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Generalized Chaplygin Gas and Varying Bulk Viscosity in Lyra Geometry

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***Abstract*—**In this paper, we have considered Friedmann-Robertson-Walker (FRW) metric with generalized Chaplygin gas which has viscosity in the context of Lyra geometry. The viscosity is considered in two different ways (i.e. zero viscosity, non-constant ρ (rho)-dependent bulk viscosity) using constant deceleration parameter which concluded that, for a special case, the viscous generalized Chaplygin gas reduces to modified Chaplygin gas. The represented model indicates on the presence of Chaplygin gas in the Universe. Observational constraints are applied and discussed on the physical and geometrical nature of the Universe.

***Keywords*—**Bulk viscosity, Lyra geometry, generalized Chaplygin gas, cosmology.

I. INTRODUCTION

ONE of the most important cosmological observations obtained by type Ia supernova is that the Universe is dominated by dark energy with negative pressure which provides the accelerated phase of expansion of the Universe [1]-[3]. Nowadays, the accelerated expansion of the Universe is a hot topic as it depends upon the theoretical model for interpretation. Also, maximum models are associated with multi-scalar fields in the presence of a cosmic fluid. The cosmic fluid (otherwise known as “dark energy”) with a negative pressure is responsible for the accelerated expansion of the Universe. There are several models to discuss on dark energy such as phantom [4], tachyon [5], holographic dark energy [6], K-essence [7] and various models of Chaplygin gas. Here, we are interested to represent the Chaplygin gas (CG) as a model of dark energy [8]-[10] and to extend the General Chaplygin gas to the modified Chaplygin gas[11]. As, we know the bulk viscosity plays an important role to drive the present acceleration of the universe. The interaction of Chaplygin gas with bulk viscosity first proposed by [12] and bulk viscous effect and Chaplygin gas in FRW cosmology for the case of flat space-time and Freidman equation due to CG which has bulk viscosity is considered by [13]. The viscous generalized Chaplygin gas model is considered as a proposed model to describe the observed accelerated expansion of the universe. Also, the effect of viscous fluid in modified gravity theories is discussed to showcase the accelerating expansion of the universe [14], [15]. In recent years, there has been a lot of interesting results on viscous generalized Chaplygin gas interaction with different modified theories of the Universe is discovered. Many researchers has investigated on generalized Chaplygin gas such as Chaubey [16], [17] has obtained the role of modified Chaplygin gas in Bianchi type-I universe, Saadat [18] has investigated on generalized Chaplygin gas with varying bulk viscosity in FRW universe, Baffou [19] studied generalized Chaplygin gas with varying bulk viscosity in f(R,T) gravity etc. Inspired by the above authors, we have studied on bulk viscous Chaplygin gas in Lyra Geometry [20]. The paper is organized as follows: In Section II, we described the necessary field equations of Lyra geometry and energy momentum tensor for the model with their derived solutions. The physical significance and brief discussion of the model are represented in Section III.

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II. FIELD EQUATIONS AND THEIR SOLUTIONS

Here, we have considered FRW metric in the form

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega^2), \tag{1}$$

where $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$and a represents the scale factor. The $\theta\ and\ \phi$ parameters are the usual azimuthal and polar angles of spherical co-ordinate with $0\leq \theta \leq \pi$ and $0\leq \varphi \leq 2\pi$, the co-ordinates (t, r, θ, φ) are called commoving coordinates.

The Einstein’s field equations for Lyra geometry proposed by Sen [21] in normal gauge can be written as

$$R_i^j - \frac{1}{2}Rg_i^j + \frac{3}{2}\varphi_i\varphi^j - \frac{3}{4}g_i^j\varphi_k\varphi^k = -T_i^j \tag{2}$$

The energy momentum tensor for bulk viscosity can be defined as

$$T_i^j = (\bar{p} + \rho)u_i u^j - \bar{p}g_i^j \tag{3}$$

where $\bar{p} = p - 3\xi H$, and ξ stands for viscosity. Here, the viscosity is taken as a function of energy density ρ in two different cases.

For the Chaplygin gas, we here introduced the EoS of Chaplygin gas

$$p = -\frac{A}{\rho^\alpha}. \tag{4}$$

φ_i is the displacement vector given by

$$\varphi_i = (0,0,0,\beta(t)) \tag{5}$$

and other symbols have their usual meaning as in Riemannian geometry.

Einstein field equations of Lyra geometry for the space-time (1) can be calculated using the fluids description of energy momentum tensor as

$$12H^2 - 3\beta^2 = 4\rho \tag{6}$$

$$8\dot{H} + 3H^2 + 3\beta^2 = -\bar{p} \tag{7}$$

where an over dot stands for derivatives of corresponding field variable w.r.t. cosmic time t. $H = \frac{\dot{a}}{a}$ is the Hubble parameter and $a(t)$ is average scale factor. The energy conservation equation leads to

$$\dot{\rho} + 3H(\rho + \bar{p}) = 0 \tag{8}$$

$$2\beta\dot{\beta} + 6H\beta^2 = 0 \tag{9}$$

Equation (9) immediately reduces to

$$\dot{\rho} + 3H\left(\rho - \frac{A}{\rho^\alpha} - 3\xi H\right) = 0 \tag{10}$$

$$\beta = \beta_0 a^{-3} \tag{11}$$

where β_0 is the integrating constant or rest displacement vector for present time. The accelerated expansion in the present epoch is usually accredited to a fluid with negative pressure and hence, the bulk viscosity has greater impact than the usual pressure. However, the pressure of an exotic dark energy form leads to a negative pressure of the universe which simulates on anti-gravity effect that drives accelerated expansion. We here considered the deceleration parameter as a constant to treat the Einstein field equations in a simpler way, which gives $H = \frac{1}{(\gamma+1)t}$. As per the recent observations, the universe is in accelerated trend. So, we can consider the deceleration parameter as a negative constant that less than one. Now, (6) and (7) together gives

$$3\beta^2 + 4\dot{H} = -2\left(\rho - \frac{A}{\rho^\alpha} - 3\xi(\rho)H\right) \tag{12}$$

Here, we proceed our study using two different cases. In case-I, we will study the model for zero viscosity and for the case-II we will study the behavior of the model by assuming the viscosity coefficient as a function of energy density that is proportional toρ^n.

A. Case-I Here We Choose $\xi(\boldsymbol{\rho}) = \mathbf{0}$

Equation (12) leads to

$$\rho^{\alpha+1} + \rho^\alpha\left(\frac{3}{2}{\beta_0}^2 t^{-\frac{6}{\gamma+1}} - \frac{2}{(\gamma+1)t^2}\right) - A = 0. \tag{13}$$

Let us consider

$$\chi(t) = \frac{3}{2}{\beta_0}^2 t^{-\frac{6}{\gamma+1}} - \frac{2}{(\gamma+1)t^2} \tag{14}$$

Here, one can notice that (13) is well defined for $\gamma \neq -1$. For $\alpha = 1$,

$$\rho = \frac{-\chi(t) \pm \sqrt{\chi^2(t) + 4A}}{2} \tag{15}$$

Since $\chi^2(t) + 4A \geq 0$, (15) leads two distinct values of ρ. The effective pressure for $\boldsymbol{\alpha} = \mathbf{1}$,

$$\bar{p} = -A\left(\frac{-\chi(t) \pm \sqrt{\chi^2(t) + 4A}}{2}\right) \tag{16}$$

For $\alpha = -1$,

$$\rho = \frac{3(\gamma+1){\beta_0}^2 t^{\frac{2(\gamma-2)}{\gamma+1}} - 4}{2(\gamma+1)t^2(A-1)} \tag{17}$$

Here, we noticed that the energy density is diverges for supper-explosion and the value of the energy density is positive later on.

The effective pressure for $\boldsymbol{\alpha} = -\mathbf{1}$,

$$\bar{p} = -\left(\frac{A\left(3(\gamma+1){\beta_0}^2 t^{\frac{2(\gamma-2)}{\gamma+1}} - 4\right)}{2(\gamma+1)t^2(A-1)}\right) \tag{18}$$

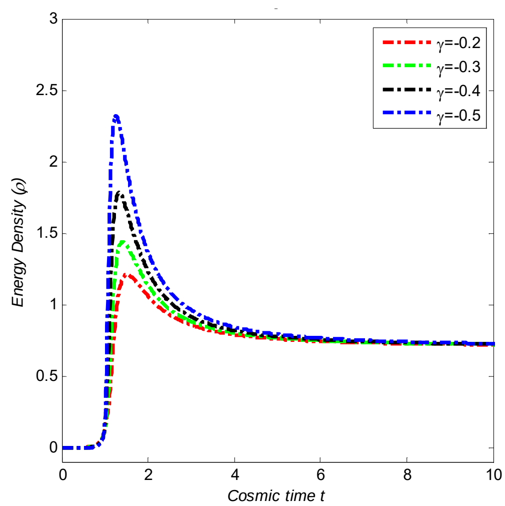

Fig. 1 Variation of energy density $\rho = \frac{-\chi(t) + \sqrt{\chi^2(t) + 4A}}{2}$, $\alpha = 1$ versus time for case-I

B. Case-II Here We Choose $\xi(\boldsymbol{\rho}) = \boldsymbol{\xi_0 \rho^n}$

Using this assumption, (12) leads to

$$\rho^{\alpha+1} + \rho^\alpha \chi(t) - \frac{3\xi_0}{(\gamma+1)t}\rho^{\alpha+n} = A \tag{19}$$

One can notice that (19) does not have a solution for$\gamma = -1$. For$\alpha = -1$ and $n = 1$,

$$\rho = \frac{3(\gamma+1){\beta_0}^2 t^{\frac{2(\gamma-2)}{\gamma+1}} - 4}{2t\{(\gamma+1)(A-1)t + 3\xi_0\}} \tag{20}$$

The effective pressure for $\alpha = -1\ and n = 1$

$$\bar{p} = -\left(\frac{\left(3(\gamma+1)\beta_0{}^2 t^{\frac{2(\gamma-2)}{\gamma+1}} - 4\right)((\gamma+1)At + 3\xi_0)}{2(\gamma+1)\{(\gamma+1)(A-1)t + 3\xi_0\}t^2}\right) \tag{21}$$

For$\alpha = -1$ and n = 2,

$$\rho = \frac{(1-A) \pm \sqrt{(1-A)^2(\gamma+1)^2 t^3 + 6\xi_0\left[3(\gamma+1)\beta_0{}^2 t^{\frac{2(\gamma-2)}{\gamma+1}} - 4\right]}}{6\xi_0\sqrt{t}} \tag{22}$$

Here, the effective pressure for$\alpha = -1$ and $n = 2$

$$\bar{p} = -\left(\frac{6(\gamma+1)At^{\frac{3}{2}}\left\{(1-A) \pm \sqrt{(1-A)^2(\gamma+1)^2 t^3 + 6\xi_0\left[3(\gamma+1)\beta_0{}^2 t^{\frac{2(\gamma-2)}{\gamma+1}} - 4\right]}\right\} + 3\xi_0\left\{(1-A) \pm \sqrt{(1-A)^2(\gamma+1)^2 t^3 + 6\xi_0\left[3(\gamma+1)\beta_0{}^2 t^{\frac{2(\gamma-2)}{\gamma+1}} - 4\right]}\right\}^2}{36(\gamma+1)\xi_0 t^2}\right) \tag{23}$$

For$\alpha = 1\ and n = 1,$

$$\rho = \frac{-\chi(t) \pm \sqrt{\chi^2(t) + 4A\chi_1(t)}}{2\chi_1(t)} \tag{24}$$

where

$$\chi_1(t) = 1 - \frac{3\xi_0}{(\gamma+1)t} \tag{25}$$

The effective pressure for $\alpha = 1$ and $n = 1$ is

$$\bar{p} = -\left(\frac{2A\chi_1(t)}{-\chi(t) \pm \sqrt{\chi^2(t) + 4A\chi_1(t)}} + \frac{3\xi_0\left(-\chi(t) \pm \sqrt{\chi^2(t) + 4A\chi_1(t)}\right)}{2(\gamma+1)\chi_1(t)t}\right) \tag{26}$$

For$\alpha = 1\ and n = 2$ (10) reduces to'

$$(\chi_1(t) - 1)\rho^3 + \rho^2 + \rho\chi(t) - A = 0 \tag{27}$$

It is a difficult task for us to find the roots of (27), as it contains highly nonlinear variable coefficients.

For representative case we considered $\beta_0 = 2.1, A = 0.5$ and $\boldsymbol{\xi_0} = 0.3$.

On observing Fig. 3, we noticed that the effective pressure is diverges initially that indicates the presence of initial singularities (may be called as Big Bang singularity) and it is negative for $t \in (1.06, 9.72)$ which is concluded that our model is fit to the recent observational data.

Here, we observed that both effective pressure and energy density maintain initial singularities. The effective pressure remain negative initially and later for a while it became positive. Further, it remains negative throughout the life span of Universe.

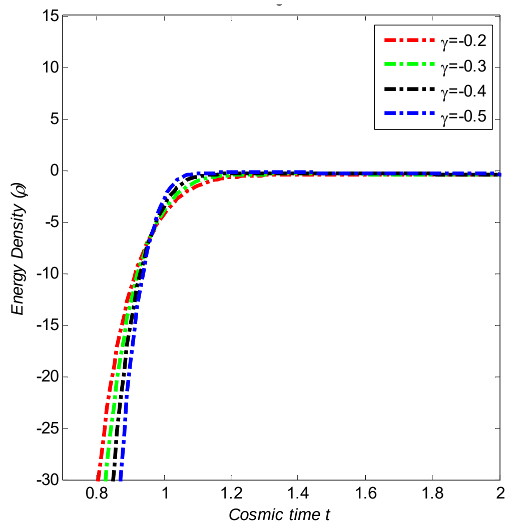

Fig. 2 The variation of $\rho = \frac{-\chi(t) - \sqrt{\chi^2(t) + 4A}}{2}$, $\alpha = 1$ versus time t for case-I

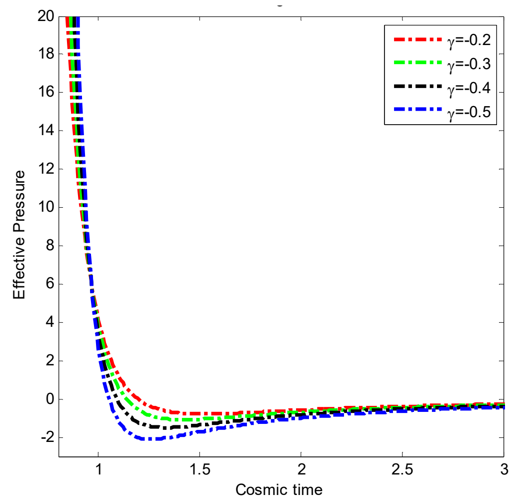

Fig. 3 The variation of Effective pressure $\bar{p}$ for $\alpha = -1$ for case-I versus time

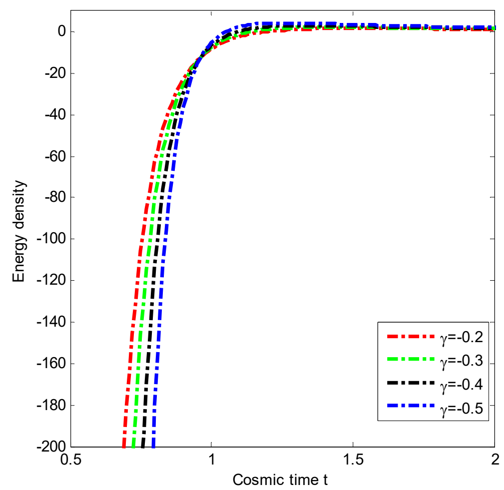

Fig. 4 Variation of Energy density ρ versus cosmic time t for $\alpha = -1$ for case-I versus time

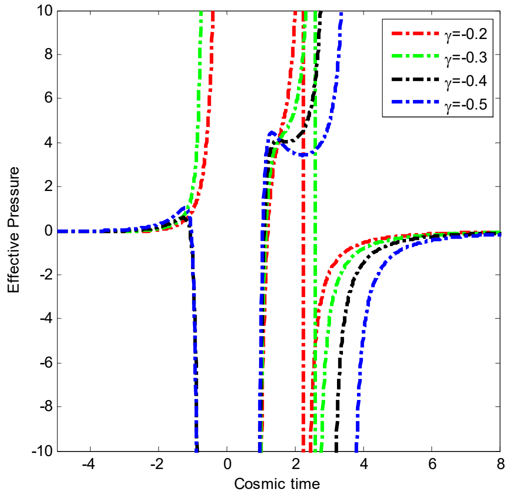

Fig. 5 Variation of effective pressure $\bar{p}$ densityρ versus time for $\alpha = -1$, n=1 for case-II

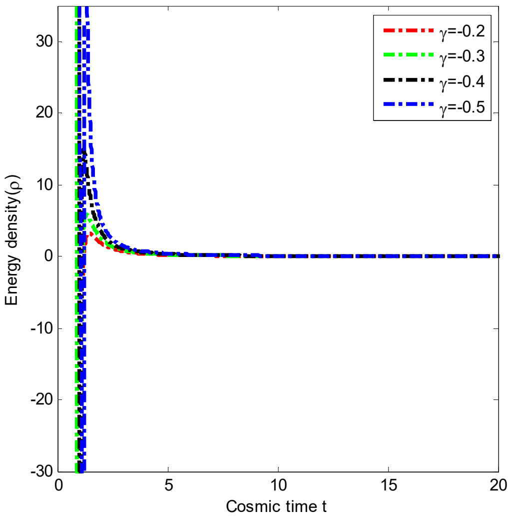

Fig. 6 Variation of Energy density ρ versus time for $\alpha = -1$, n=1 for case-II

III. Conclusions

In this paper, we have investigated on the model using the generalized Chaplygin gas with bulk viscosity with the help of Bianchi Type-I Universe in the context of Lyra geometry. In case-I, we considered a zero-viscosity model at $\alpha = 1$ and obtained two representations for energy density ρ (See Fig.1 and Fig. 2) and observed in Fig. 1, initially the energy density is diverging in nature, and after the explosion of the Universe behaves as an increasing function of time and remain positive throughout the Universe. But in Fig. 2, we found the energy density is divergent at the beginning and leads to an increasing function for a period of time. Hence, the first one concludes to a consistent result. For $\alpha = -1$ in case-I, the effective pressure and energy density is studied in Figs. 2 and 3. From Fig. 2, we observed the effective pressure and energy density are diverge at the beginning for all value of γ. But for $\gamma > 3.6$ both the energy density and effective pressure are consistent. The case-II represents an unique model which concludes (see Figs. 3 and 4) that both the effective pressure and energy density maintain initial singularity which coincides with the recent observational data, i.e. Supernova Ia (otherwise known as Big-bang singularity) and later on it oscillates for some time, furthermore, it approaches to a constant.

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