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Lattice Boltzmann Simulation of MHD Natural Convection Heat Transfer of Cu-Water Nanofluid in a Linearly/Sinusoidally Heated Cavity

Bouchmel Mliki, Chaouki Ali, Mohamed Ammar Abbassi

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Abstract**—**In this numerical study, natural convection of Cu–water nanofluid in a cavity submitted to different heating modes on its vertical walls is analyzed. Maxwell-Garnetts (MG) and Brinkman models have been utilized for calculating the effective thermal conductivity and dynamic viscosity of nanofluid, respectively. Influences of Rayleigh number (Ra = 10^3–10^6), nanoparticle volume concentration (ϕ = 0-0.04) and Hartmann number (Ha = 0-90) on the flow and heat transfer characteristics have been examined. The results indicate that the Hartmann number influences the heat transfer at Ra = 10^6 more than other Raleigh numbers, as the least effect is observed at Ra = 10^3. Moreover, the results show that the solid volume fraction has a significant influence on heat transfer, depending on the value of Hartmann, heat generation or absorption coefficient and Rayleigh numbers.

Keywords**—**Heat transfer, linearly/sinusoidally heated, Lattice Boltzmann Method, natural convection, nanofluid.

I. INTRODUCTION

MHD natural convection in enclosures occurs in numerous applications and has been studied extensively in the literature putting emphasis on the parameters that influence the flow and heat transfer [1]-[23].

Various investigations on natural convection were implemented by researchers with different numerical methods. Especially, in order to understand buoyancy-driven heat transfer of nanofluids in a cavity, several investigations have been numerically and experimentally conducted. Putra et al. [24] conducted an experiment for observation on the natural convective characteristics of water based on Al_2O_3. It was revealed that the heat transfer rate decreased with the increment of the volume fraction of nanoparticles. Mliki et al. [25] applied the lattice Boltzmann method to investigate buoyancy driven heat transfer enhancement in an L-shaped cavity utilizing nanofluids. They analyzed the effect of different parameters such as Rayleigh number (Ra =10^3-10^6), aspect ratio of the L-shaped enclosure (AR = 0.2-0.6) and nanoparticle volume concentration (ϕ = 0-0.05) on the flow and temperature fields. They reported an enhancement in heat transfer when the nanofluid (Cu/water) was used as the working fluid compared with water. Also, they concluded that the Nusselt number decreases by an increase in volume concentration, especially for higher Rayleigh numbers.

Khanafer et al. [26] studied numerically the natural convection heat transfer of a nanofluid in a square enclosure for different Grashof number and nanoparticle volume fractions. It was revealed that the heat transfer rate increased with the increase of particle fraction at any given Grashof number. Dehnavi and Rezvani [27] studied the alumina/water nanofluid free convection in a Γ-shaped cavity. They showed that for low Rayleigh numbers, nanoparticles with higher thermal conductivity caused more enhancements in heat transfer. Kefayati [28] conducted a research dealing with the effect of a magnetic source on natural convection of ferrofluid in a linearly heated cavity utilizing Lattice Boltzmann Simulation. The investigation demonstrated that the heat transfer decreased by the increment of the particle volume fraction for various Rayleigh numbers. Kefayati [29] investigated numerically natural convection in a square cavity with sinusoidal temperature distribution and in the presence of magnetic field. Results showed that the heat transfer decreased by the increment of Hartmann number for various Rayleigh numbers. Kefayati [30] studied laminar mixed convection of non-Newtonian nanofluids in a two sided lid-driven square cavity. The results of this study show that the augmentation of Richardson number causes a heat transfer drop. Also, the addition of nanoparticle increases heat transfer for multifarious studied parameters.

Mahmoudi et al. [31] numerically investigated Magneto-Hydro-Dynamic (MHD) natural convection heat transfer in an open cavity. They demonstrated that the average Nusselt number is an increasing function with nanoparticles volume fraction for all Rayleigh numbers, generation/absorption coefficient and Hartmann numbers.

Mejri et al. [32] have performed a numerical study using the Lattice Boltzmann method in a square cavity filled with nanofluid and heated sinusoidally on both walls sides. They used the lattice Boltzmann Simulation in conjunction with the SIMPLER algorithm to couple velocity and temperature fields. The presented results show that for Ra = 5×10^4 and Ha = 20 the heat transfer rate and entropy generation respectively increase and decrease with the increases of volume fraction. In addition, the authors have concluded that, the proper choice of Ra, Ha, γ and ϕ could be able to maximize heat transfer rate simultaneously minimizing entropy generation. Wu et al. [33] investigated numerically natural convection in a rectangular

Chaouki Ali is with the Unité de Recherche Matériaux, Université of Sfax Laboratory of Electro-Mechanical System-LASEM, LR99ES36 3038, Sfax, Tunisia (e-mail: chaouki.ali.fsg@gmail.com).

Bouchmel Mliki and Mohamed Ammar Abbassi are with the Unité de Recherche Matériaux, Energie et Energies Renouvelables (MEER), Faculté des Sciences de Gafsa, B.P.19, Zarroug, Gafsa, 2112, Tunisie(e-mail: bouchmelmliki@hotmail.com, medammar.Abbassi@enim.rnu.tn).

enclosure with sinusoidal temperature distributions on both walls sides using a non-equilibrium thermal model. Their results showed that the value of the maximum stream function decreases with the increase of Raleigh numbers.

MHD effect on nanofluid free convection in a cavity filled with CuO-water nanofluid, having a wall heated sinusoidally was studied numerically by Sheikholeslami et al. [34]. They exhibited that the dimensionless wall temperature amplitude had effects on the flow and heat transfer rates in the cavity. In this study, the effective thermal conductivity and viscosity of nanofluid are calculated by KKL (Koo-Kleinstreuer-Li) correlation. The influence of the Rayleigh number (*Ra*), dimensionless amplitude of the sinusoidal wall temperature (*a*), Hartmann number (*Ha*) and volume fraction of nanoparticles (ϕ) are examined. The results presented show that the Nusselt number is an increasing function of nanoparticles volume fraction, dimensionless amplitude of the sinusoidal wall temperature and Rayleigh number while it is a decreasing function of Hartmann number. Sheikholeslami et al. [35] studied the problem of MHD free convection in a horizontal cylindrical enclosure with an inner triangular cylinder. The effect of nanoparticle volume fraction on the enhancement of heat transfer has been investigated for several value sets of Rayleigh and Hartmann numbers. Their results indicate that the value of the maximum stream function decreases with the increase of Hartmann number. In addition, the results indicate that Nusselt number increases with the increase of Rayleigh number. Effect of thermal radiation on MHD, nanofluid (Al_2O_3-water) flow and convective heat transfer utilizing the two phase model was conducted by Sheikholeslami et al. [36]. The study was carried out by using physical parameters governing flow, such as, Reynolds number, magnetic parameter, rotation parameter, thermophoretic parameter, Brownian parameter and radiation parameter on heat and mass characteristics. The results show that Nusselt number has direct relationship with radiation parameter and Reynolds number while it has reverse relationship with other active parameters. It can also be found that nanoparticle concentration boundary layer thickness decreases with the increase of radiation parameter. In another study, Kalteh and Hasani [37] applied lattice Boltzmann method to investigate the natural convection flows utilizing nanofluids in an L-shaped cavity. Their results showed an increase in average Nusselt number with an increase in nanoparticle volume concentration, for all the considered Rayleigh numbers and aspect ratios. Squeezing of unsteady nanofluid flow and heat transfer has been studied by Sheikholeslami et al. [38]. They showed that for the case in which two plates are moving together, the Nusselt number increases with an increase of the nanoparticle volume fraction and Eckert number while it decreases with the growth of the squeeze number. Some relevant studies on the topic can be seen from the list of references [39]-[43].

The aim of the present study is to identify the ability of Lattice Boltzmann Method (LBM) for solving nanofluid, magnetic field simultaneously in the presence of a linearly/ sinusoidally boundary condition. The effects of Hartmann number, nanoparticle volume fraction and Rayleigh number on the flow and heat transfer characteristics have been examined. The results will be presented via streamlines, isotherms and Nusselt numbers.

II. Problem Description

The considered physical geometry with related parameters and coordinates are shown in Fig. 1. The rectangular cavity (LxH) is filled with water and nanoparticles of Cu. A magnetic field with uniform strength *Bo* is applied in the horizontal direction. The thermo-physical properties of the base fluid and the nanoparticles are given in Table I. It is further assumed that the Boussinesq approximation is valid for buoyancy force.

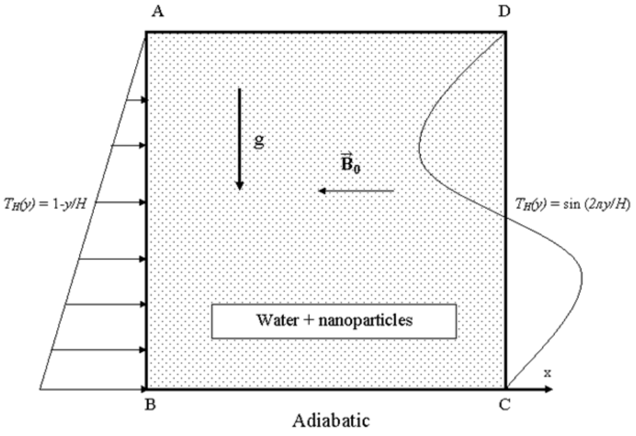


Fig. 1 Geometry of the problem

TABLE I
THERMO PHYSICAL PROPERTIES OF FLUID AND NANOPARTICLES

Physical Properties	H_2O	Cu
C_p *(J/kg.K)*	4179	385
ρ *(kg/m³)*	997.1	8933
k (W/m.K)	0.613	400
$\beta\times10^{-5}$ *(1/K)*	21	1.6

III. Mathematical Formulation

The continuity equation (1), the momentum equations (2) and (3) and the energy equation (4) for MHD natural convection by macroscopic variables are written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{nf}\,(u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\mu_{nf}\,(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}) \tag{2}$$

$$\rho_{nf}(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\mu_{nf}(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})+(\rho\beta_T)_{nf}\,g(T-T_c)-B_0{}^2.\sigma_{nf}v \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})+\frac{Q_0}{(\rho C_p)_{nf}}\left(T-T_C\right) \tag{4}$$

In these equations the non-dimensional parameters are defined by

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{uL}{\alpha_{nf}},\quad V=\frac{vL}{\alpha_{nf}},\quad \theta=\frac{T-T_c}{T_h-T_c},\quad P=\frac{pL^2}{\rho_{nf}\alpha_{nf}^2}$$
$$Pr=\frac{\nu_{nf}}{\alpha_{nf}},\quad Ra=\frac{g\beta(\mathrm{T}-\mathrm{T}_c)\mathrm{L}^3}{\nu_f\alpha_f},\quad q=\frac{\mathrm{Q}_0L^2}{(\rho c_p)_{nf}\,\alpha_{nf}},\quad Ha=HB_0\sqrt{\frac{\sigma_{nf}}{\mu_{nf}}} \tag{5}$$

where σ_{nf} is electrical conductivity of nanofluid, B_0 is the magnitude of the magnetic field and Q_0 is the heat generation or absorption coefficient.

The effective density, heat capacitance, the thermal expansion coefficient, thermal diffusivity and the thermal expansion coefficient of the nanofluid are respectively defined by:

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_p \tag{6}$$

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_p \tag{7}$$

$$\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}} \tag{8}$$

$$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_p \tag{9}$$

IV. SOLUTION METHOD

In the LBM simulations the viscosity is calculated by:

$$\nu_f=NMac_s\sqrt{\frac{\Pr}{\mathrm{Ra}}} \tag{10}$$

To guarantee an incompressible flow the Mach number should be less than $Ma = 0.3$.

The heat transport rate is described by the local Nusselt number Nu:

$$Nu_l=\frac{hH}{k_f} \tag{11}$$

The thermal conductivity of the nanofluid is defined by:

$$k_{nf}=-\frac{q_w}{\partial T/\partial x} \tag{12}$$

Therefore the heat transport coefficient is given by:

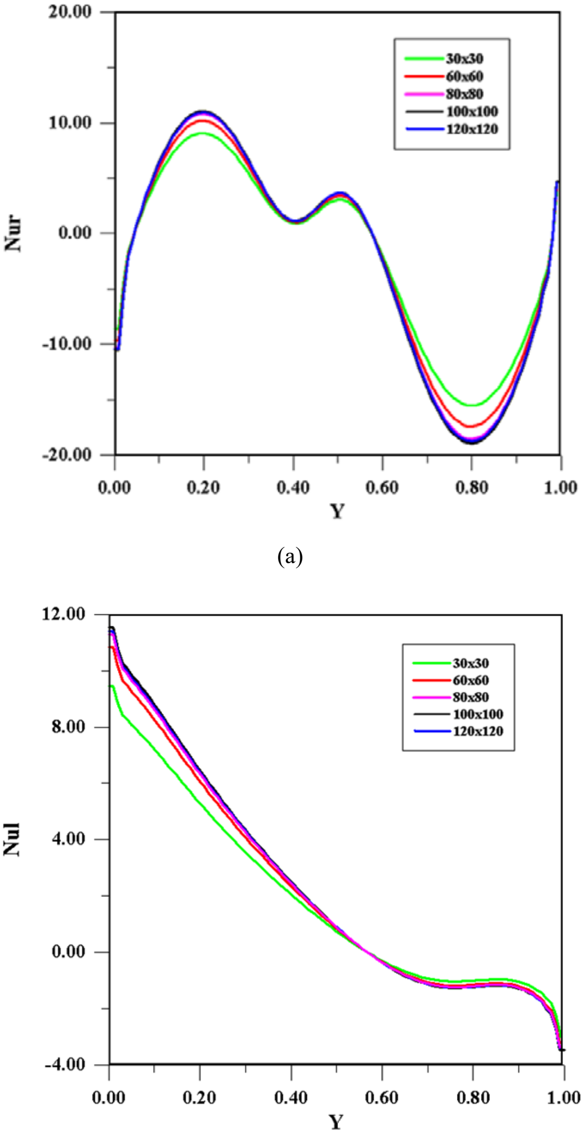
$$h=\frac{q_w}{T_h-T_C} \tag{13}$$

Substituting (12) and (13) into (11), and using the dimensionless quantities, we get the local Nusselt number along the left walls (AB)is calculated by:

$$Nu_{AB}=-\frac{k_{nf}}{k_f}\left(\frac{\partial\theta}{\partial X}\right)\bigg|_{X=0} \tag{14}$$

V. GRID INDEPENDENT TEST AND VALIDATION

First, we examine the dependence of the results to the grid size, and then we validate the numerical methodology used to solve the governing equations. The grid independence test was performed successively using five size grids (30x30, 60x60, 80x80, 100x100 and 120x120) with $\phi = 0.04$, at $Ra = 10^5$.



(a)

(b)

Fig. 2 Local Nusselt number on left and right walls for different uniform grids

As it can be seen in Fig. 2, the local Nusselt number on the right and left walls of the cavity obtained for 100x100 and 120x120 grids are very close. Indeed, the maximal discrepancy is about 0.09%. According to this figure, 100 × 100 lattices number is chosen as the final independent lattice numbers. The comparison of isotherms for $Ra = 10^4$ obtained showed a very good agreement with those of Kefayati [29] (Fig. 3).

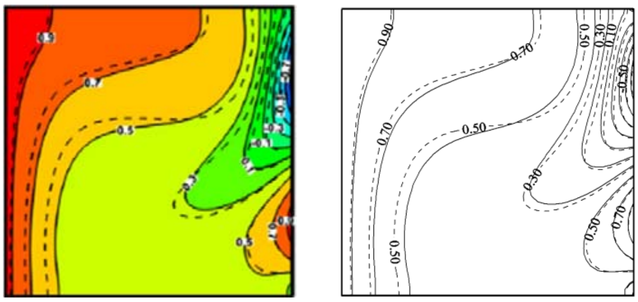


Fig. 3 Validation of Contours of isothermal contour for base fluid (lines) and nanofluid with $\phi = 0.06$ (dashed lines) at $Ra = 10^4$

VI. Results and Discussion

Numerical results are made for four volume fraction of the nanoparticles (ϕ = 0, 0.02, 0.04 and 0.06), Hartmann number (Ha = 0, 30, 60, 90), Rayleigh number ($Ra = 10^3$, 10^4, 10^5 and 10^6), and a wide range of dimensionless heat generation or absorption coefficients (q = -10, -5, 0, 5, 10). In addition, the results for the distribution of u and v velocities in the middle of the cavity, at various volume fractions of the nanoparticles will be presented, illustrated and discussed.

Figs. 4 and 5 illustrate the effect of Hartmann number on the isotherms and streamlines with $\phi = 0.04$ for various Rayleigh numbers as the heat generation or absorption coefficients q is equal to 0. Examining (3), the sign of Ha is opposite to the Ra^* one in source term. Therefore, there is an opposite effect of Ra^* and Ha on flow regime and heat transfer. For all Rayleigh and Hartmann numbers considered in this study, the flow structure is described by a clockwise cell with elliptical core. The intensity of these cells, characterized by $|\psi_{max}|$ increases with the Rayleigh number and decreases as the Hartmann number increases. This is because when the Hartmann number increases (Ha = 30, 60 and 90), the Lorentz force due to the magnetic field effect becomes higher than the buoyancy force which causes to reduce the flow circulation intensity and as a result the convection effect begins to diminish.

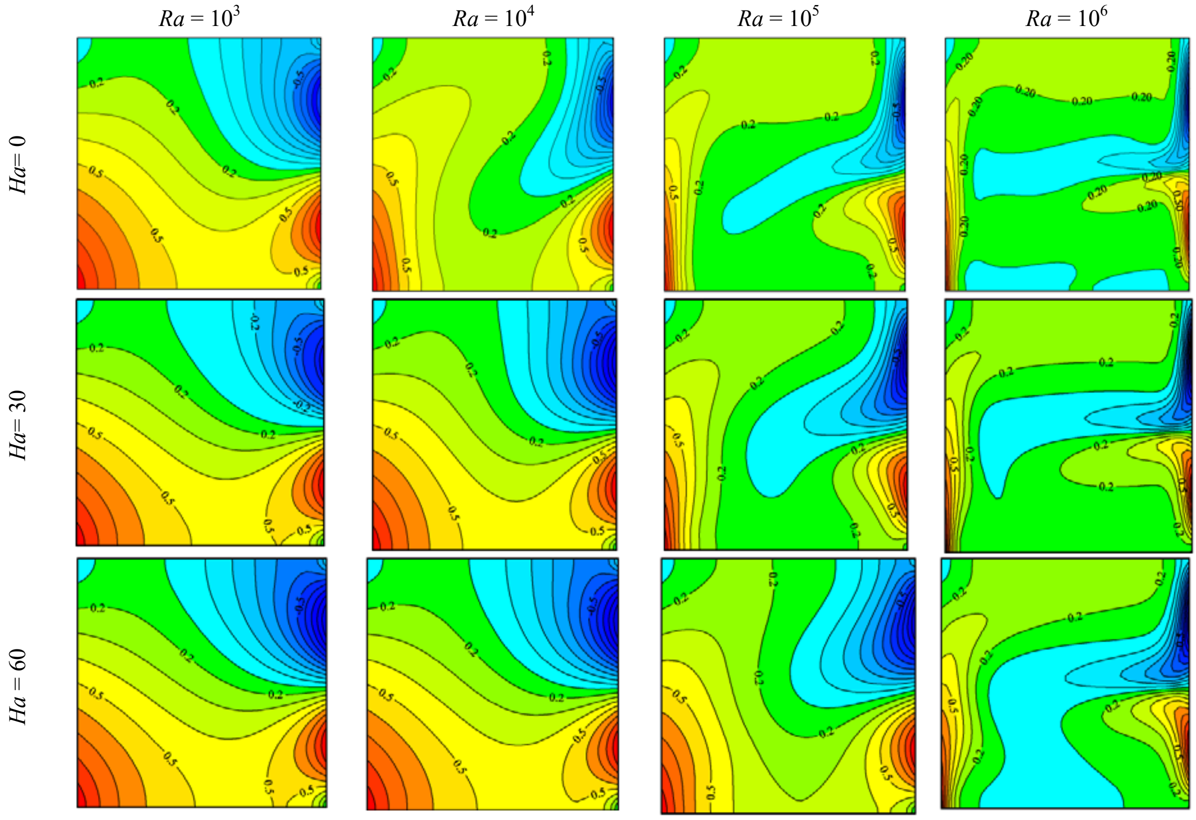


Fig. 4 Isotherms for different Ra and Ha at $\phi = 0.04$ (water-Cu nanofluid)

$Ra = 10^3$ $Ra = 10^4$ $Ra = 10^5$ $Ra = 10^6$

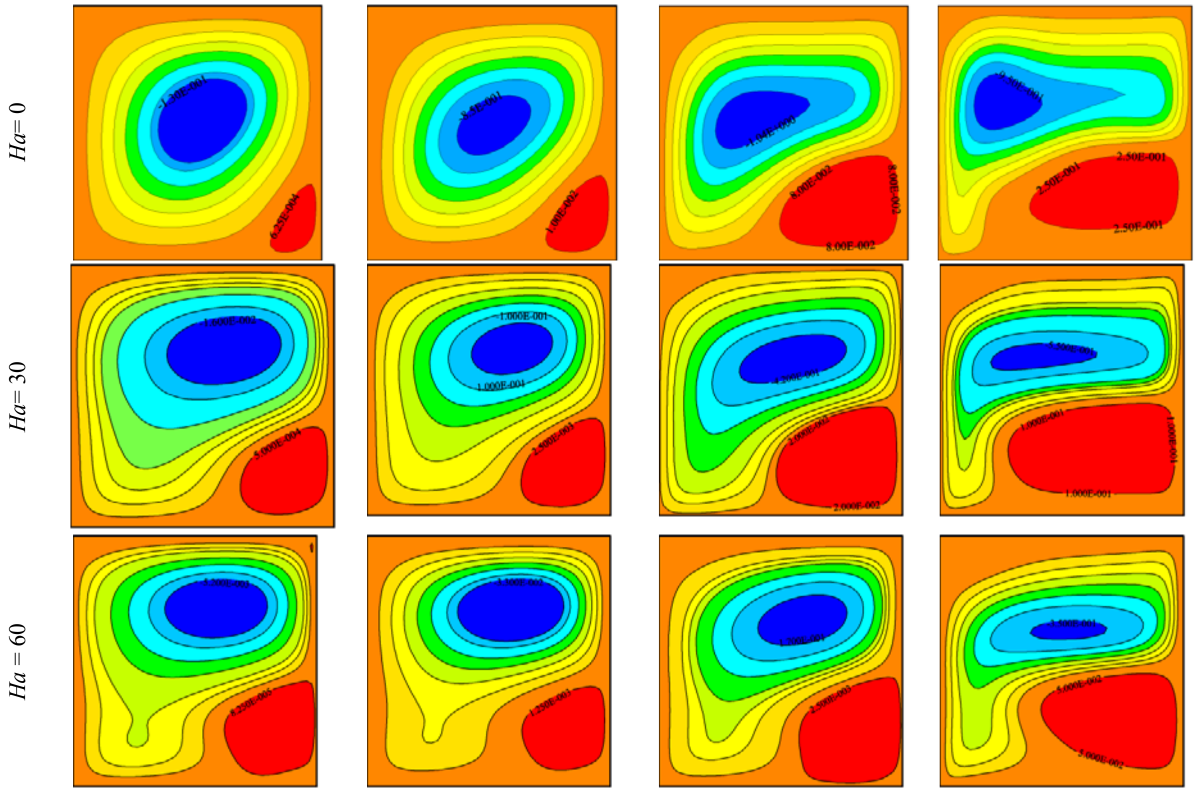


Fig. 5 Streamlines for different *Ra* and *Ha* at ϕ= 0.04 (water-Cu nanofluid)

At *Ra* = 10^6, the effect of the Hartmann number on the isotherms is more noticeable than *Ra* = 10^3 where the difference between the compared isotherms augments markedly. On the other hand, as the Hartmann number increases a reduction on the temperature gradients near the cavity wall occurs. This is an indication for the approach of the quasi-conduction regime within the cavity. This occurs due to the opposite effect of Ra and Ha on heat transfer. On the contrary, by increasing of the Rayleigh number, the effect of the convective heat transfer becomes more significant, and distinct boundary layers form along the active walls (AB) of the cavity. In addition, for all Rayleigh numbers, the effects of Hartmann number on the streamlines were illustrated by the form of the streamlines central circulation. As Hartmann number increases, the streamlines stretch upward whereas the power of the secondary bigger circulation in the bottom right corner of the cavity rises. Globally, increasing Hartmann number causes Lorenz force to increase and leads to a substantial suppression of the convection. Also, the results obtained show that the intensities of the temperature of the fluid in the cavity decrease as a result of the application of the magnetic field, especially in higher Rayleigh numbers.

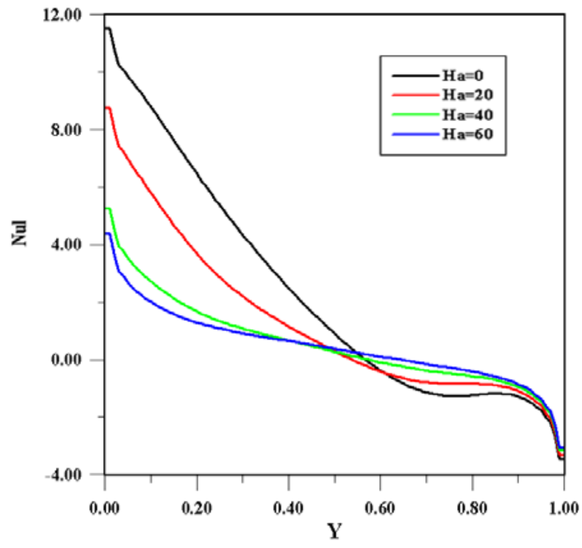
The effects of the Hartmann numbers on the average Nusselt number at left and Right sidewalls have been studied for *Ra* = 10^5 and ϕ = 0.04 on Fig. 6. It is clear that the local Nusselt number decreases with an increase in the Hartmann number. This is due to the magnetic damping effect that suppresses the overall heat transfer in the enclosure. In addition, the maximum effect of the Hartmann number on the local Nusselt number were obtained in different positions where it is at y = 0 and 0.8 at left and right sidewalls respectively.

VII. CONCLUSIONS

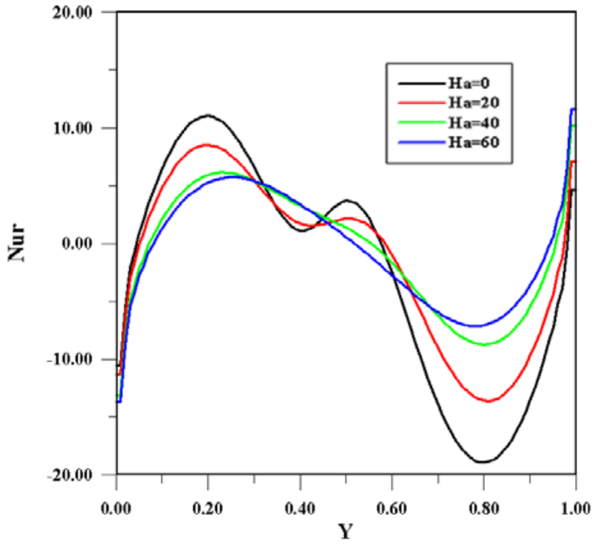
This paper, MHD natural convection heat transfer performance in a cavity filled with Cu–water nanofluids is investigated numerically. The effects of Rayleigh and Hartmann numbers on the flow and heat transfer characteristics have been examined.

1) The maximum stream function values increase with the Rayleigh number and decrease as the Hartmann number increases.
2) The magnetic field reduces the heat transfer and the fluid flow intensity.
3) The maximum stream function values increase with the Rayleigh number and decrease as the heat generation/ absorption coefficient increases.
4) The trends of temperature profiles depict that the effect of heat generation/absorption coefficient decreases strongly with the increase of Rayleigh number.
5) For *Ra* = 10^5 and $q > 0$, the nanofluid temperature is higher than that of the hot wall. Therefore, the hot vertical wall receives heat from the nanofluid. This leads to negative Nusselt number at left wall. Conversely, an opposite effect occurs in the heat absorption condition $q < 0$.
6) For *Ra* = 10^6, the effect of heat generation or absorption coefficient is vanished.

7) The addition of the Cu–particle provokes the magnitude of velocity in the cavity to plummet noticeably.



(a)



(b)

Fig. 6 Local Nusselt number on left and right walls for different *Ha* at $Ra = 10^5$ and $\phi = 0.04$ (water-Cu nanofluid)

NOMENCLATURE

B	Magnetic field, *Tesla = N/A.m²*
d_p	diameter of particle, *nm*
Ha	Hartmann number
k	Thermal conductivity, *W/m.K*
Ma	Mach number
Nu	Local Nusselt number
P	Pressure, N/m^2
Pr	Prandtl number
Ra	Rayleigh number
T	Temperature, *K*
$\mathbf{u}(u, v)$	Velocities, *m/s*
$\mathbf{x}(x, y)$	Lattice coordinates, *m*
H	height of cavity, *m*

Greek Symbols

α	Thermal diffusivity, m^2/s
β	Coefficient of thermal expansion, K^{-1}
k_b	Boltzmann constant, $J.\ K^{-1}$
Δx	Lattice spacing
Δt	Time increment
ϕ	Solid volume fraction
μ	Dynamic viscosity, *kg/m.s*
ρ	Fluid density , kg/m^3
θ	Non-dimensional temperature
ν	Kinematic viscosity, m^2/s
σ	Electrical conductivity, $(\Omega.m)^{-1}$

Subscripts

c	cold surface
f	fluid
h	hot surface
nf	nanofluid
p	particle

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