

# Modeling Exponential Growth Activity Using Technology: A Research with Bachelor of Business Administration Students

V. Vargas-Alejo, L. E. Montero-Moguel

**Abstract**—Understanding the concept of function has been important in mathematics education for many years. In this study, the models built by a group of five business administration and accounting undergraduate students when carrying out a population growth activity are analyzed. The theoretical framework is the Models and Modeling Perspective. The results show how the students included tables, graphics, and algebraic representations in their models. Using technology was useful to interpret, describe, and predict the situation. The first model, the students built to describe the situation, was linear. After that, they modified and refined their ways of thinking; finally, they created exponential growth. Modeling the activity was useful to deep on mathematical concepts such as covariation, rate of change, and exponential function also to differentiate between linear and exponential growth.

**Keywords**—Covariation reasoning, exponential function, modeling, representations.

## I. INTRODUCTION

UNDERSTANDING the mathematical concept of function requires that the students develop their covariational reasoning [1]. References [2] and [3] mention that to develop a deep understanding of mathematical ideas, students must be familiar with and fluent in multiple representations (including those constructed by technology) and must acquire the ability to move from one to another representation. Teachers should promote learning of mathematics through the use of learning environments that include authentic, realistic and pertinent tasks or problems. Research based on the Models and Modeling Perspective [MMP] has shown that the use of Models Eliciting Activities [MEAs] can help students to understand mathematical concepts as function, in particular, exponential function [4]. The students can revise, modify, expand, and refine their knowledge.

In this article, the following research questions are answered: What models do management and accounting students build to solve the Population growth activity? What representations and concepts do they use? It is important to mention that part of the results presented here was published in the proceedings of an international event [5]. In this article, the models created by the students are presented, as well as the way in which the six principles proposed for the design of the MEA [6] were used to analyze the activity.

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## II. THEORETICAL FRAMEWORK

The theoretical framework used was the MMP [7], [2]. Learning mathematics is a process of developing conceptual systems, which change continuously, are modified, extended and refined based on the student's interactions with their environment (teachers and peers), and when solving problems [2]. The MMP proposes to structure experiences for the students, in which they can express, test and refine their ways of thinking and develop significant mathematical constructions [3]. The MMP suggests the use of MEAs in the classroom to encourage that the student manipulates, shares, modifies and reuses conceptual tools, to build, describe, explain, manipulate, predict or control mathematically significant systems [3]. During the modelling cycles involved in working MEAs, students are repeatedly revising or refining their conception of the given problem [8].

MEAs encourage students to develop models that include iterative design cycles similar to those that occur in everyday life, or in their professional lives. They enable students to carry out mathematization processes, since resolution involves students quantifying, dimensioning, coordinating, categorizing, symbolizing algebraically and systematizing relevant objects, relationships, actions, patterns and regularities [3]. Technology plays an important role because of its potential for building representations.

Six principles are proposed [6] for the design of MEAs: personal meaningfulness (reality principle), model construction, self-evaluation, model externalization (model documentation principle), simple prototype, and model generalization. The MMP was the theoretical framework used for the design of the activity, the implementation and the analysis of the collected data. The principles were used for the design of the GP activity.

## III. METHODOLOGY

The methodology was qualitative. The participants were five undergraduate students (adults immersed in the labor field) who were studying Mathematics applied to business in the first semester of the Bachelor of Business Administration and Bachelor of Accounting. The activity was implemented in a technological environment. The session took place in a computer room where each student had access to one computer. The students worked individually, in teams, and in the whole group. Team 1 included three students (S1, S2, S3), and Team 2 two students (S4, S5). S1, S2, and S5 were

undergraduate students in Administration and S3 and S4 were undergraduate students in Accounting. S1 was the highest-scoring student in the group (in traditional classes) and S4 was the lowest-scoring student; therefore, S4 participated less in math classes. She worked as an insurance saleswoman overtime. The ages of the students were between 24 and 34 years old. The PG activity, called Population Growth in the metropolitan area (Figs. 1-3), was designed based on the increased vehicular traffic problem derived from population growth in Guadalajara metropolitan area. The data were extracted from government sources [9]. Recursive tabular, tabular (functional relationship), graphical, and algebraic procedures can be used to solve the PG activity.

The PG activity was implemented in a period of three and a half hours, in two sessions. The phases were: 1) individual and group phase, to read the newspaper article, 2) individual, team, and group phase, to solve the problem and 3) individual phase, to solve a textbook exercise. The analysis criteria used to analyze the results obtained by students were the six principles, and the type of model constructed.

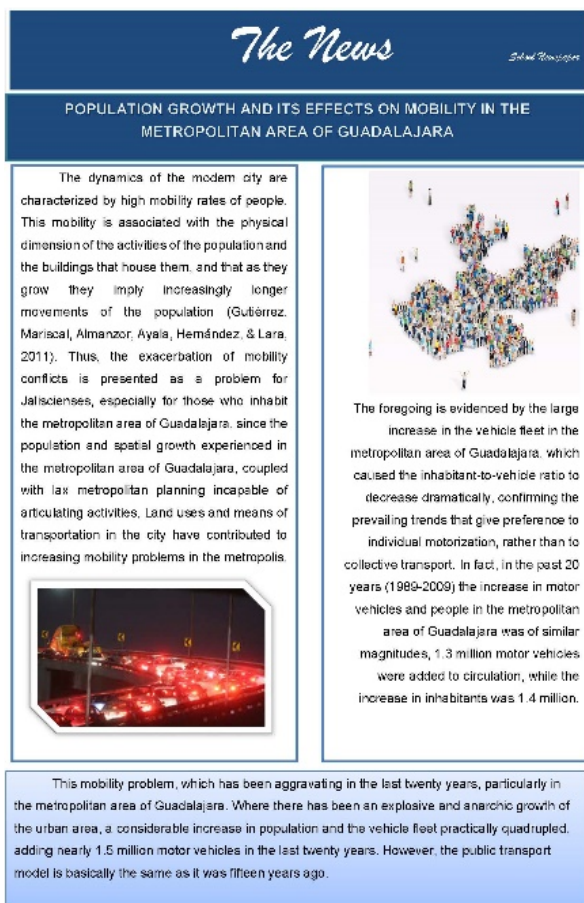


Fig. 1 Newspaper of the PG activity

The role of the teacher was facilitator. His interventions were to ask questions such as: Is the problem clear? Is it similar to those that you usually solve in your classes? What information is included? Why do you think this model is useful? Finally, he helped the students reflect on the

conclusions in the group session, and to build the algebraic representation with the aim of the generalization of the procedures in order to the students could use it in other different contexts.

**Answer and argue your answers**

- 1.- What effect does population growth produce in the metropolitan area of Guadalajara according to the journalistic note?
- 2.- What measures should the state government take so that the effects of population growth do not affect mobility problems as much?
- 3.- Since when has mobility problems worsened in the metropolitan area of Guadalajara?

Fig. 2 Warm up questions of PG activity

Santiago Vázquez, a graduate student of the management career of our campus UVM GDL SUR, is doing his Masters in Local Development and Territory. His teacher asked him to prepare a letter addressed to the Department of road infrastructure on the population growth of the metropolitan area of Guadalajara, so that it can be taken into account in the next road infrastructure projects. Santiago investigated and found that in 2018 the population of the metropolitan area of Guadalajara reached 4,299 million and that the average growth is 1.7% per year. For the letter to have the necessary impact in the Secretariat, Santiago needs to send a procedure that allows knowing what the population would be in the years 2020, 2022, 2024, 2030, 2040, 2041, 2100, expressing if the variation is constant and knowing in what year there will be 6 million inhabitants, 7,560 million, 8,232 million and for what year the population will double compared from 2018. Help Santiago write the letter. Describe the procedure in such a way that it can be useful to describe the population growth of any other city or the world.

Fig. 3 Problem situation of PG activity

**IV. RESULTS AND DISCUSSION OF RESULTS**

Based on the six principles for the design of MEAs [6], the models elaborated by students with the use of Excel are described. The potential of PG activity is analyzed in terms of the representations, conjectures, arguments, mathematical knowledge, and beliefs that emerged.

*A. The Personal Meaningfulness Principle*

The individual reading of the newspaper allowed the students to make sense of the problem [6]. They mentioned their concern on population growth, and its relationship with the roads of Guadalajara metropolitan area (this is denoted in one of the written letters, Fig. 6). For example, they indicated that there were many inhabitants in the city, public transport was insufficient, there were more and more deteriorated streets, and this would have consequences if the situation was not addressed.

*B. The Model Construction, Model Externalization and Model Self-Evaluation Principle*

**1. Initial Models**

Students S1, S2, and S3 (Team 1) used Excel to elaborate

procedures (Figs. 4 and 5) to answer the questions posed in the problem. Students S4, and S5 (Team 2) used calculators to perform operations on their notebook. Subsequently, they used Excel to build tabular (Fig. 6) and graph (Fig. 7) representations. While performing the procedures, the students continued with their concerns and reflections such as the following: What will happen to the roads when we are twice as many inhabitants?

A	B	C	D	E
4	año		anual	
5	2018	4.299	1.70%	0.073083
6	2019	4.372083	1.70%	0.07432541
7	2020	4.44640841	1.70%	0.07558894
8	2021	4.52199735	1.70%	0.07687396
9	2022	4.59887131	1.70%	0.07818081
10	2023	4.67705212	1.70%	0.07950989
11	2024	4.75656201	1.70%	0.08086155
12	2025	4.83742356	1.70%	0.0822362
13	2026	4.91965976	1.70%	0.08363422
14	2027	5.00329398	1.70%	0.085056
15	2028	5.08834998	1.70%	0.08650195
16	2029	5.17485193	1.70%	0.08797248
17	2030	5.26282441	1.70%	0.08946801

Fig. 4 Data table produced by team 1 (year=año)

A	B	C	D	E
4	año		anual	
5	2018	4.299	0.017	=C5*D5
6	2019	=C5+E5	0.017	=C6*D6
7	2020	=C6+E6	0.017	=C7*D7
8	2021	=C7+E7	0.017	=C8*D8
9	2022	=C8+E8	0.017	=C9*D9
10	2023	=C9+E9	0.017	=C10*D10
11	2024	=C10+E10	0.017	=C11*D11
12	2025	=C11+E11	0.017	=C12*D12
13	2026	=C12+E12	0.017	=C13*D13
14	2027	=C13+E13	0.017	=C14*D14
15	2028	=C14+E14	0.017	=C15*D15
16	2029	=C15+E15	0.017	=C16*D16
17	2030	=C16+E16	0.017	=C17*D17

Fig. 5 Data table produced by team 1 (formulas shown, year=año)

#### i) Covariation and Rate of Change in Team 1 Model

The table presented by team 1 (Fig. 4) shows how the students (row 5, columns B, C, D, E) identified that in 2018 the population was 4,299 million inhabitants and calculated the increase in the number of inhabitants per year according to the rate of growth (column E). In the table there are only two columns with titles: year (cell B4) and annual (cell D4). Column C corresponds to the population corresponds to each year (column B). The table is not described in detail, but it is observed that the students detected the annual variation in the population (P); they identified a pattern of behavior and wrote a recursive formula. Fig. 6 shows in more detail the organization of the data in the Table. The conjecture of the S1 and S3 students was that the population growth was linear since the rate (1.7) was constant. They did not recognize the exponential relationship.

S1: the variation is constant because the rate is constant, and therefore, the growth is constant.

Also, the letter shows the idea of linear growth: "And in the following years, both constant and non-constant variations can be observed, but an increase of more than 100,000 inhabitants per year is always observed." The students believed that the rate of change was constant: approximately 100,000 inhabitants per year.

A	B	C	
1	AÑO	POBLACIÓN (MILLONES)	
2	2018	4.299	
3	2019	4.372	0.073
4	2020	4.446	0.074
5	2021	4.522	0.076
6	2022	4.599	0.077
7	2023	4.677	0.078
8	2024	4.757	0.080
9	2025	4.837	0.081
10	2026	4.920	0.082
11	2027	5.003	0.084
12	2028	5.088	0.085
13	2029	5.175	0.087
14	2030	5.263	0.088

Fig. 6 Data table produced by team 2 (year=año, population= población)

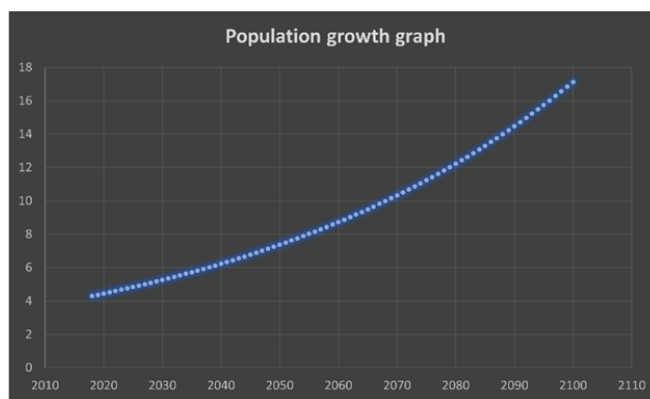


Fig. 7 Graph prepared by team 2

#### ii) Covariation and Rate of Change in Team 2 Model

Team 2 also had difficulty identifying whether the rate of change was constant or not. S5 conjectured that growth was linear, because the growth rate was constant and equal to 1.7.

S4: Ok, so, you say that in the year 2018 we have 4,299 million and the annual growth is 1.7; that is... it would be as a compound interest or what?

S5: That is, what it says, express if the variation is constant [S5 refers to the question included in the activity]

S4: mmm

S5: If it is constant or compound

S4: And how can we know?

S5: I think it is constant because every year, it increases the same every year [refers to the 1.7% rate]

S4, unlike its partner S3, detected that the variation was not constant after performing a couple of operations, and that he had to perform a table

S5: hey, and it's not easier ... Every year it will increase this [quantity]. From 2018 to 2020 are two years

S4: ujum

S5: if we multiply this by two? [the student thought in a linear behavior]

S4: I already knew you were going to tell me that. But no, because it is 1.7 of each certain difference. Then you have to subtract it from each new quantity. So, I told you it was like ... this ...

S5: a data table

S4: uh-huh, a little data table. We can do it fast in Excel,

right?

S5: yes

S4: because we have to calculate about 80 years

S4 identified that the procedure was similar to what she did, daily, in her job as an insurance saleswoman.

S4: It is that ... I do this frequently at work

Team 2 identified a recursive relation (Fig. 4). Unlike Team 1, the students synthesized all the operations and wrote the formula:  $(B2 * 0.017) + B2$ . That is, they used the population of the given (current) year to determine that of the next year. The students wrote the formula:  $B3-B2$  in cell C3 to analyze what the variation between the quantities was like. This team had less difficulty identifying exponential behavior. Although S1 (team 1) heard S4 mention that the variation was not constant, he ignored it; the above, because S4 was the lowest performing student in mathematics.

### iii) Inverse Function in Team 2 Model

Both teams had difficulty identifying the year in which there would be six million inhabitants, 7,560 and 8,232.

S5: 7560 ... is the same. It exceeds. It is not 2052 [the operations they carried out did not allow them to find the exact value of 7560 inhabitants]

S4: Oh ... so ... you know what? Something must not have gone well

S5: 7561 [student continued to perform operations]

S4: why can't we get 7560

The students did not know how to find the year in which the metropolitan area would have six million inhabitants, 7560 and 8232; they only used the data in the table.

## 2. The Final Model of the Teams

As mentioned in [10], the first model of the students (linear model) was barren, but it was modified and refined into an exponential model through interaction between students, in teams, and in a group. That is, in the group discussion each team read their letter (Fig. 8). They described the procedures performed (Figs. 4, 8 and 9), which were discussed. When Team 2 presented the graph, S1 said out loud:

S1: it is true, the growth is not constant [The group had defined at that time that constant growth was to grow the same every year].

Using Excel allowed all students to organize the information given in the problem and write a relationship between the quantities using the Spreadsheet language. Dragging the formula made it easier for students to review how the quantities varied. In [11] the recursive relationship is considered to be far from supporting the understanding of the covariation between two quantities and the algebraic representation of a function. However, according to the NCTM [12], it is important that recursive relationships, as well as functions, arise in the classroom to promote understanding of the advantages and limitations of both. This was later discussed by the students and the teacher.

### C. Simple Prototype and Model Generalization Principle

Team 2, particularly S4, identified the usefulness of the exponential model to solve problems with another type of

context, but not the other students.

To whom it May concern:

Through this letter, I extend my concern about the population increase due to the fact that in the recent year there is a population of 4,299 million, an increase in the population of 1.7% per year can be perceived, which is an alarming figure since for the 2020 it is perceived that the population will increase to 4,446 million inhabitants and in the coming years such as:

year	Population in millions	Annual	Population increase
2018	4.299	1.70%	0.073083
2020	4.44640841	1.70%	0.07558894
2022	4.59887131	1.70%	0.07818081
2024	4.75656201	1.70%	0.08086155
2030	5.26282441	1.70%	0.08946801
2040	6.22914456	1.70%	0.10589546
2041	6.33504001	1.70%	0.10769568
2100	17.1271691	1.70%	0.29116187

And in the following years, both constant and non-constant and non-constant variations can be observed, but an increase of more than 100,000 inhabitants per year is always observed.

year	Population	Annual growth	Growth for years
2038	6.023	1.70%	0.102
2051	7.498	1.70%	0.127
2052	7.626	1.70%	0.130
2056	8.158	1.70%	0.139
2057	8.296	1.70%	0.141

And by 2067 there will be a population of 9 million 820 thousand inhabitants.

To arrive at the results, the constant annual growth of 1.7% was taken into account and it was tabulated in annual growth until the year 2100, taking the percentage of the population in 2018 (4,299 million) having a final increase of 120,690 inhabitants.

We await your prompt response. |

Fig. 8 Letter written by team 1

### Appreciable Secretary of Road Infrastructure.

To whom it May concern.

Through this we inform you that, through a study carried out in the Guadalajara metropolitan area, we use a method that will allow you to know the increase in population in future years.

Taking into account the current year, an average of constant growth of 1.7% per year was obtained. For example, in 2018 having 4,299 million inhabitants, it is estimated that in 2020 we will have 4,446 million, in table years it will increase 0.300 million.

It is also estimated that in 42 years the population will double compared to 2018, in 20 years there will be more than 6,000 million inhabitants, without forgetting that it is 2100 the population will be seen at 17,127 million, almost 298.39%

Which we consider that it would be valuable to use for the metropolitan planning of our city, or the world.

Fig. 9 Letter written by team 2

## 1. Extension of Student Knowledge with Teacher-Led Support

After the validation of the models during the group session, the teacher [T] gave a short interactive class. The tables were analyzed, in terms of the covariation. The teacher promoted the emergence of an algebraic representation  $P(n) = 4.299(1.017)^n$  and the inverse function.

T: Do you think there is any way to write the function so that it allows analyzing the growth of other cities with different initial populations and with a different exchange rate? Let's see if we can do something that works, okay?

The teacher's objective was to support reflection on how to use an exponential model as a simple prototype to solve problems with another type of context and, therefore, to generalize the model.

In a later session, the teacher proposed to the students to solve a compound interest textbook word problem. The students solved the problem without difficulties and explained



the exponential growth behavior involved. That is, they managed to use their knowledge to describe a situation in another context. This problem was used to analyze the inverse function.

#### V. CONCLUSIONS

Using Excel to create tabular representations to solve the problem allowed the students to focus on reviewing how quantities varied by dragging formulas. The use of graphic representation enabled team 1 to modify conjectures about linear behavior and identify exponential behavior. Although the students did not build the algebraic relationship without the teacher's help, they identified the recursive relationship in the language of the Spreadsheet, allowing them to answer various questions, and also understand the situation.

The teacher's support was essential to build and make sense of the algebraic relationship and observe the dependency between variables. So, the students analyzed mathematical concepts such as: covariation, rate of change and exponential function using different representations.

One aspect that was not explored in the classroom was the modification of initial quantities, as well as the rate of change, to generate a family of problems. In other words, it is necessary to take more advantage of the potential of technology. However, the students observed patterns, relationships, and regularities, which is important in learning mathematics.

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