-(17), which are:

\[
N_e = \frac{u_v + \gamma - u_h}{\beta + \gamma} N
\]

\[
N_i = \frac{u_h - u_v + \beta}{\beta + \gamma} N
\]

The solutions of the departure rates under AVE and NVE are depicted in Fig. 3, where \(T'(t), D'(t)\) and \(T'\) represent queuing time, length of queue and departure time to arrive at work on time under NVE, respectively. Based on Fig. 3 and (13)-(22), the following propositions can be obtained:

**Proposition 2.** Compared with NVE, the utility of in-vehicle activities under AVE has no effect on \(T_q\) and \(T_{q'}\), and \(N_e\) and \(N_i\) under AVE.

**Proposition 3.** Compared with NVE, under AVE, the \(r_e\) becomes larger, while the \(r_i\) becomes smaller. In addition, commuters need to leave in advance to arrive at work on time. These results are consistent with Proposition 2. We also notice that \(r_e\) under AVE is greater than that under NVE, while \(r_i\) under AVE is less than that under NVE. Moreover, commuters need to travel 3 minutes in advance under AVE to arrive workplace on time compared with that under NVE, verifying Proposition 3.

**Proposition 4.** Compared with NVE, the utility of in-vehicle activities under AVE increases the length of queue, meanwhile increases the net utility of commuters and total net utility of the system.

### IV. NUMERICAL STUDY

In this section, a numerical experiment is used to illustrate the properties of the proposed model. In this paper, the parameters are derived from [19], and some adjustments are made. The input parameters of the proposed model are listed in Table I. This application assumes that a fixed number of commuters who depart from home to work is 12000, that is, \(N = 12000\), and the capacity of bottleneck is 12000 vehicles per hour. Therefore, the duration of the morning peak is 1 hour. In addition, we set \(T^* = 8:00\), \(T' = 0.4\) hours and \(T = 12:00\). For simplicity, it is assumed that the bottleneck is located near the workplace, that is, the commuter leaving the bottleneck arrives at the workplace at the same time.

| TABLE I MODEL INPUT PARAMETERS |
|-------------------------------|------------------|-----------------|-----------------|-----------------|
| \(N\) (veh)                  | \(s\) (veh/h)    | \(T'\) (h)      | \(a\) ($/h)    | \(\beta\) ($/h) |
| 12000                        | 12000            | 0.4             | 10              | 6               |
| \(\gamma\) ($/h)            | \(u_v\) ($/h)   | \(u_e\) ($/h)  | \(u_w\) ($/h)  |
| 14                            | 10               | 6               | 12              |

**A. Analysis of Commuters’ Departure Pattern**

Commuters’ departure pattern is analyzed as follows. Fig. 4 shows the cumulative departure individuals under the AVE (\(u_v = 6\)) and NVE (\(u_v = 0\)) in the domain of \((t_q, t_{q'})\), respectively. The solid line represents the departure rate curve under AVE, and the dashed line represents the departure rate curve under NVE. As can be seen in Fig. 4, \(t_q\) and \(t_{q'}\) are the same in both environments (\(t_q = 06:42, t_{q'} = 07:42\)) which indicates that in-vehicle activities has no effect on \(t_q\) and \(t_{q'}\). In addition the number of individuals \((N_e = 10800)\) who arrive early is the same, and that of individuals \((N_i = 1200)\) who arrive late is also the same in both environments.

**B. Analysis of the Queue Length and the Net Utility**

The length of queue and the net utility are displayed in Fig. 5. It shows how the queue length vary with the time under AVE and NVE, respectively. As can be seen in Fig. 5, the queue starting time (at 07:06) and the queue ending time (at 08:06) are the same in both environments. The time to reach the maximum queue length, however, is different. In AVE, the length of queue increases linearly during the period 07:06-07:51, and the maximum queue length (1800 individuals) is reached at 07:51. After that, the queue length decreases linearly, and the queue disappears at 08:06. In NVE, however, the length of queue increases linearly during the period 07:06-07:54, and the
maximum queue length (1200 individuals) is reached at 07:54.
After that, the queue length decreases linearly, and the queue also disappears at 08:06. Moreover, the length of queue in AVE is larger than that in NVE at each moment, indicating that in-vehicle activities will increase the queuing length at bottleneck. These results are consistent with Proposition 3.

In the future, this paper can be extended from the following aspects: (i) We assume that all vehicles are AVs. However, a mixed vehicle environment which commuters use AVs or NVs to commute will exist for a period. So, it is necessary to extend the model to capture the travel patterns of the mixed vehicle environment. (ii) We assume that the activity utility is a constant. In the future research, a more complex utility function needs to be considered to replace the constant utility function. (iii) We only analyze the propositions of user equilibrium, and do not give the corresponding congestion management strategies. In the subsequent research, the congestion toll and tradable credit scheme will be studied based on the equilibrium model.

V. CONCLUSIONS AND DISCUSSION

In this study, based on the activity method, the equilibrium trip scheduling is analyzed when commuters travel with AVs. Different from commuters who use NVs, commuters who use AVs can get in-vehicle activity utility during travel. Therefore, the in-vehicle activity utility has to be considered in modeling the commuter travel pattern. In the proposed model, we firstly present an activity-travel scheduling model under AVE by using an activity-based approach. Then, the properties of the equilibrium model under AVE are analytically explored and compared with those under NVE. Finally, a numerical experiment is performed to verify the propositions. The findings demonstrate that the user equilibrium travel pattern in AVE is different from that in NVE. In particular, the in-vehicle activities in AVE change the departure rate, the length of queue, the queuing time and the net utility for individuals and system. This analysis has shed light on future traffic management with AVs.

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