

Improving Lubrication Efficiency at High Sliding Speeds by Plasma Surface Texturing

Wei Zha, Jingzeng Zhang, Chen Zhao, Ran Cai, Xueyuan Nie

Abstract—Cathodic plasma electrolysis (CPE) is used to create surface textures on cast iron samples for improving the tribological properties. Micro craters with confined size distribution were successfully formed by CPE process. These craters can generate extra hydrodynamic pressure that separates two sliding surfaces, increase the oil film thickness and accelerate the transition from boundary to mixed lubrication. It was found that the optimal crater size was 1.7 μm , at which the maximum lubrication efficiency was achieved. The Taguchi method was used to optimize the process parameters (voltage and roughness) for CPE surface texturing. The orthogonal array and the signal-to-noise ratio were employed to study the effect of each process parameter on the coefficient of friction. The results showed that with higher voltage and lower roughness, the lower friction coefficient can be obtained, and thus the lubrication can be more efficiently used for friction reduction.

Keywords—Cathodic plasma electrolysis, friction, lubrication, plasma surface texturing.

I. INTRODUCTION

FRICITION and wear are the severe problems which significantly influence the energy consumption [1] and lifetime [2] of the mechanical components. Hard coating is conventionally applied to reduce the friction and wear by thermal spray [3]-[5], electroplating [6] and hot dipping [7]. But some limitations are related with these techniques such as high cost, complicated process, inability to treat workpieces with irregular shapes and the environmentally unfriendly electrolytes [8]. The use of lubricant is an effective method to control wear and friction. Lubrication could be divided into three regimes: boundary, mixed and hydrodynamic (elastohydrodynamic) lubrication [9]. In boundary lubrication, the two sliding surfaces fully contact with each other and the friction is completely determined by the properties of the surfaces, such as roughness, skewness and kurtosis [10]. In mixed lubrication, the oil film thickness boosts to a value that is high enough to partially support the applied load and the friction is determined by the properties of lubricant and asperities together [11]. In hydrodynamic lubrication, the two sliding surfaces are completely separated by the lubricant. Since no asperities contact occur, the friction is only related to the viscosity of lubricant and the wear can be almost avoided [12]. Similar to hydrodynamic lubrication, in elastohydrodynamic lubrication the lubricant fully supports the

applied load. However, since the extremely high pressure is applied, the significant elastic deformation of the surfaces occurs and influences the thickness of the lubricant film [13].

For last decades, surface texturing was developed to improve the efficiency of lubrication and thus more effectively reduce the wear and friction [14]. The effects of surface texturing vary when the sliding system is in different lubrication regimes. The craters act as micro reservoirs for lubricant so that when two surfaces slide in boundary lubrication, the lubricant can be supplied from the craters to the sliding interface, although the lubricant is squeezed out from the contact region due to the high pressure [11]. Besides that, the craters can trap the wear debris to prevent the severe abrasive wear until they are filled [15]. In hydrodynamic lubrication, the negative pressure in the diverging region of a dimple is limited by the cavitation, while the positive pressure in the converging region is not limited, leading to the positive net pressure that support the applied load [16], [17]. Above effects work together in mixed lubrication to reduce the friction and wear [18].

So far, surface texturing has been applied in many fields to control the friction, such as cutting tools [19]-[21], mechanical seals [22]-[24], journal bearings [25]-[27], cylinder liners [28]-[30] and piston rings [31]-[33]. The textured surface exhibits low friction coefficient and high wear resistance. Laser surface texturing [34], [35] is one of the most popular techniques that are being used. It can produce accurate textures with the help of computer software. However, the expensive high-energy-laser sources, complicated control software, and masks used to create the certain patterned textures limit the application of Laser surface texturing in massive industrial production. Micro-electric discharging machining [20] and chemical etching [36] could produce surface textures without changing the mechanical properties of the workpieces or causing no harm to the target surface, but the material removal rate is low.

This paper aims to apply CPE on cast iron samples to improve their tribological behavior. During CPE surface texturing process, the surfaces of cast iron samples (cathode) are covered by the gas bubbles, resulting in the accumulation of the electrons. The applied voltage across the electrodes mainly drops in the gaseous envelope, generating high electric field. As the electric field strength reaches to a critical value (106 ~ 108 V/m), the gas starts to ionize, forming uniformly distributed plasma discharges on the whole sample surface [37]. Heated by the extremely high temperature plasma core (6000-8000 K) [38], the gas bubbles explode melting the metal surface and forming the craters. The effects of applied voltages and surface roughness on the friction coefficient are studied by the Taguchi method. The relationship between the crater size and the

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lubrication efficiency is also investigated.

II. EXPERIMENTAL DETAILS

A. Sample Preparation

Cast iron (Fe: 93%, C: 3%, Mg: 0.8%, P: 0.1%, Si: 2%, S: 0.05%) rings (outer diameter: 100 mm and inner diameter: 70 mm) were polished with grit 1200 sand papers and ultra-sonic cleaned with acetone. As shown in Fig. 1, during CPE process the sample and the nozzle acted as cathode and anode, respectively. The electrolyte (25-30 g/L Na_2CO_3 dissolved in deionized water) was pumped (1.5 L/min) to the nozzle and uniformly sprayed to the sample surface. The cathodic plasma discharging process was conducted for 60 s under DC mode at 180 V (at which the plasma discharges initiated), 200 V and 220 V (above which abnormal arcing occurred), respectively. The gap between the nozzle and sample surface was set to be about 6 mm. When uniform plasma envelope was built, the current density was 1.5 A/cm². During the whole process, the temperature of the electrolyte was kept at 35-40 °C with the chiller. A blank sample with no treatment was also prepared as a reference. The roughness of samples prepared at 180, 200 and 220 V were 0.80, 1.01 and 1.25 μm , respectively.

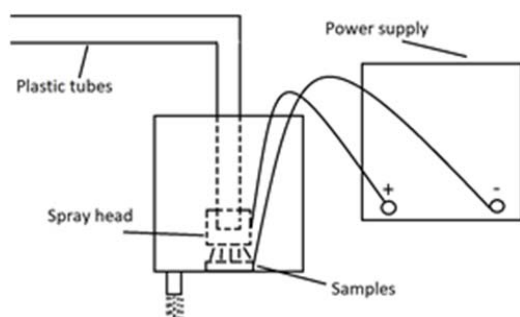


Fig. 1 Experimental setup of CPE

B. Tribotests

For the tribotests, all samples were polished to three levels of surface roughness: 0.7, 0.6 and 0.5 μm . The surface morphologies were observed with the optical microscope. The crater size was obtained from the microscopic images with a photo analysis software. The tribological behavior of the CPE surface textured samples were studied by the Pin-on-disk sliding tester. The rotational speed of the tester could reach 960 rpm corresponding to sliding velocity of about 4.5 m/s. All the tests were conducted at room temperature with SAE 5W20 as the lubricant. AISI 52100 steel balls (62 HRC, 6 mm in diameter) were used as pins with a normal load of 5 N. During the tribotests, the sliding velocity increased from 0 to 4.5 m/s, and the coefficient of friction (COF) vs. sliding velocity curves were recorded.

III. RESULTS

A. Surface Morphology

The surface morphologies of CPE surface textured samples after polishing are shown in Fig. 2. As demonstrated in Fig. 2 (a), the crater size was very small at 180 V, which was the

critical voltage for initiation of the plasma. Thus, the energy of the gas bubble explosion was not high enough to generate large craters. As the voltage increased by 20 V, the crater size increased to about 1.8 μm , as illustrated in Fig. 2 (b). However, occasionally large crater with diameter of 4 μm could be found in Fig. 2 (c) when the voltage increased to 220 V. Under such high voltage, more electrons accumulated on the gas bubbles, leading to the high energy explosion. It has been verified that the crater size of the sample treated by higher voltage was larger than the one treated by lower voltage after they are polished to the same roughness. Similar results can be observed in Figs. 2 (d)-(f) and (g)-(i).

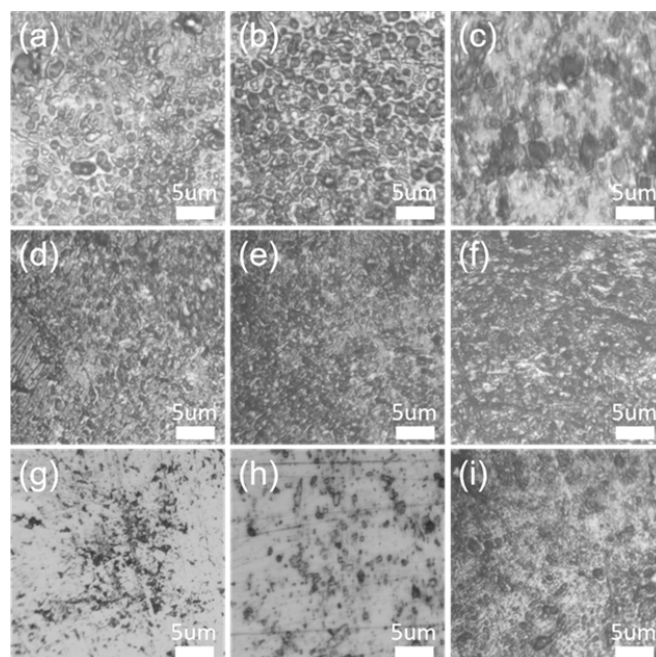


Fig. 2 The surface morphologies of CPE treated samples polished to obtain the roughness of 0.7 μm at (a) 180 V, (b) 200 V and (c) 220 V; the surface morphologies of CPE treated samples polished to obtain the roughness of 0.6 μm at (d) 180 V, (e) 200 V and (f) 220 V; the surface morphologies of CPE treated samples polished to obtain the roughness of 0.5 μm at (g) 180 V, (h) 200 V and (i) 220 V

B. Frictional Behaviour

The COF vs. sliding velocity curve of the reference sample is shown in Fig. 3 (a). It can be found that although the velocity increased from 0 to 4.5 m/s, the COF barely decreased, which means the tribosystem was working in boundary lubrication. Figs. 3 (b)-(d) show the COF vs. velocity curves of CPE surface textured samples. When the velocity was very low, the COF stayed at a relatively high value indicating that the lubricating system was in the boundary regime. As the velocity increased, the COF dropped steeply, and the lubrication was transitioned into mixed regime. At maximum sliding velocity, the COF still decreased but with a much lower rate, which means the tribosystem was still in mixed lubrication but going to enter elastohydrodynamic lubrication (EHL) if the velocity kept increasing. It was noticed that at certain velocity, the COF of each sample was different. The lowest value of COF of each

sample was collected and used as the responses for Taguchi design.

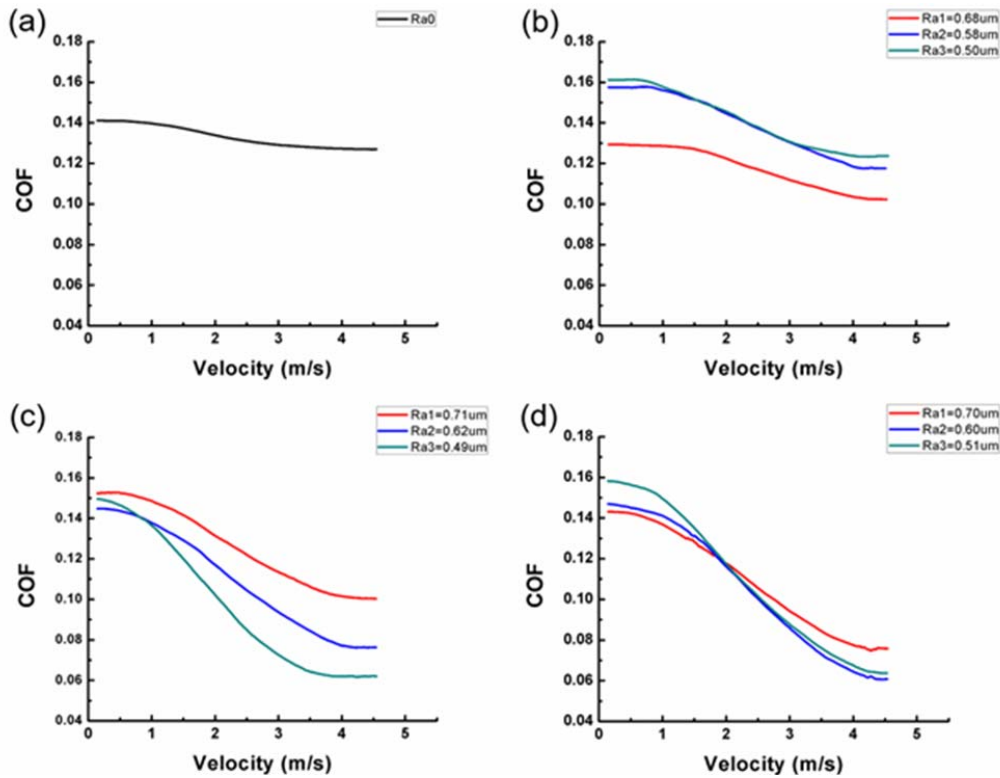


Fig. 3 The COF vs. Sliding velocity curves of (a) reference, (b)-(d) CPE treated samples at 180 V, 200 V and 220 V

C. Taguchi Design

Run	Voltage (V)	Roughness (μm)	Lowest COF	S/N ratio
1	180	0.68	0.108	19.33
2	180	0.58	0.125	18.06
3	180	0.50	0.130	17.72
4	200	0.71	0.090	20.91
5	200	0.62	0.075	22.49
6	200	0.49	0.060	24.43
7	220	0.70	0.075	22.49
8	220	0.60	0.058	24.73
9	220	0.51	0.060	24.43

Fig. 4 The results for COFs and S/N ratios

To select an appropriate orthogonal array, the total degrees of freedom of process parameter *s* (applied voltage and surface roughness) need to be calculated first. In this work, each process parameter had three levels. So the degree of freedom of each parameter was 2. Since the interaction between the process parameters was neglected, the total degrees of freedom were 4. Basically, the degrees of freedom for the orthogonal array should be larger than that of the process parameters. So a L9 orthogonal array was chosen. Only nine experiments needed

to be conducted to study the entire combination of the process parameters.

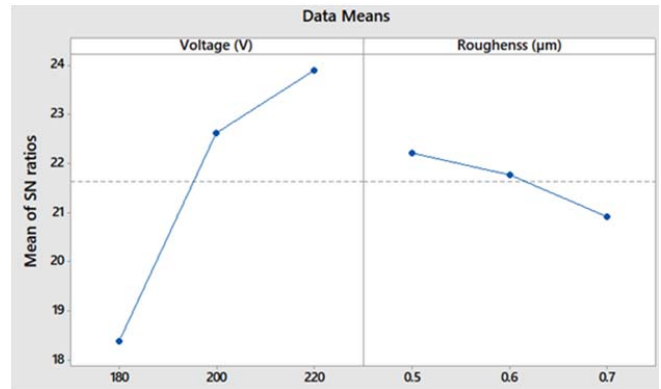


Fig. 5 Plots of effects of main S/N ratios

To obtain optimal friction performance, the lower-the-better formula: $S/N = -10\log(y^2)$ for COF was used [39]. The experimental results for COF and corresponding S/N ratio are shown in Fig. 4. The effect of each process parameter at different levels can be separated out by taking the mean value of S/N ratio for the certain level. The mean S/N ratio for 180 V, 200 V and 220 V can be calculated by taking the average of the S/N ratio for the experiments 1-3, 4-6 and 7-9, respectively. The mean S/N ratio for each level of roughness can be

computed in the same way. Fig. 5 shows the mean S/N ratio graph for COF. It can be found that the mean S/N ratio increases non-linearly with the increase of voltage and the decrease of roughness. When the voltage increased from 180 V to 200 V, the mean S/N ratio dramatically increased. But when the voltage increased to 220 V, the mean S/N ratio increased slowly. It indicates that there might be an optimal value of crater size that results in the lowest COF.

IV. DISCUSSION

Hamrock and Dowson [40] proposed to calculate the film thickness between two smooth sliding surfaces with an empirical equation:

$$h_c = 2.69U^{0.67}G^{0.53}W^{-0.067} \left[1 - 0.61 \exp \left(-0.73D^{-2/\pi} \right) \right]$$

where the dimensionless speed parameter $U = \frac{u\eta_0}{E'R_x}$, the dimensionless load parameter $W = \frac{\omega}{E'R_x^k}$ and the dimensionless load parameter $G = \alpha E'$. In the current work, dynamic viscosity of lubricant η_0 is 0.1, the effective elastic modulus of the surfaces E' is 68.3 GPa, reduced radius of curvature R_x is 5.35×10^{-5} m, The external load ω is 5 N, k is 2 for elliptical contact, lubricant pressure-viscosity coefficient α is 1.3×10^8 .

The film thickness ratio $\lambda = hc/Ra$ was developed to determine in which lubrication regimes the sliding system is [41] where hc is film thickness and Ra is surface roughness. If $\lambda < 1$, it is in boundary lubrication; if $1 < \lambda < 3$, it is in mixed lubrication; and if $\lambda > 3$, it is in hydrodynamic lubrication. For the reference sample ($Ra = 0.16$), the film thickness ratio was calculated as $\lambda = 0.85$ even the sliding speed goes up to 4.5 m/s, which verifies that the reference sample stayed in boundary lubrication during the whole tribotest.

The λ of those CPE surface textured samples were also less than 1 for the whole velocity range, but the drastic decrease of COF along with increasing sliding velocity indicated that those tribosystems had already entered mixed lubrication. This phenomenon could be explained by the extra hydrodynamic pressure generated by the craters, which were not considered in Hamrock and Dowson's theory. This extra hydrodynamic pressure could lift the counterpart, so that the film thickness would increase even the speed was not very high. Therefore, the divergence between the experimental results and Hamrock and Dowson's theory represented the strength of extra hydrodynamic pressure and could be used to scale the effectiveness of the surface texturing that can improve the lubrication efficiency. To evaluate the divergence, the critical film thickness ratio λ_c , at which the transition between boundary and mixed lubrication occurred, was calculated based on the COF vs. sliding velocity curves. The lower λ_c , the higher divergence and thus the higher lubrication efficiency. The curve of λ_c vs. crater size is shown in Fig. 6. It can be found that the λ_c value of all samples was much less than 1, which means that surface texturing can significantly improve the lubrication efficiency. Moreover, as the crater size increased, the λ_c decreased to about 0.1 and then increases to about 0.16. Thus,

the optimal crater size was about 1.7 μm . If the crater size is less than this value, the hydrodynamic pressure is weaker and can only support the two sliding surfaces at higher speed. If the crater size is larger than that, the film becomes unstable since the deep craters might destroy the film.

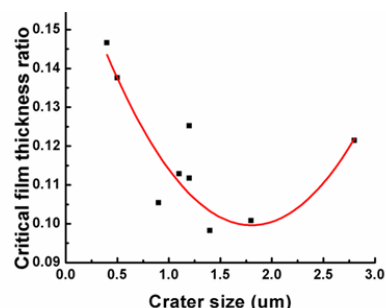


Fig. 6 Mapping nonlinear data to a higher dimensional feature space

V. CONCLUSION

CPE was used to produce surface textures on cast iron samples. The Taguchi experimental design was used to investigate the optimal process parameters (applied voltage and roughness) that resulted in the lowest COF at high sliding speed. It was concluded that the lower COF could be obtained at higher voltage and lower roughness. Since a larger crater size produced by a higher voltage could generate a larger hydrodynamic pressure, the result of the enlarged crater could lead to occurrence of separation of the sliding contact surfaces shifting to a lower sliding velocity. In contrast to a rougher cratered surface, a smooth surface would benefit the formation and stability of lubricant film. The optimal voltage and roughness were 220 V and 0.5 μm analyzed by the S/N ratio diagram where a minimum COF of about 0.06 could be obtained at velocity of 4.5 m/s. The critical film thickness ratio λ_c at which the tribosystem was transitioned from boundary to mixed lubrication was used to determine the lubrication efficiency. Better lubrication efficiency could be obtained if lower critical film thickness ratio was calculated, since the tribosystem was transitioned into mixed lubrication at lower velocity. It was concluded that the lubrication efficiency was improved for all CPE surface textured samples compared with the untextured one. Increasing the crater size could improve the lubrication efficiency before the highest value was obtained at optimal crater size of about 1.7 μm . After that, the further increase in the crater size would lead to the decrease of lubrication efficiency.

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