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Modeling Non-Darcy Natural Convection Flow of a Micropolar Dusty Fluid with Convective Boundary Condition

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Abstract—A numerical approach of the effectiveness of numerous parameters on magnetohydrodynamic (MHD) natural convection heat and mass transfer problem of a dusty micropolar fluid in a non-Darcy porous regime is prepared in the current paper. In addition, a convective boundary condition is scrutinized into the micropolar dusty fluid model. The governing boundary layer equations are converted utilizing similarity transformations to a system of dimensionless equations to be convenient for numerical treatment. The resulting equations for fluid phase and dust phases of momentum, angular momentum, energy, and concentration with the appropriate boundary conditions are solved numerically applying the Runge-Kutta method of fourth-order. In accordance with the numerical study, it is obtained that the magnitude of the velocity of both fluid phase and particle phase reduces with an increasing magnetic parameter, the mass concentration of the dust particles, and Forchheimer number. While rises due to an increment in convective parameter and Darcy number. Also, the results refer that high values of the magnetic parameter, convective parameter, and Forchheimer number support the temperature distributions. However, deterioration occurs as the mass concentration of the dust particles and Darcy number increases. The angular velocity behavior is described by progress when studying the effect of the magnetic parameter and microrotation parameter.

Keywords—micropolar dusty fluid, convective heating, natural convection, MHD, porous media.

I. Introduction

Micropolar fluids are treated as a model of non-Newtonian fluids models which do not adhere to the Newtonian law of the viscosity. As well micropolar fluids are seen as consisting of suspended particles in a viscous medium and these particles are in the form of solid particles, spherical particles or random-oriented with their private spins and microrotations. In addition, micropolar fluids are characterized by microstructures. Eringen [1], [2] introduced the theory of thermomicropolar and micropolar fluids which can be utilized to portray fluid conduct in a lot of practical applications. Out of many of these applications animal blood, liquid crystal and the mathematical model for fluids with the suspensions as real fluids, colloidal fluids, and polymeric fluids. It is worth mentioning that a comprehensive and excellent review of microbial fluid mechanics and its applications was displayed by Ariman et al. [3], [4]. When studying micropolar fluids, it was found that there are many mathematical models for the natural convection flow of a micropolar fluid. The problem of natural convection boundary layer flow of micropolar fluids over a vertical cylinder has been analyzed by Rani and Kim [5]. Cheng [6] investigated boundary layer flow around natural convection heat transfer of a micropolar fluid with the power-law variance in surface temperature near a vertical truncated cone. A numerical discussion was prepared by Damseh et al. [7] about unsteady natural convection in the boundary layer flow of a micropolar fluid with fixed heat flux over a vertical surface. The natural convection flow problem of a micropolar fluid over a vertical plate and permeability in a porous medium with uniform heat flux has been studied by Hassanien et al. [8]. Ferdows and Liu [9] presented the natural convective flow of micropolar fluid past a vertical plate in the presence of a magnetic field effect. Recently, Unsteady MHD natural convection boundary layer flow over a radiated stretching sheet in a micropolar fluid with viscous dissipation and thermal radiation has been grasped by Rao et al. [10].

Based on the many MHD applications, it is studied to understand fluid motion behavior where it is studying the fluid attitude electrically conducting and the magnetic properties. Among of MHD applications are mention as plasma, power generation, liquid metals, etc. Numerous analyses have been executed with various aspects of unsteady MHD flows of Newtonian and non-Newtonian fluid [11]-[15]. Sheri and Shamshuddin [16] discussed the influences of a chemical reaction and viscous dissipation on mass and heat transfer flow of magnetohydrodynamic (MHD) of micropolar fluid. An investigation has been done by Shehzad et al. [17] into the magnetohydrodynamic flow problem of non-Newtonian nanofluid with convective boundary conditions.

Darcy's law has been discovered to study the flow of fluids through porous media in 1856 and the Darcy equation is defined as strong to describe this flow. According to Lee and Yang [18], the flow of fluids through a bank of circular cylinders was studied and they modeled it as Darcy-Forchheimer drag. As for the equations of Darcy-Brinkman-Forchheimer have been applied by Prasad and Kladias [19] to solve the heat transfer problem in horizontal porous layers which are heated from below under the impacts of boundary viscous diffusion and Inertia. Dye et al. [20] presented a description of fluid flow over the systems of porous medium for a set of Reynolds numbers congruous

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to both Darcy and non-Darcy regimes. Mixed convection problem from a vertical plate in a non-Darcy porous medium saturated with a non-Newtonian fluid under the effects of both magnetic field and melting has been analyzed by Prasad and Hemalatha [21]. It can be noted that a public pattern of the porous media is utilized to investigate the problem of the flow of Darcy and non-Darcy regimes in an axisymmetric porous cavity by Nithiarasu et al. [22]. Bakier [23] explained the problem of mass and heat transfer by natural convection with thermophoresis and radiation effects in a micropolar fluid through a non-Darcy porous medium. He found out that the dimensionless velocity and the function of mass transfer increase with enhancement thermophoresis, while reduces angular velocity and temperature. There are researches that have been displayed [24]-[26] about the flow of non-Darcy natural and mixed convection.

On the other hand, dusty fluid can be defined as due to the addition of dust particles in the size of the micrometer to the base fluids in nature. In addition, the dust fluid improves the thermal conductivity and heat transfer process of these fluids. The phenomenon of natural convection was associated with dusty fluid flow in problems addressed by Siddiqa et al. [27]-[29]. Also, Unsteady natural convection flow problem in a rectangular channel of a dusty fluid has been studied by Dalal et al. [30]. The problem of flow MHD boundary layer over a semi-infinite surface of dusty fluid under induced magnetic field effects was examined by Silu et al. [31]. There is an issue that deals with the influences of Biot number and magnetic field on the flow of the boundary layer and heat transfer over a stretching surface of dusty fluid containing silver (Ag) nanoparticles have been investigated by Gireesha et al. [32]. Recently, Krishnamurthy [33] presented a study on MHD flow of nano micropolar fluid in the presence of dust particles over a permeable stretching sheet in a porous medium. Finally, the objective of the current investigation is to present a discussion about magnetohydrodynamic (MHD) natural convection boundary layer flow and mass and heat transfer of a dusty micropolar fluid in a non-Darcy porous regime in the presence of a convective boundary condition. In other words, it is to present a discussion about the impacts of a magnetic field and a convective boundary condition on incompressible boundary layer flow and heat transfer of a dusty micropolar fluid through a non-Darcy porous regime. Non-linear equations of momentum, heat and mass transfer and angular momentum are solved numerically using the fourth-order Runge-Kutta method by software algebraic Matlab.

II. Model Formulation

The mathematical description of the current problem of natural convection laminar boundary layer flow of a micropolar fluid in a non-Darcy porous medium in the presence of dust particles and magnetic field with a convective boundary condition is explained in this section. Characterized by this fluid as viscous, incompressible and electrically conducted because of an applied magnetic field B_0 in the $\hat{y}$ direction, as well as the flow is in two dimensional and stable. Fig. 1 displays the physical diagram and coordinate system of this problem. Cartesian coordinates $\hat{x}$ and $\hat{y}$ are used such that $\hat{x}$ is along the vertical surface and $\hat{y}$ perpendicular it. It is assumed that T_∞ and C_∞ are the ambient temperature and species concentration, respectively, n is a constant lay between 0 and 1 $(0 \leqslant n \leqslant 1)$, T_w is the surface temperature which will determine later result from a convective heating process which is described by heat transfer coefficient h_f and temperature T_f, and C_w represents stationary value of C at $\hat{y} = 0$. Based on using approximations of the boundary-layer and Oberbeck-Boussinesq and assumption that number density of the dust particle remains constant during the flow and the dust particles have the same size, the governing equations of the fluid phase and particle phase follow the next system of equations:

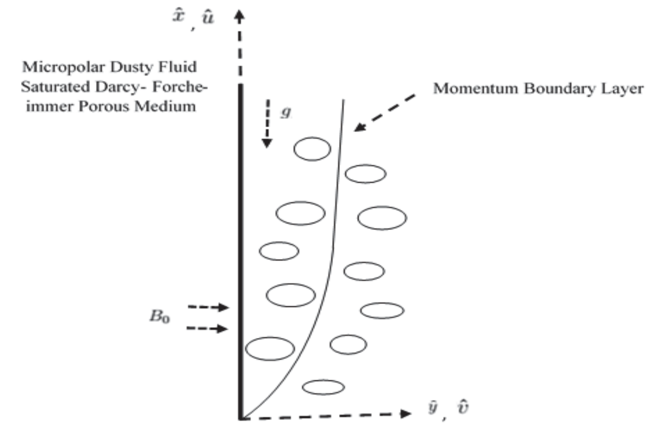

Fig. 1 Physical diagram representation for the problem

For the fluid phase:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0 \tag{1}$$

$$\begin{aligned} \hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} &= \nu \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + \frac{k^*}{\rho}\frac{\partial E}{\partial \hat{y}} + g\beta(T - T_\infty) + \\ g\beta^*(C - C_\infty) &- \frac{\nu}{K}\hat{u} - \frac{\sigma B_0^2}{\rho}\hat{u} - \frac{b}{K}\hat{u}^2 + \frac{\rho_p}{\rho \tau_m}(\hat{u_p} - \hat{u}) \end{aligned} \tag{2}$$

$$\hat{u}\frac{\partial E}{\partial \hat{x}} + \hat{v}\frac{\partial E}{\partial \hat{y}} = \frac{\varepsilon}{\rho j}\frac{\partial^2 E}{\partial \hat{y}^2} - \frac{k^*}{\rho j}\left(2E + \frac{\partial \hat{u}}{\partial \hat{y}}\right) \tag{3}$$

$$\begin{aligned} (\rho c_p)_f \left(\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}}\right) &= k\frac{\partial^2 T}{\partial \hat{y}^2} + \frac{\rho_p c_s}{\tau_T}(T_p - T) + \\ \frac{\rho_p}{\tau_m}(\hat{u_p} - \hat{u})^2 & \end{aligned} \tag{4}$$

$$\hat{u}\frac{\partial C}{\partial \hat{x}} + \hat{v}\frac{\partial C}{\partial \hat{y}} = D\frac{\partial^2 C}{\partial \hat{y}^2} \tag{5}$$

For the dust phase:

$$\frac{\partial \hat{u_p}}{\partial \hat{x}} + \frac{\partial \hat{v_p}}{\partial \hat{y}} = 0 \tag{6}$$

$$\hat{u}_p\frac{\partial \hat{u}_p}{\partial \hat{x}} + \hat{v}_p\frac{\partial \hat{u}_p}{\partial \hat{y}} = -\frac{1}{\tau_m}(\hat{u_p} - \hat{u}) \tag{7}$$

$$\rho_p c_s \left(\hat{u}_p \frac{\partial T_p}{\partial \hat{x}} + \hat{v}_p \frac{\partial T_p}{\partial \hat{y}} \right) = -\frac{\rho_p c_s}{\tau_T} (T_p - T) \qquad (8)$$

Here, $\hat{u}$, $\hat{v}$, $\hat{u}_p$ and $\hat{v}_p$ are the components of velocity in two phases fluid and dust along $\hat{x}$ and $\hat{y}$ axes, respectively, ρ and ρ_p reference the density of fluid and dust particles, respectively, k^* and ν are the coefficient of rotational and kinematic viscosity, respectively, g refers to the acceleration of gravity vector, σ signify to the fluid electrical conductivity, B_0 gives the strength of the magnetic field, β and β^* present the coefficient of thermal and concentration expansion, respectively, b points out the Forchheimer constant, K represents the permeability of the porous medium, k gives the thermal conductivity of the fluid, E refers to the angular velocity, C refers to the species concentration, D points out the coefficient of mass diffusion, j is the density of micro-inertia, ε represents the viscosity of spin gradient, T and T_p determine the temperature of fluid and dust particles, respectively, c_p and c_s give the specific heat of fluid and dust particles, respectively, $(\rho c_p)_f$ indicates the heat capacity of the fluid, τ_m and τ_T aim the velocity relaxation time and thermal relaxation time of the dust particle, respectively and $\tau_T = \frac{3}{2}\frac{c_s}{c_p} Pr \tau_m$. Based on the following boundary conditions:

$$\begin{aligned} &\hat{u} = U_0, \quad \hat{v} = 0, \quad -k\frac{\partial T}{\partial \hat{y}} = h_f\,(T_f - T) \\ &C = C_w, \quad E = -n\frac{\partial \hat{u}}{\partial \hat{y}}, \quad \hat{y} = 0 \end{aligned} \qquad (9)$$

$$\begin{aligned} &\hat{u}, \hat{u_p} \longrightarrow 0, \quad \hat{v_p} \longrightarrow \hat{v}, \quad T, T_p \longrightarrow T_\infty \\ &C \longrightarrow C_\infty, \quad E \longrightarrow 0, \quad \hat{y} \longrightarrow \infty \end{aligned} \qquad (10)$$

The dimensionless form for a micropolar dusty fluid flow equations of momentum, angular momentum, energy, and concentration with previous conditions can be reached by introducing the following transformations:

$$\begin{aligned} &\psi = \left(\sqrt{2\nu\hat{x}U_0}\right) f(\eta) = \left(\sqrt{2\nu\hat{x}U_0}\right) F(\eta), \\ &\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_f - T_\infty}, \\ &\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad E(\eta) = \left(\sqrt{\frac{U_0}{2\nu\hat{x}}}\right) U_0 S, \\ &\eta = \left(\sqrt{\frac{U_0}{2\nu\hat{x}}}\right) \hat{y} \end{aligned} \qquad (11)$$

where η represents the transverse coordinate in non-dimension form, ψ is the stream function such that $\hat{u} = \frac{\partial\psi}{\partial\hat{y}}$ and $\hat{v} = -\frac{\partial\psi}{\partial\hat{x}}$, f and F are the stream function in non-dimension form for fluid phase and particle phase, respectively, ϕ points out concentration function in non-dimension form, θ and θ_p indicate the temperature function in non-dimension form for fluid phase and particle phase, respectively, S gives the angular velocity in non-dimension form.

Now the dimensionless form of the system of above equations with the corresponding boundary conditions are:

For the fluid phase:

$$\begin{aligned} &f''' + ff'' + BS' + Gr\theta + Gm\phi - \frac{1}{DaRe}f' - Mf' - \\ &\frac{Fs}{Da}f'^2 + D_\rho\alpha_d(F' - f') = 0 \end{aligned} \qquad (12)$$

$$\lambda S'' - 2\frac{\lambda}{G^*}(2S + f'') + f'S + fS' = 0 \qquad (13)$$

$$\theta'' + Prf\theta' + \frac{2}{3}D_\rho\alpha_d(\theta_p - \theta) + PrD_\rho\alpha_d Ec(F' - f')^2 = 0 \qquad (14)$$

$$\phi'' + Scf\phi' = 0 \qquad (15)$$

For the dust phase:

$$FF'' + \alpha_d(f' - F') = 0 \qquad (16)$$

$$F\theta_p' + \frac{2}{3}\frac{1}{\Gamma Pr}\alpha_d(\theta - \theta_p) = 0 \qquad (17)$$

Transformed boundary conditions are:

$$\begin{aligned} &f' = 1, \quad f = 0, \quad \theta' = -Bi(1 - \theta) \\ &\phi = 1, \quad S = -nf'', \quad \eta = 0 \end{aligned} \qquad (18)$$

$$\begin{aligned} &f', F' \longrightarrow 0, \quad F \longrightarrow f, \quad \theta, \theta_p \longrightarrow 0 \\ &\phi \longrightarrow 0, \quad S \longrightarrow 0, \quad \eta \longrightarrow \infty \end{aligned} \qquad (19)$$

where $\lambda = \frac{\varepsilon}{\rho j \nu}$ and $G^* = \frac{\varepsilon U_0}{k^*\nu\hat{x}}$ represented the micropolar material parameters, $B = \frac{k^*}{\rho\nu}$ expresses the coupling constant parameter, $Da = \frac{K}{L^2}$ points out the Darcy number, $Re = \frac{U_0 L^2}{2\nu\hat{x}}$ gives the local Reynolds number, $M = \frac{\nu\sigma B_0^2}{\rho U_0^2}$ refers to the magnetic parameter, $Fs = \frac{2b\hat{x}}{L^2}$ denotes the Forchheimer number, $Sc = \frac{\nu}{D}$ presents the Schmidt number, $Pr = \frac{\mu c_p}{k}$ defines the Prandtl number, $Gr = \frac{g\beta\nu(T_f - T_\infty)}{U_0^3}$ and $Gm = \frac{g\beta^*\nu(C_w - C_\infty)}{U_0^3}$ indicate the local Grashof number and modified Grashof number, respectively, $Ec = \frac{U_0^2}{c_p(T_f - T_\infty)}$ Known as the Eckert number, $\alpha_d = \frac{2\hat{x}}{U_0}\frac{1}{\tau_m}$ determines fluid particle interaction parameter, $D_\rho = \frac{\rho_p}{\rho}$ signify to the mass concentration of the dust particles, $\Gamma = \frac{c_s}{c_p}$ represents the specific heat ratio of the mixture and $Bi = \frac{h_f}{k}\sqrt{\frac{2\nu\hat{x}}{U_0}} = \frac{h_f}{k}\frac{L}{\sqrt{Re}}$ gives the Biot number.

III. Numerical Method

The set of ordinary and nonlinear differential equations (12)-(17) with the boundary conditions (18) and (19) have been solved numerically utilized the Runge-Kutta method of fourth-order. Our equations are dealt with in a Matlab based on the fact that each nth-order equation is mutated into n of the first-order equations. Then using the bvp4c function, the system of first-order equations is solved. The edge of the boundary layer at infinity (η_∞) was selected in this method equal to $\eta_{max} = 8$ that correctly assures that each numerical solutions approach the asymptotic values. Additionally, numerical solutions have not a salient significant change when rising values of η_{max}. In order to clarify this method more, we rewrite the momentum equation (12) as an example as follow:

TABLE I
VALUES OF $-\theta'(0)$ AND $f''(0)$ FOR DIFFERENT VALUES OF THE PARAMETERS

Pr	M	Gr	Bi	n	D_ρ	B	Da	$-\theta'(0)$	$f''(0)$
0.7	2	6	10	0.5	1	0.5	0.3	0.42101	-0.94268
0.8								0.44981	-0.96026
0.9								0.47688	-0.97672
1								0.50249	-0.99222
0.7	0	6	10	0.5	1	0.5	0.3	0.43955	-0.44823
	2							0.42101	-0.94268
	4							0.40443	-1.38461
	6							0.38960	-1.78554
0.7	2	5	10	0.5	1	0.5	0.3	0.40739	-1.19047
		6						0.42101	-0.94268
		7						0.43294	-0.69666
		8						0.44346	-0.45223
0.7	2	6	1	0.5	1	0.5	0.3	0.28648	-1.32278
			5					0.39950	-1.00117
			10					0.42101	-0.94268
			∞					0.44527	-0.87766
0.7	2	6	10	0	1	0.5	0.3	0.42225	-0.81574
				0.3				0.42101	-0.94268
				0.5				0.42101	-0.94268
				1				0.41931	-1.11652
0.7	2	6	10	0.5	0	0.5	0.3	0.43908	-0.92639
					1			0.42101	-0.94268
					4			0.36992	-0.99253
					10			0.28059	-1.09575
0.7	2	6	10	0.5	1	0.5	0.3	0.42101	-0.94268
						5		0.40258	-0.57653
						7		0.38523	-0.88054
						9		0.37554	-0.81323
0.7	2	6	10	0.5	1	0.5	0.3	0.42101	-0.94268
							0.4	0.44243	-0.32777
							1	0.48892	1.16705
							1.1	0.49196	1.28521

$$f''' = -ff'' - BS' - Gr\theta - Gm\phi + \frac{1}{DaRe}f' + Mf' + \frac{Fs}{Da}f'^2 - D_\rho \alpha_d (F' - f') \tag{20}$$

The following system of equations represents how to place the momentum equation in MATLAB.

$$\begin{aligned} &F(1) = Y(2),\\ &F(2) = Y(3),\\ &F(3) = -Y(1)Y(3) - BY(9) - GrY(4) - GmY(6) +\\ &(1/DaRe)Y(2) + MY(2) + (Fs/Da)Y^2(2) -\\ &D_\rho \alpha_d (Y(11) - Y(2)) \end{aligned} \tag{21}$$

where

$$\begin{aligned} &f = Y(1),\\ &f' = Y(2) = \frac{dY(1)}{d\eta} = F(1),\\ &f'' = Y(3) = \frac{dY(2)}{d\eta} = F(2),\\ &f''' = \frac{dY(3)}{d\eta} = F(3),\\ &S' = Y(9),\\ &\theta = Y(4),\\ &\phi = Y(6),\\ &F' = Y(11) \end{aligned} \tag{22}$$

Values of the local Nusselt number $-\theta'(0)$ and skin friction coefficient $f''(0)$ for some parameters at $Gm = 6, G^* = 0.2, Re = 0.4, Fs = 0.5, Ec = 2, Sc = 5, \alpha_d = 0.1, \lambda = 1, \Gamma = 0.1$ have been estimated in Table I.

IV. RESULTS AND DISCUSSION

Herein we examine the induced results from the impact of dimensionless governing parameters on different physical quantities in the dimensionless form such the velocity and temperature for fluid phase and particle phase, too on species concentration function and angular velocity (micro-rotation) function. Our parameters values range as follows magnetic parameter ($0 \leqslant M \leqslant 6$), mass concentration of the dust particles ($0 \leqslant D_\rho \leqslant 10$), fluid particle interaction parameter ($0.01 \leqslant \alpha_d \leqslant 0.1$), Forchheimer number ($0.5 \leqslant Fs \leqslant 5$), Darcy number ($0.3 \leqslant Da \leqslant 1.1$), convective parameter ($1 \leqslant Bi \leqslant \infty$), local Grashof number ($5 \leqslant Gr \leqslant 8$), modified Grashof number ($1 \leqslant Gm \leqslant 6$), Prandtl number ($0.7 \leqslant Pr \leqslant 1$), constant parameter (microrotation parameter) ($0 \leqslant n \leqslant 1$) and coupling constant parameter ($0.5 \leqslant B \leqslant 9$). But the other parameters remain constant in our numerical computations as follows the micropolar material parameters $\lambda = 1$ and $G^* = 0.2$, local Reynolds number $Re = 0.4$, Eckert number $Ec = 2$, Schmidt number $Sc = 5$ and the specific heat ratio of the mixture $\Gamma = 0.1$.

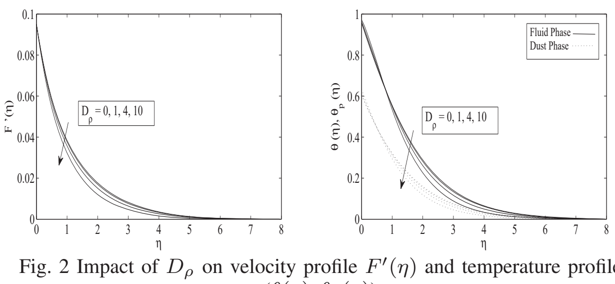

Fig. 2 Impact of D_ρ on velocity profile $F'(\eta)$ and temperature profile $(\theta(\eta), \theta_p(\eta))$

Foremost we display the effecting of dust parameters on the velocity and temperature profiles. Fig. 2 represents the extent of influence of mass concentration of the dust particles D_ρ versus η on the magnitude of the dimensionless velocity and temperature distribution for fluid phase and particle phase. It is

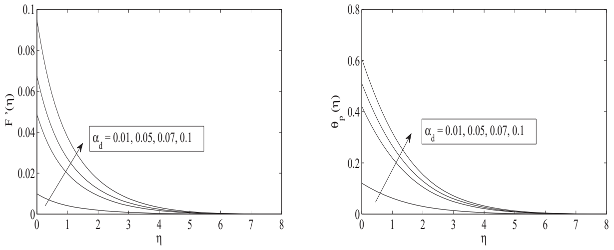

Fig. 3 Impact of α_d on velocity profile $F'(\eta)$ and temperature profile $\theta_p(\eta)$

observed that the velocity (F') and temperatures (θ, θ_p) reduce with an increase in values of D_ρ. In other words, increasing the mass concentration of the dust particles leads to the lack of movement and increase the rate of heat transfer as a result of improved thermal conductivity of the particle phase. It is interesting to observe that putting $D_\rho = 0$ represents viscous micropolar fluid without dust particles. Fig. 3 refers that the velocity and temperature of the dust particles enhance as fluid-particle interaction parameter α_d increases. This behavior may be attributed to the fact that the interaction between fluid and particles is large and hegemony of thermal conductivity of the particle phase then the particle phase reduces the velocity of the fluid till it reaches the same fluid velocity. This means a decrease in fluid velocity and an increment in dust particles velocity.

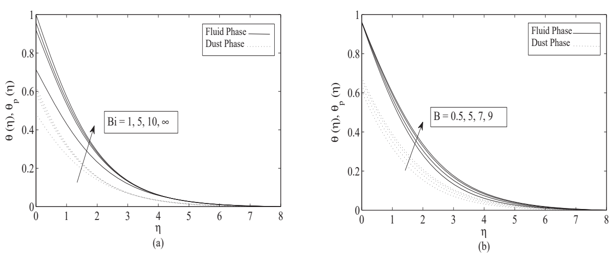

Fig. 4 (a-b) Impact of Bi and B on temperature profile $(\theta(\eta), \theta_p(\eta))$

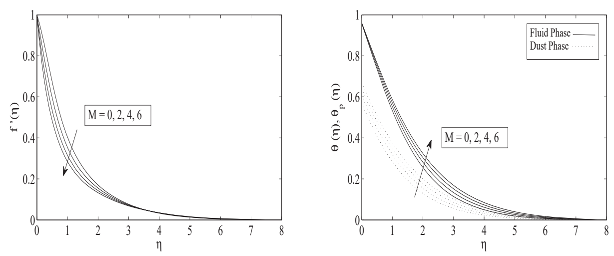

Fig. 5 Impact of M on velocity profile $f'(\eta)$ and temperature profile $(\theta(\eta), \theta_p(\eta))$

As shown in Fig. 4 (a), Biot number Bi improves the temperature profiles in two phases fluid and dust (θ, θ_p). This shows that increasing the convective parameter leads to that fluid temperature approaching from the isothermal case. Hence the thickness of the thermal boundary layer rises due to interchange in convective heat. The temperature distributions (θ, θ_p) for various values of coupling constant parameter B are presented in Fig. 4 (b). From this depiction, it is clearly shown that there is progress in the temperatures under the influence of B. The dimensionless velocity and temperature profiles in both phases fluid and dust for different values of the magnetic parameter M are displayed in Fig. 5. It is obvious that there is shrinking in the dimensionless velocity (f') with rising values of M. While the dimensionless temperatures (θ) and (θ_p) growing with M. Physically this behavior is due to that an application of a magnetic field produce Lorentz force which represents the opposite force to the flow and this force increases with the enhancement of magnetic field parameter which leads to diminishing motion and improves the thermal boundary layer thickness.

When studying the Darcy number Da and Forchheimer number Fs on the velocity and temperature profiles for fluid and particle phases in Figs. 6 and 7, it is found that the dimensionless velocity (f') rises with the enhancement of Darcy number but reduces with Forchheimer number. Also, it is noted from these figures Darcy number minimizes the dimensionless temperatures while Forchheimer number supports the temperature profiles. This behavior can be indicated by the use of Forchheimer number, which represents inertial drag and its rise leads to increase the flow resistance making the movement in a low state, as for the increase in the temperatures is due to that the energy deployed in the medium represented in the form of heat.

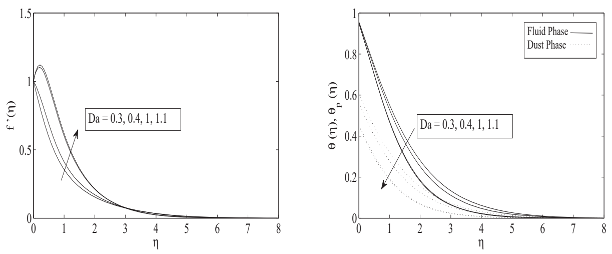

Fig. 6 Impact of Da on velocity profile $f'(\eta)$ and temperature profile $(\theta(\eta), \theta_p(\eta))$

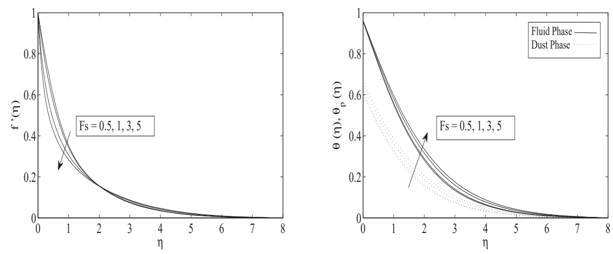

Fig. 7 Impact of Fs on velocity profile $f'(\eta)$ and temperature profile $(\theta(\eta), \theta_p(\eta))$

With the help of Grashof number Gr in Fig. 8, it is reached that the Grashof number supports the velocity profiles (f') and leads to shrinking the temperature profiles (θ, θ_p). The temperature behavior can be attributed to the fact that temperatures drop rapidly near the surface of the plate because high values of Gr transfers heat away from the plate. In Fig. 9, the velocity of dust particles (F') and temperatures (θ, θ_p) decrement with an increase of modified Grashof number Gm.

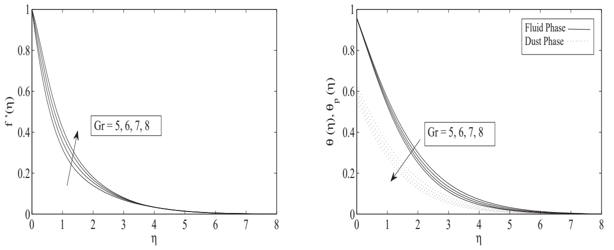

Fig. 8 Impact of Gr on velocity profile $f'(\eta)$ and temperature profile $(\theta(\eta), \theta_p(\eta))$

For different values of Prandtl number Pr as shown in Fig. 10, we recognize that the excess in Pr leads to a fall in the velocity profiles (f') and the temperature profiles (θ, θ_p). Deflation behavior in the velocity and temperature profiles is due to the thickness of the large fluid that tends to the contraction of velocity due to the viscosity of the large fluids and the thermal boundary layer thickness reduced (small thermal conductivity) by increasing Pr, respectively.

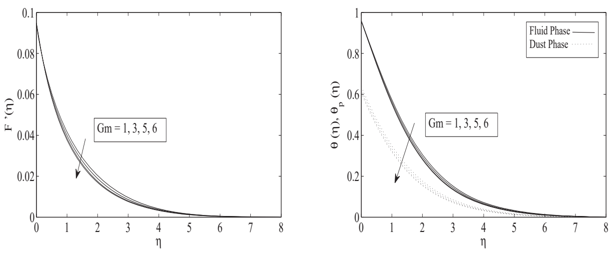

Fig. 9 Impact of Gm on velocity profile $F'(\eta)$ and temperature profile $(\theta(\eta), \theta_p(\eta))$

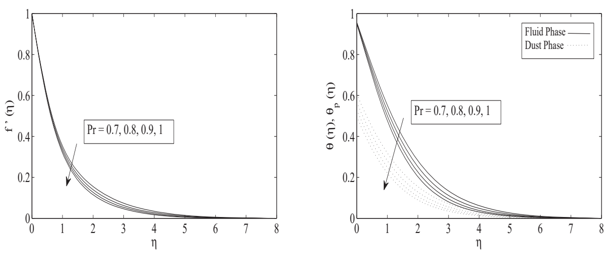

Fig. 10 Impact of Pr on velocity profile $f'(\eta)$ and temperature profile $(\theta(\eta), \theta_p(\eta))$

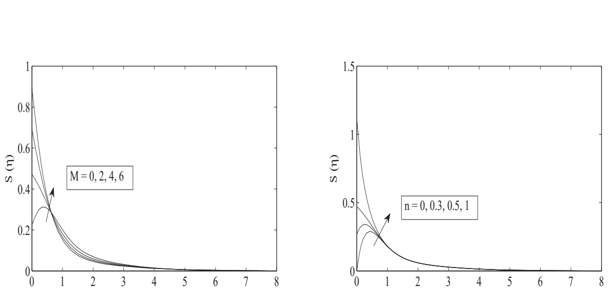

Fig. 11 Impact of M and n on angular velocity S.

Finally, we present the graphs that explain the behavior of the angular velocity when studying some parameters such as magnetic parameter M, constant parameter (microrotation parameter) n, and Biot number Bi. In Fig. 11 the values of the angular velocity mount with high values of M and n. Taking $n = 0$ leads to $S = 0$ (no-spin condition) according to the condition at the wall $S(0) = -nf''(0)$ indicates that the microelements in the flow of concentrated particles near the wall surface can not able to rotate. Taking $n = 0.5$ (S can not equal zero when $f''(0) \neq 0$) refers that the part antisymmetric from the stress tensor disappears and represents a weak concentration. The particle spin for fine particle suspensions is to be equal to the velocity of the fluid at the wall. Using $n = 0.5$ it is potential to clarify the possibility of reducing the ruling equations to the equations of the Newtonian fluid $(k^* = 0)$. Taking $n = 1$ for the purpose of modeling the flows of the turbulent boundary layer. The results obtained in Fig. 12 shows that the effect of Bi tends to dwindle the values of angular velocity.

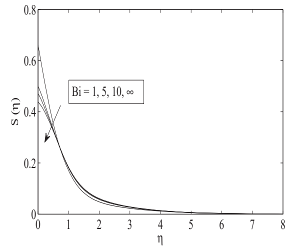

Fig. 12 Impact of Bi on angular velocity S

V. Conclusion

The problem of MHD natural convection heat and mass transfer of a dusty micropolar fluid in a non-Darcy porous medium with a convective boundary condition has been studied in the present paper. To solve the dimensionless boundary layer equations we used the Runge-Kutta method of fourth-order. A parametric study of our problem was conducted to elucidate and discuss the effects of some parameters on the magnitude of the velocity and temperature distribution for the fluid phase and particle phase and on the angular velocity. Important findings of this study are displayed as follow:

1) The profiles of velocity for fluid phase f' reduce with increasing magnetic parameter, Forchheimer number, and Prandtl number. While an enhancement in values of Darcy number and Grashof number increases the dimensionless velocity.
2) The profiles of velocity for dust phase F' increment with rising values of fluid-particle interaction parameter and deteriorates by an enhancement in both the mass concentration of the dust particles and modified Grashof number.
3) The increment in magnetic parameter, Biot number, Forchheimer number, and coupling constant parameter lead to improving the profiles of temperature for both fluid and dust phase (θ, θ_p). But a reduction takes place with higher values of mass concentration of the dust particles, Darcy number, Grashof number, modified Grashof number, and Prandtl number.

4) There is an elevation in the behavior of θ_p at studying fluid-particle interaction parameter.
5) An increase in Biot number tends to diminish the dimensionless angular velocity while this velocity progress by augmentation the magnetic parameter and microrotation parameter.

References

[1] A. C. Eringen, "Theory of thermomicropolar fluids", J. Math. Appl. voI. 38, PP. 480-495, 1972.
[2] A. C. Eringen, "Theory of micropolar fluids", J. Math. Mech. voI. 16, pp. 1-18, 1966.
[3] T. Ariman, M. A. Turk, N. D. Sylvester, "Microcontinuum fluid mechanicsa review", Int. J. Eng. Sci. voI. 11, PP. 905-929, 1973.
[4] T. Ariman, M. A. Turk, N. D. Sylvester, "Applications of microcontinuum fluid mechanics", Int. J. Eng. Sci. voI. 12, PP. 273-293, 1974.
[5] H. P. Rani, C. N. Kim, "A transient natural convection of micropolar fluids over a vertical cylinder", Heat Mass Transfer voI. 46, PP. 1277-1285, 2010.
[6] C. Y. Cheng, "Natural convection of a micropolar fluid from a vertical truncated cone with power-law variation in surface temperature", Int. Commun. Heat Mass Transfer voI. 35, PP. 39-46, 2008.
[7] R. A. Damseh, T. A. Al-Azab, B. A. Shannak, M. Al Husein, "Unsteady natural convection heat transfer of micropolar fluid over a vertical surface with constant heat flux", Turk. J. Eng. Environ. Sci. voI. 31, PP. 225-233, 2007.
[8] I .A. Hassanien, A. H. Essawy, N .M. Moursy, Natural convection flow of micropolar fluid from a permeable uniform heat flux surface in porous medium, Appl. Math. Comput. voI. 152, PP. 323-335, 2004.
[9] M. Ferdows, D. Liu, "Natural convective flow of a magneto-micropolar fluid along a vertical plate", Propul. Power. Rese. voI. 7, PP. 43-51, 2018.
[10] N. V. K. Rao, C. Srinivasulu, C. S. K. Raju, B. Devika, "Thermal natural convection of magneto hydrodynamics micropolar unsteady fluid over a radiated stretching sheet with viscous dissipation", J. Nanofluids voI. 8, PP. 550-555, 2019.
[11] L. Rundora, O. D. Makinde, "Unsteady MHD flow of non-Newtonian fluid in a channel filled with a saturated porous medium with asymmetric navier slip and convective heating", Appl. Math. Inform. Sci. Int. J. voI. 12, PP. 483-493, 2018.
[12] A. Mahdy, "Unsteady MHD slip flow of a non-Newtonian Casson fluid due to stretching sheet with suction or blowing effect", J. Appl. Fluid Mech. voI. 9, PP. 785- 793, 2016.
[13] A. Mahdy, S. A. Ahmed, "Unsteady MHD convective flow of non-Newtonian Casson fluid in the stagnation region of an impulsively rotating sphere", J. Aero. Eng. voI. 30, PP. 04017036 (8 pages), 2017.
[14] F. M. Hady, A. Mahdy, R. A. Mohamed, Omima A. Abo Zaid, "Effects of viscous dissipation on unsteady MHD thermo bioconvection boundary layer flow of a nanofluid containing gyrotactic microorganisms along a stretching sheet", World J. Mech. voI. 6, PP. 505-526, 2016.
[15] S. A. Ahmed, A. Mahdy, "Unsteady MHD double diffusive convection in the stagnation region of an impulsively rotating sphere in the presence of thermal radiation effect", J. Taiwan Institu. Chemical Eng. voI. 58, PP. 173-180, 2016.
[16] S. R. Sheri, "Heat and mass transfer on the MHD flow of micro polar fluid in the presence of viscous dissipation and chemical reaction", Procedia Eng. voI. 127, PP. 885-892, 2015.
[17] S. A. Shehzad, T. Hayat, A. Alsaedi, "MHD flow of Jeffrey nanofluid with convective boundary conditions", J. Braz. Soc. Mech. Sci. Eng. voI. 37, PP. 873-883, 2015.
[18] S. L. Lee, J. H. Yang, "Modeling of Darcy-Forchheimer drag for fluid flow across a bank of circular cylinders", Int. J. Heat Mass Transfer voI. 40, PP. 3149-3155, 1997.
[19] V. Prasad, N. Kladias, "Non-Darcy natural convection in saturated porous media, In: S Kaka, B Kilkis, FA Kulacki and F Arin (eds)", Convective Heat Mass Transfer Porous Media. voI. 196, PP. 173-224, 1991.
[20] A. L. Dye, J. E.McClure, C. T. Miller, W. G. Gray, "Description of non-Darcy flows in porous medium systems", Phys. Rev. E voI. 87, PP. 033012 (14 pages), 2013.
[21] J. S. R. Prasad, K. Hemalatha, "A study on mixed convective, MHD flow from a vertical plate embedded in non-Newtonian fluid saturated non- Darcy porous medium with melting effect", J. Appl. Fluid Mech. voI. 9, PP. 293-302, 2016.
[22] P. Nithiarasu, K. N. Seetharamu, T. Sundararajan, "Non-Darcy double-diffusive natural convection in axisymmetric fluid saturated porous cavities", Heat Mass Transfer voI. 32, PP. 427-433, 1997.
[23] A. Y. Bakier, "Natural convection heat and mass transfer in a micropolar fluid- saturated non-Darcy porous regime with radiation and thermophoresis effects", Therm. Sci. voI. 15, PP. S317-S326, 2011.
[24] F. M. Hady, R. A. Mohamed, A. Mahdy, "Non-Darcy natural convection flow along a vertical wavy plate embedded in a non-Newtonian fluid saturated porous medium", Int. J. Appl. Mech. Eng. voI. 13, PP. 91-100, 2008.
[25] F. M. Hady, R. A. Mohamed, A. Mahdy, Omima A. Abo-Zaid, "Non-Darcy natural convection boundary layer flow over a vertical cone in porous media saturated with a nanofluid containing gyrotactic microorganisms with a convective boundary condition", J. Nanofluids voI. 5, PP. 765-773, 2016.
[26] R. A. Mohamed, A. Mahdy, S. Abo-Dahab, "Effects of thermophoresis, heat source/sink, variable viscosity and chemical reaction on non-Darcian mixed convective heat and mass transfer flow over a semi-infinite porous inclined plate in the presence of thermal radiation", J. Computational Theoretical Nanoscie. voI. 10, PP. 1366-1375, 2013.
[27] S. Siddiqa, N. Begum, Md. A. Hossain, R. S. R. Gorla, "Natural convection flow of a two-phase dusty non-Newtonian fluid along a vertical surface", Int. J. Heat Mass Transfer voI. 113, PP. 482-489, 2017.
[28] S. Siddiqa, N. Begum, M. A. Hossain, R. S. R. Gorla, "Numerical solutions of natural convection flow of a dusty nanofluid about a vertical wavy truncated cone", J. Heat Transfer voI. 139, PP. 022503 (11 pages), 2017.
[29] S. Siddiqa, N. Begum, M. A. Hossain, R. S. R. Gorla, "Two-phase natural convection flow of a dusty fluid", Int. J. Numer. Meth. Heat Fluid Flow voI. 25, PP. 1542-1556, 2015.
[30] D. C. Dalal, N. Datta, S. K. Mukherjea, "Unsteady natural convection of a dusty fluid in an infinite rectangular channel", Int. J. Heat Mass Transfer voI. 41, PP. 547-562, 1998.
[31] S. M. Silu, M. Wainaina, M. Kimathi, "Effects of magnetic induction on MHD boundary Layer flow of dusty fluid over a stretching sheet", Global J. Pure Appl. Math. voI. 14, PP. 1197-1215, 2018.
[32] B. J. Gireesha, R. S. R. Gorla, M. R. Krishnamurthy, B. C. Prasannakumara, "Biot number effect on MHD flow and heat transfer of nanofluid with suspended dust particles in the presence of nonlinear thermal radiation and non-uniform heat source/sink", Acta Et Commentationes Universitatis Tartuensis De Mathematica voI. 22, PP. 91-114, 2018.
[33] B. J. Gireesha, R. S. R. Gorla, M. R. Krishnamurthy, B. C. Prasannakumara, "MHD flow and radiative heat transfer of micropolar dusty fluid suspended with alumina nanoparticles over a stretching sheet embedded in a porous medium", JNNCE J. Eng. Manag. voI. 2, PP. 30-45, 2018.