# Probabilistic Approach of Dealing with Uncertainties in Distributed Constraint Optimization Problems and Situation Awareness for Multi-agent Systems

Sagir M. Yusuf, Chris Baber



***Abstract*****—**In this paper, we describe how Bayesian inferential reasoning will contributes in obtaining a well-satisfied prediction for Distributed Constraint Optimization Problems (DCOPs) with uncertainties. We also demonstrate how DCOPs could be merged to multi-agent knowledge understand and prediction (i.e. Situation Awareness). The DCOPs functions were merged with Bayesian Belief Network (BBN) in the form of situation, awareness, and utility nodes. We describe how the uncertainties can be represented to the BBN and make an effective prediction using the expectation-maximization algorithm or conjugate gradient descent algorithm. The idea of variable prediction using Bayesian inference may reduce the number of variables in agents' sampling domain and also allow missing variables estimations. Experiment results proved that the BBN perform compelling predictions with samples containing uncertainties than the perfect samples. That is, Bayesian inference can help in handling uncertainties and dynamism of DCOPs, which is the current issue in the DCOPs community. We show how Bayesian inference could be formalized with Distributed Situation Awareness (DSA) using uncertain and missing agents' data. The whole framework was tested on multi-UAV mission for forest fire searching. Future work focuses on augmenting existing architecture to deal with dynamic DCOPs algorithms and multi-agent information merging.

***Keywords*****—**DCOP, multi-agent reasoning, Bayesian reasoning, swarm intelligence.

## I. Introduction

DCOPs involve tasking agents to effectively assign variables to themselves under constraints in order to minimize/maximize costs [1]-[4]. DCOP was applied in solving many real-world problems such as the allocation of patients to doctors [5], [6], management of sensors to utilize their energy [7], multi-agent searching [8], mobile phone control [9], etc. Works in the literature focus attention on homogeneous agents operating in a static environment [1], [10]-[12]. In this paper, we are trying to understand the issues concerning formatting DCOP with uncertainties in a highly changing environment.

Uncertainty in data means the lack of assurance in the data values which could be solved through prediction and estimation of the variables [13], [14]. Agents need to utilize the data at hand to make decisions that support collaborative behaviours and productivity maximization [15]-[17]. The prediction made has to be good enough to support the optimization of the global cost functions and agents' collaborative behaviours. We draw an analogy between the need to predict future states (based on a history of previous states) and the concept of Situation Awareness from human factors. This views situation awareness as a process of attention management in which the environment is sampled to create a 'model' of what is happening and why. A variation of this concept is DSA of [18], [19] which assumes that the model is spread across agents, and can best be thought of as a system-level, emergent property of a collection of agents. A potential benefit of this approach that DSA can arise with minimal communication of agents (which multi-agent system could have an energy cost in supporting commons). A potential problem could arise from the partial form of views held by each agent. To date, the concept of DSA has not been formally described. So we use the BBN to characterise DSA and relate this to DCOP, its dynamism, and uncertainties.

Probabilistic inferential reasoning provides a statistical way of predicting variables from observed data [20]. The question here is: in the multi-agent system, how sure are we that the predictions made are right and within target? It makes the problem to be questioning the probability of occurrence of the predicted probabilities. Bayesian reasoning uses conditional probabilities given by (1) [21]:

$$P(X_i(t)|Y(t)) = \frac{P(X_i(t)) * P(Y(t)|X_i(t))}{\sum_i^n P(X_i(t)) * P(Y(t)|X_i(t))} \quad (1)$$

where $X_1(t)$, $X_2(t)$, $X_3(t)$, ..., $X_n(t)$ is the set of mutually exclusive events at a given time. Therefore from the probability of an event occurrence, one can predict other event occurrence probabilities (1). The agents' knowledge can be represented using the BBN. BBN is a popular tool for modelling variables and their causal relationship [16]. It consists of a Directed Acyclic Graph and a conditional probability table expressing the causal relationship. The directed arrow from the parent node to a child means the parent node causes the child node. Fig. 1 describes how Bayesian inferential reasoning can be applied to DCOP.

Fig. 1 describes the merging of DCOP to Bayesian inferential reasoning in multi-agent systems. Cost functions of DCOP can be mapped to utility nodes of the BBN. The situation of the agents can be model as the BBN traditional node (nature node), and the awareness can be BBN's decision

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nodes. Luckily enough, the uncertainties in DCOP (missing data or data with doubts) are accepted by BBN and Bayesian learning algorithms (expectation-maximization and gradient descent algorithm) with promising output.

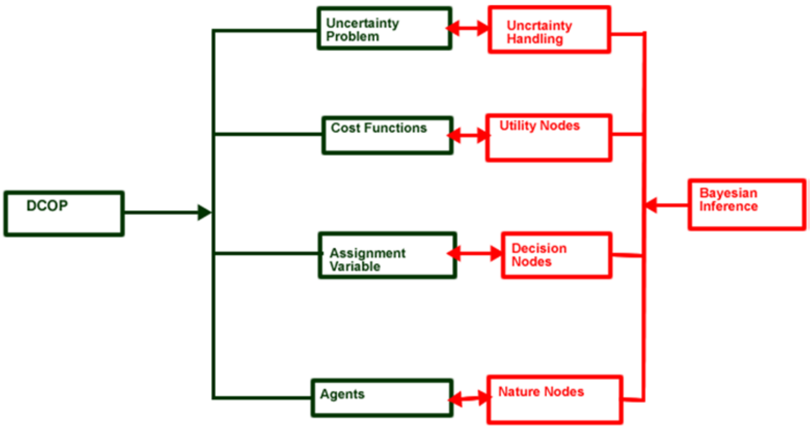


Fig. 1 DCOP and Bayesian Inference Coupling

In this paper, we modelled DCOP in a dynamic and uncertain situation using Bayesian inferential reasoning. Uncertainties and dynamism in the DCOP are handled by Bayesian learning algorithms to make perfect predictions for agents' situation-awareness. Therefore we use Bayesian inference to formalise multi-agents Situation Awareness in DSA and pose that as DCOP.

## II. Background

### A. DCOP

DCOP involves the efficient assignment of variables to agents in order to optimize global cost functions. It can be described as the tuple S [1], [10]:

$$S = \{A, V, D, C, \alpha\} \tag{2}$$

where A is the set of agents; V is the set of variables for agents; D is the variable domain; C is the set of cost functions to be optimized; α is the function for the assignment of variables to agents. Based on environmental behaviours, DCOP can exist in different forms such as dynamic, multi-objective, probabilistic DCOPs, etc. [1], [10], [22]. Multi-objective DCOP optimizes a set of varying cost functions while probabilistic DCOP uses probability distributions to model environmental changes [1]-[3]. DCOPs have various real-world applications such as sensor scheduling [7], UAV planning [8], patient scheduling [6], disaster management [1], [23], [24], supervision allocation, and so on. While there are many algorithms for solving DCOP, ongoing challenges relate to solving DCOP in dynamic and uncertain environments [8], [10]-[13], [25], [26]. Our work is to understand how the introduction of that uncertainty affects the fitting of Bayesian inference with DCOP.

### B. Bayesian Inference

Bayesian inference provides the statistical way of making an inference from the available data (prior values) to predict future values of related variables. A clear introduction and example on Bayesian inference were discussed in Section I (Fig. 1). BBN provides a graphical way of modelling nodes and their causal relationships [16], [17]. BBN is an acyclic graph G(V, E). V={v1,v2,v3,..vn} E = {e1,e2,..,en} V is the set of nodes and E are the directed edges in the graph [20], [27]. It was applied in modelling uncertainties in many areas such as forensic science as mean of making logical decisions using the little records at hand [16], human inference grading [28], [29], data mining [30], [31], logical reasoning [17], multi-agent sensor conflict resolution [32], and DCOP [3]. This paper aimed at formatting the Bayesian inferential reasoning to handle DCOP's dynamism and uncertainties in multi-agent coordination.

### C. Fitting Bayesian Inference with DCOP

To extend the application of Bayesian inference to DCOP algorithms and DSA, our BBN will consist of three types of nodes, i.e., nature (situation), utility, and decision (awareness) nodes. Situation nodes (traditional BBN nature nodes) represent ordinary variables or agents. Utility nodes describe how happy the system is with the predicted value (i.e., costs optimization). Decision (awareness) nodes represent the choice of our inference (Fig. 1). Let us give a clear example of merging DCOP and Bayesian inferential reasoning. Assume a team (aerial and ground robots) with different sensor profiles and capacity tasked to conduct wildfire search. The agents need to monitor their belief in the presence of fire and other exogenous variables (wind speed, wind direction, etc.), which can be the situation of agents modelled as ordinary nodes. The agents' awareness is the decision on such variables, for example, to regenerate waypoint for search, to go and recharge, etc., which is the decision nodes for the modelled BBN. The agents are needed to optimize the mission cost (e.g., energy, time, communication link, etc.), which can be modelled as the BBN's utility node (how happy the agents are with their decision. Fig. 2 describes a sample BBN for the agents.

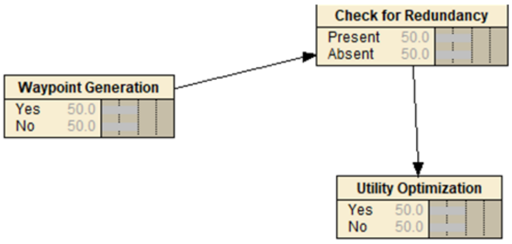


Fig. 2 BBN with Utility and Decisions

From Fig. 2, the BBN node labelled “Waypoint generation" is an awareness node (decision node), which is to decide that the agent needs to regenerate another waypoint after visiting the current waypoint. The node labelled "Check for Redundancy" is a situation node (nature node) to check the current condition of the agents' network, whether there are too close (redundant) waypoints or not. If the redundancy exists for sure, the agents' team will not be happy with the optimization (modelled as the utility node labelled “Utility Optimization”) as repeated searching consumes energy, time, communications links etc. Each node has a conditional probability table (CPT) to be by the agents or experts for making future prediction purposes.

As discussed earlier, DCOP agents have global cost functions to be optimized over time. From (2), the cost function C can be assigned as the utility node and their assignment decision function α is the decision (awareness) nodes. Therefore the BBN for DCOP is in the form of the acyclic graph.

$$G(V_{s,u,a},E). \qquad \text{For } s\neq u\neq a \tag{3}$$

The variables s,u, a correspond to situation, utility, and awareness nodes, respectively. In the case of dynamic, multi-objective, and probabilistic DCOPs, it could be modelled as the tuple $S_t$.

$$S_t=(R^t,V^t,\psi^t,\vec{C}^t,\Upsilon^t,\Delta^t,\delta^t,P^t). \tag{4}$$

where R is the set of agents. $V^t = \{v^t_1,v^t_2, v^t_3, \ldots, v^t_n\}$ are the set variables for allocation. $\psi^t$ is the domain for $V^t$. $\vec{C}^t = [c^t_1,c^t_2,c^t_3,\ldots,c^t_n]^T$ set of conflicting cost functions to minimize or maximized at a given time base on the current task (they correspond to utility nodes in the BBN). $\Upsilon^t$: $V^t\backslash\Delta^t \rightarrow R^t_i$ function for the variables assignments by considering uncertainties (they correspond to the decision nodes). $\Delta^t$ is a set of uncertainties formulated in the BBN. $P^t$ is the probability distribution for the uncertainties in the variable (i.e., uncertainties in $\Delta^t$). Therefore the BBN task is to find the solution $\Phi^{t+1}$ over time by applying (1) for each agent mission data

$$\Phi^{t+1}:=\operatorname{argmin}/\max_{\Phi 1,\Phi 2,\Phi 3,\ldots,\Phi T}\ \varphi\left[\sum_{t=0}^{T}\sum_{ci}\tau(\vec{C}_i(\Phi^t_{V_i}\backslash\Delta))\right]^T \tag{5}$$

Uncertainties in the agents’ beliefs (data) can be as a result of missing data or soft findings [33]. Soft findings are data that contain some degree of uncertainties due to unreliability of the source or sensor fault. We can present the uncertainty degree base on the level of assurance in one of the following ways [33]:

(a) Restricted or unrestricted range: in this approach, we can give ranges of DCOP values to the BBN. For example, temperature = [$25^0$-$30^0$], i.e., the value of the temperature is between $25^0$-$30^0$ or temperature > $30^0$.
(b) Possibility or impossibility list: Setting a list for the possible values or negating the list to show impossibilities in these values. For example temperature = {$20^0$, $25^0$, $30^0$}.
(c) Likelihood: the set of probabilities can be attached to the possible variables in restricted or unrestricted form. For instance, temperature = {$20^0$ .8, $25^0$ .1+-1, $30^0$ .1}
(d) Complete or incomplete certainty: It happens when the BBN has full uncertainty by setting the variable as unknown, or it has no doubt on the variable by providing its value to the BBN. Figure 3 describes a perfect DCOP and Bayesian inference mapping composites mapping.

As described in Fig. 1, by providing the data to the BBN, the network can solve (3) using the conjugate gradient algorithm or expectation maximization algorithms [33]. The expectation-maximization algorithm provides an optimal solution in two steps (i) using (1) to compute the expected value of soft findings and missing data (ii) using the feed data to maximize the utilities [34], [35]. Conjugate gradient descent set an objective function by negating the log-likelihood of the network’s prediction and changing its parameters to know the way the steepness of the objective function (negated likelihood) is changing [21]. The choice of the algorithm depends on the resources and type of data at hand. Therefore having our dynamic, uncertain, multi-objective, or probabilistic DCOP, we can solve it through the use of this algorithm. Fig. 3 can be used in fitting any DCOP with Bayesian inference.

Fig. 3 describes the procedure of merging DCOP elements to BBN in a multi-agent environment pose as DSA. Agents can use the learned BBN in making decisions and utility optimization (because the agents can predict the future and avoid non-utilities optimization tasks). It can also be used as an input to other DCOP algorithms in [1], [2], [4], [10]-[12], [24]-[26], [36] as it handles uncertainties, missing data, and dynamism.

## III. Related Work

DCOP is one of the areas that are dragging the attention of multi-agent researchers due to its variety of applications. Constraint Satisfaction Problem (CSP) is the early multi-agent optimization problem. In CSP, agents are limited within constraint and task for assigning their variables [1], [37]-[41]. Distributed Constraint Satisfaction Problem (DisCSP) allows the agents to be independent and act within local capacities [39], [42], [43]. CSP and DisCSP aimed at the variable assignments under constraint while DCOP is to find the best assignment [44]. Different algorithms were developed to solve DCOP in dynamic, multi-objective, probabilistic, and classical DCOP. The ongoing challenge is formulating DCOP in dynamic and uncertain forms [1], [2], [10]-[12].

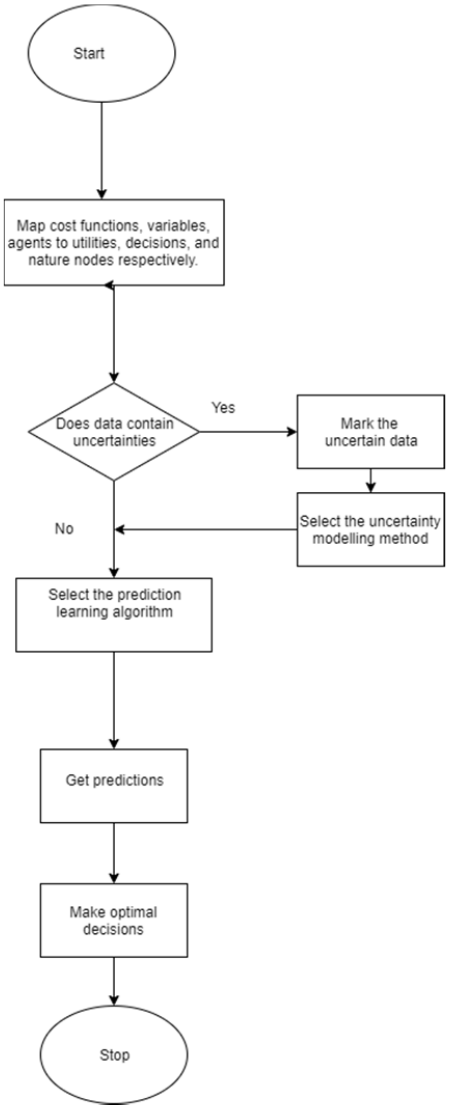


Fig. 3 Flowchart for mapping DCOP with Bayesian Reasoning

Reference [44] describes an algorithm for solving DCOP based on Distributed Pseudotree Optimization Procedure (DPOP). The algorithm arranges the agents in a tree structure; children of every node send their utilities for optimization to their respective parent. The parents are responsible for optimal computing values for their offspring. Reference [3] augmented the DPOP algorithm by introducing Bayesian DPOP (B-DPOP). In B-DPOP, after agents arranged themselves in pseudo-tree form, the agents use Bayesian optimization on their cost functions before forwarding their updated values to their agents. Reference [1] introduces a multi-thread variable optimization on DCOP using DPOP. Agents optimize multiple variables by applying parallel execution. The algorithms improve DCOP performance by allowing agents to handle more than one variable at a time

In the Distributed Breakout Algorithm [45], agents exchange costs with neighbouring agents and change their values randomly if these fail to meet criteria. Maximum Gain Message and Stochastic Coordination Algorithms [25], [26], agents start with assigning random variables to themselves and informing their neighbours about the costs. Agents with efficient cost (i.e., below the fixed threshold) are tasked with the optimization of the other agents' values. In the extension algorithms, agents are divided into leaders and followers according to the optimality of their values. Reference [16] describes a probability adjusting way of making a conclusion from the available crime evidence and prioritizing them. The priority came from the belief in the data saliency.

In the above-mentioned kinds of literature, Bayesian inferential reasoning provides a suitable way of concluding from the little data at hand under severe constraints and uncertainty. This paper wants to show how dynamic and uncertainties of DCOP will fit inferential reasoning. That is, how to extend DCOPs algorithm to handle uncertainties, improve agents' situation awareness, and describe a formal relationship between DCOP and Bayesian inference.

## IV. Experimental Results

Considering our BBN in Fig. 2, we generated training data using Netica simulation API [33]. We monitor the perfection of the prediction of the learning algorithms (expectation-maximization and gradient descent) with and without uncertainties, and also by dividing the number of samples into two portions. We then feed these values to conjugate gradient descent and the expectation-maximization algorithms. The performance of both algorithms seems to be the same. We keep multiplying the number of samples. In each sample generated, we test 0%, 25%, and 50% (i.e., of the nodes contain uncertain data) uncertain samples and monitor the prediction error. Figs. 4 and 5 describe the effect of the number of data with uncertainties in the sample.

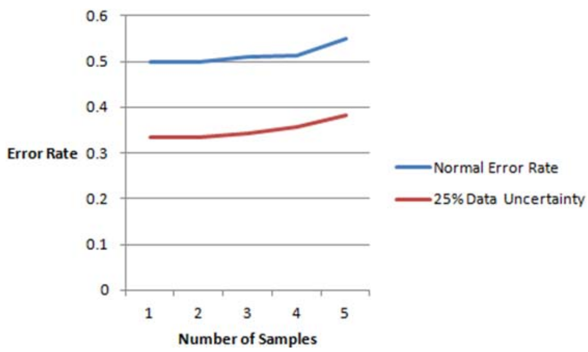


Fig. 4 Normal Data Error Rate versus 25% Data Uncertainty

From Figs. 4 and 5, we generated the number of the sample from $10^2$, $10^3$, $10^4$, $10^5$, and $10^6$. We made the system to be predicting the node "Redundant search occurrence" (from Fig. 2). The error rate is the perfection of the prediction from 0 to 1, with 0 is the best [33]. Surprisingly, if the sample contains 25% uncertain data, the algorithms (both expectation-maximization and conjugate gradient descent) make good predictions than providing with the normal data (i.e., data with no uncertain samples) without uncertainty as described in

Figs. 4 and 5. However, if the whole entry of a state contains a full uncertain sample, the prediction generates a high prediction error. Therefore the algorithms work perfectly by mixing the uncertainties in the DCOP data; that is, it has to be spread across the nodes' entries. The outcome of the training algorithms is a learned network (of Fig. 2) that can be used by the agents for prediction of event occurrence and utility optimization. Therefore in a multi-agent mission modelled as DCOP, the agents will be using the learned network to be predicting the outcome of their future action's outcomes on their utility optimization. When the environment is changing so frequently, the learning process needs to prioritise recent cases over old cases.

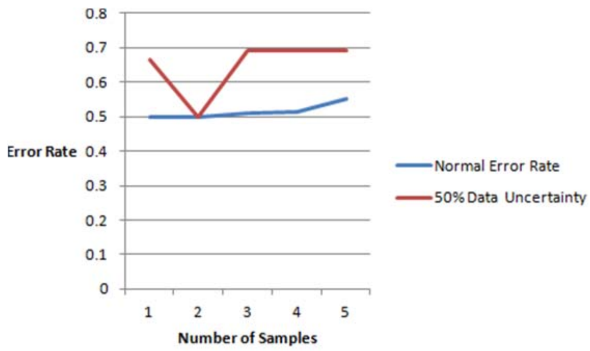


Fig. 5 Normal Data Error Rate versus 50% Data Uncertainty

## V. Conclusion

We describe how Bayesian inferential reasoning could be applied in the DCOP problem with uncertainties. The BBN merges the respective DCOP entries to its decision (awareness), utility, and situation (traditional BBN nature) nodes and applies gradient descent or expectation-maximization algorithm in making optimal predictions by considering the uncertainties of the data. We set a simulation experiment on Netica and generate random test cases in order to test the prediction perfection. Amazingly, experiment results show that optimal predictions were possible in the presence of uncertainty in data than without uncertainties in both algorithms (gradient descent and expectation-maximization). Results show that, for optimal results, the uncertainties have to spread across both BBN nodes options. Therefore, we argue that Bayesian inferential reasoning can be extended to DCOP and multi-agent situation-awareness with uncertainties. We also claim that Bayesian inferential reasoning performs better predictions with the uncertain data because it gives the learning algorithms a wider range for prediction, than with perfect samples (i.e. samples with certain data). Thus, Bayesian inferential reasoning can solve the uncertainties in DCOP and formalise DSA The output of the learned network can also be used for multi-agents' future situation-awareness predictions and utility optimizations. This is a continuation of our work in [13] by introducing priority based Bayesian learning and more formal coupling between Situation Awareness, DCOP, and Bayesian reasoning and learning.

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