World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:13, No:11, 2019

Metric Dimension on Line Graph of Honeycomb Networks

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Abstract—Let $G = (V, E)$ be a connected graph and distance between any two vertices a and b in G is $a-b$ geodesic and is denoted by $d(a, b)$. A set of vertices W resolves a graph G if each vertex is uniquely determined by its vector of distances to the vertices in W. A metric dimension of G is the minimum cardinality of a resolving set of G. In this paper line graph of honeycomb network has been derived and then we calculated the metric dimension on line graph of honeycomb network.

Keywords—Resolving set, metric dimension, honeycomb network, line graph.

I. INTRODUCTION

CHEMICAL graph theory is used to model molecules to gain an accurate and deep understanding of physical properties of chemical compounds. Chemical graph describes the structure of chemical compounds in terms of graph theory. In Chemical Graphs, vertices are represented by different type of atoms and edges are denoted by bond between atoms.

Metric dimension is one of the most significant fields of graph theory. It has several applications in various fields of life, for instance image processing, network theory, pattern recognition, optimization, robot navigation, network discovery and verification [28], geometrical routing protocols [29], join in graphs [30] and coin weighting problems [31] etc. A moving point along a graph may be traced by calculating the distance from a point to a conglomeration of sonar stations positioned clearly in the graph. The representation of chemical compound is evaluated by more than one suggested structures drawn along a graph, which expresses the physical and chemical properties of compound. This mathematical representation for disparate chemical compounds is of utmost significance for chemists in drug discovery. As mentioned in [4], [5], the structure of a chemical compounds is drawn by a labeled graph where vertex labels specify the atom and edge specifies bond types. Hence, a graph's theoretic description of this problem is to illustrate interpretation for the vertices of a graph in a way that has specific representations. This is the affairs of [2], [5], [13].

Through considering G as connected graph, $d(u, v)$ denotes u-v geodesic. Assume set $B = \{b_1, b_2, ..., b_k\}$ being an ordered subset of $V(G)$ (vertex set of G). The representation $r(u|B)$ of u w.r.t. B is the $k-tuple$ $(d(u, b_1), d(u, b_2), d(u, b_3), ..., d(u, b_k)\}$ where B is called a resolving or locating set $[21, 16]$, if each graph vertex is uniquely identified by its distances from the vertices of B. The minimum set of vertices in a resolving set is commonly known as the basis for graph G and cardinality of set of basis element is considered as the metric dimension of G, which is represented as $dim(G)$ [10]. A fairish literature related to metric basis is discussed in [1], [7], [13]. We have an ordered set of vertices $B = \{b_1, b_2, ..., b_k\}$ of a graph G, the $d(u, b_i)$ is zero iff $u = b_i$. If $r(s|B) \neq r(t|B)$ for each pair of distinct vertices s, t belongs to $(V(G)|B)$ then B is called a resolving set.

Slater was the first mathematician who introduced the concept of metric dimension in [16]. Melter et al. also investigated the same concept independently in [7]. Raj et al. pored over metric dimensions of various chemical networks as well as star of David network $SD(n)$ in [22], [23]. The metric dimension of a connected graph changes when the number of vertices is altered in the graph and turns infinite when numbers of vertices are infinite and is known as unbounded metric dimension. Likewise, metric dimension remains finite when altering in number of vertices is finite and is called bounded metric dimension. Finally, if the metric dimension sticks around same for all number of vertices in a connected graph G, then it is called a constant metric dimension [24]. The metric dimension of path graph is 1 in [5]; cycles have metric dimension 2 for every $n \geq 3$. Rooted product of two graphs F and J is stated as take $u = |V(F)|$ copies of J, and for each vertex u_j of F, identify u_i with the root node of the j^{th} copy of J. Godsil et al. [25] rooted product of Harary graphs $H_(m, n)$, Jahangir graphs, antiprism A_n and generalized Petersen graphs $P(n, 2)$ by path and cycle would be calculated and metric dimensions of line graph of certain families of graphs would be determined. It is also of interest to determined the rooted product of graphs and then find out the metric dimension of rooted product of graphs by path and cycles. Manuel et al. [12] determined the constant metric dimension of honeycomb networks. After gain some idea of Manuel, metric dimension of line graph of honeycomb network would be determined.

A. Honeycomb

There are various designs in which hexagons tender to build honeycomb network. If $HC(1)$ is a hexagonic honeycomb network, we will add six hexagons to the exterior edges of $HC(1)$ in order to obtain the honeycomb network $HC(2)$. Leading on, the honeycomb network is obtained from $HC(n-1)$, when we added a layer of hexagon on the boundary of honeycomb network $HC(n-1)$, we get honeycomb network $HC(n)$. The hexagons between the centre and boundary of $HC(n)$ firm about the parameter n of $HC(n)$.

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Honeycomb networks are extendedly used in computer graphics [31], cell phone base stations [31], image processing and in chemistry as the representation of benzene hydrocarbons. Honeycomb networks are better in terms of diameter, degree, total number of links, cost and connected planner graphs. Stojmenovic [32] investigated the topological properties of Honeycomb network, routing in Honeycomb network and Honeycomb tourus networks.

Theorem1. For $G \cong L[HC(n)]$; $n \geq 2$ then G has metric dimension greater than 2.

Proof. Here, it is essential to show that there does not exist any set B with two vertices that is a resolving set of G. Let on contrary G has metric dimension equal to 2. Let $H = \{c_{i+1}, a^i_{j+1}\}$ be a resolving set then

$$r(v^i_j|H) = r(u^i_{j+3}|H)$$

so H is not resolving set for the graph. Let $H = \{u^{j+1}_{j+3}, u^{i+2}_{j+4}\}$ be a resolving set then

$$r(u^{i+2}_{j+2}|H) = r(u^{i+1}_{j+5}|H)$$

so H is not resolving set. Let $H = \{c_{i+2}, u^{i+2}_j\}$ be a resolving set then

$$r(u^{i+1}_{j+2}|H) = r(u^{i+1}_{j+3}|B)$$

so H is not resolving set. Let $H = \{v^{i+1}_{j+1}, u^{i+2}_{j+5}\}$ be a resolving set then

$$r(b^i_{j+3}|H) = r(b^i_{j+4}|H)$$

so H is not resolving set. Let $H = \{b^i_{j+3}, u^{i+2}_{j+1}\}$ be a resolving set then

$$r(b^i_{j+2}|H) = r(u^{i+2}_{j+4}|H)$$

so H is not resolving set. Let $H = \{a^i_{j+3}, v^{i+2}_{j+1}\}$ be a resolving set then

$$r(a^i_{j+2}|H) = r(v^{i+2}_{j+4}|H)$$

so H is not resolving set. Let $H = \{v^{i+1}_{j+3}, v^{i+2}_{j+4}\}$ be a resolving set then

$$r(v^{i+2}_{j+2}|H) = r(v^{i+2}_{j+5}|H)$$

so H is not resolving set. Let $H = \{a^i_{j+1}, b^i_{j+3}\}$ be a resolving set then

$$r(v^i_{j+5}|H) = r(u^i_{j+2}|H)$$

so H is not resolving set. Let $H = \{a^{i+1}_{j+6}, a^{i+3}_{j+5}\}$ be a resolving set then

$$r(v^{i+2}_{j+10}|H) = r(v_{j+13}|H)$$

so H is not resolving set. Let $H = \{c_{i+7}, b^i_{j+8}\}$ be a resolving set then

$$r(u^i_{j+14}|H) = r(u^i_{j+10}|H)$$

so H is not resolving set. Let $H = \{c_{i+5}, v^{i+1}_{j+9}\}$ be a resolving set then

$$r(v^{i+1}_{j+8}|H) = r(v^{i+1}_{j+10}|H)$$

so H is not resolving set. Similarly there is no set H with two vertices is a resolving set for $L[HC(n)]$ network so its metric dimension is greater than 2.

Theorem2. For $G \cong L[HC(n)]$; $n \geq 2$ then G has metric dimension 3.

Proof. The $L[HC(n)]$ with one vertex between every two vertices has vertex set,

$$V(L(HC(n))) = \{v^i_j:\ 1 \leq i \leq n, 1 \leq j \leq k, j = 4n-2i\}$$

$$\cup\{a^i_j:\ 1 \leq i \leq n-1, 1 \leq j \leq k, j = 2n-i\}$$

$$\cup\{u^i_j:\ 1 \leq i \leq n, 1 \leq j \leq k, j = 4n-2i\}$$

$$\cup\{b^i_j:\ 1 \leq i \leq n-1, 1 \leq j \leq k, j = 2n-i\}$$

$$\cup\{c_i:\ 1 \leq i \leq 2n\}$$

$$\lambda(v^i_j) = \begin{cases} 2i+j-2, 4n-j-1, 4n-j-4 \\ for\ \ 1 \leq i \leq n, 1 \leq j \leq k-4, \\ j, 4n-i-j, 4n-i-j-1 \\ for\ \ 1 \leq i \leq n, 1 \leq j \leq k-4, \\ 2i+j-2, 4n-j, 4n-j-4 \\ for\ \ 1 \leq i \leq n, k-4 \leq j \leq k-3, \\ 2i+j-2, 4n-j+2, 4n-j-4 \\ for\ \ 1 \leq i \leq n, j \leq k, \\ 2i+j-2, 4n-j+1, 4n-j-4 \\ for\ \ j \leq k-1, \\ j, 4n-j, 4n-j-2i \\ for\ \ j \leq k-3, \\ j, 4n-j, 4n-i-j \\ for\ \ j \leq k-2, \\ j, 4n-j+i, 4n-j-i \\ for\ \ j \leq k-1, \\ j, 4n-j+2i, 4n-j \\ for\ \ j \leq k, \end{cases}$$

$$\lambda(a_j^i) = \begin{cases} 2i+2j-2, 4n-2j, 4n-2j-3 \\ for \ \ 1 \leq i \leq n-1, 1 \leq j \leq k-2, \\ \\ 2j, 4n-2j, 4n-2j-2i \\ for \ \ 1 \leq i \leq n-1, j \leq k-2, \\ \\ 2i+2j-2, 4n-2j+1, 4n-2j-3 \\ for \ \ 1 \leq i \leq n-1, j \leq k-1, \\ \\ 2i+2j-2, 4n-2j+3, 4n-2j-3 \\ for \ \ 1 \leq i \leq n-1, j \leq k, \\ \\ 2j, 4n-2j+1, 4n-2j-2i \\ for \ \ 1 \leq i \leq n-1, j \leq k-1, \\ \\ 2j, 4n-2j+3, 4n-2j-1 \\ for \ \ 1 \leq i \leq n-1, i \leq k, \end{cases}$$

$$\lambda(u_j^i) = \begin{cases} 2i+j-2, 4n-j-4, 4n-j-1 \\ for \ \ 1 \leq i \leq n, 1 \leq j \leq k-4, \\ \\ j, 4n-i-j-1, 4n-i-j \\ for \ \ 1 \leq i \leq n, 1 \leq j \leq k-4, \\ \\ 2i+j-2, 4n-j-4, 4n-j \\ for \ \ 1 \leq i \leq n, k-4 \leq j \leq k-3, \\ \\ 2i+j-2, 4n-j-4, 4n-j+2 \\ for \ \ 1 \leq i \leq n, j \leq k, \\ \\ 2i+j-2, 4n-j-4, 4n-j+1 \\ for \ \ 1 \leq i \leq n, j \leq k-1, \\ \\ j, 4n-j-2i, 4n-j \\ for \ \ 1 \leq i \leq n, j \leq k-3, \\ \\ j, 4n-j-i, 4n-i \\ for \ \ 1 \leq i \leq n, j \leq k-2, \\ \\ j, 4n-j-i, 4n-j+i \\ for \ \ 1 \leq i \leq n, j \leq k-1, \\ \\ j, 4n-j, 4n-j+2i \\ for \ \ 1 \leq i \leq n, j \leq k, \end{cases}$$

$$\lambda(b_j^i) = \begin{cases} 2i+2j-2, 4n-2j-3, 4n-2j \\ for \ \ 1 \leq i \leq n-1, 1 \leq j \leq k-2, \\ \\ 2j, 4n-2j-2i, 4n-2j \\ for \ \ 1 \leq i \leq n-1, j \leq k-2, \\ \\ 2i+2j-2, 4n-2j-3, 4n-2j+1 \\ for \ \ 1 \leq i \leq n-1, j \leq k-1, \\ \\ 2i+2j-2, 4n-2j-3, 4n-2j+3 \\ for \ \ 1 \leq i \leq n-1, j \leq k, \\ \\ 2j, 4n-2j-2i, 4n-2j+1 \\ for \ \ 1 \leq i \leq n-1, j \leq k-1, \\ \\ 2j, 4n-2j-1, 4n-2j+3 \\ for \ \ 1 \leq i \leq n-1, j \leq k, \end{cases}$$

$$\lambda(c_j) = \begin{cases} 2j-1, 4n-2j, 4n-2j \\ for \ \ j \leq 2n-2 \\ \\ 2j-1, 4n-2j+i, 4n-2j+i \\ for \ \ i \leq 1, j \leq 2n-1 \\ \\ 2j, 4n-2j+3i, 4n-2j+3i \\ for \ \ i \leq 1, j \leq 2n \end{cases}$$

Let v_s^i and v_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(v_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s-1, 4n-s-4) = (2i+t-2, 4n-t-1, 4n-t-4) \Rightarrow s=t, \ \ s=t, \ \ s=t$ which is contradiction. Let v_s^i and v_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(v_t^i|H)$$

$\Rightarrow (s, 4n-i-s, 4n-i-s-1) = (t, 4n-i-t, 4n-i-t-1) \Rightarrow s=t, \ \ s=t, \ \ s=t$ which is contradiction. Let v_s^i and v_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(v_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s, 4n-s-4) = (2i+t-2, 4n-t, 4n-t-4) \Rightarrow s=t, \ \ s=t, \ \ s=t$ which is contradiction. Let v_s^i and v_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(v_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s+1, 4n-s-4) = (2i+t-2, 4n-t+1, 4n-t-4) \Rightarrow s=t, \ \ s=t, \ \ s=t$ which is contradiction. Let v_s^i and v_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(v_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s+2, 4n-s-4) = (2i+t-2, 4n-t+2, 4n-t-4) \Rightarrow s=t, \ \ s=t, \ \ s=t$ which is contradiction. Let a_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+2s-2, 4n-2s, 4n-2s-3) = (2i+2t-2, 4n-2t, 4n-2t-3) \Rightarrow s=t, \ \ s=t, \ \ s=t$ which is contradiction. Let a_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2s, 4n-2s, 4n-2s-2i) = (2t, 4n-2t, 4n-2t-2i) \Rightarrow s=t, \ \ s=t, \ \ s=t$ which is contradiction. Let a_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+2s-2, 4n-2s+1, 4n-2s-3) = (2i+2t-2, 4n-2t+1, 4n-2t-3) \Rightarrow s=t, \ \ s=t, \ \ s=t$ which is contradiction. Let a_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i + 2s - 2, 4n - 2s + 3, 4n - 2s - 3) = (2i + t - 2, 4n - 2t + 3, 4n - 2t - 3) \Rightarrow s = t, \quad s = t, \quad s = t$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i + s - 2, 4n - s - 1, 4n - s - 4) = (2i + 2t - 2, 4n - 2t, 4n - 2t - 3) \Rightarrow s = 2t, \quad s = 2t, \quad s = 2t$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+2s-2, 4n-2s, 4n-2s-3) = (t, 4n-i-t, 4n-i-t-1) \Rightarrow s = \frac{t-2i+2}{2}, \quad s = \frac{t+i}{2}, \quad s = \frac{t+i-2}{2}$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i + s - 2, 4n - s, 4n - s - 4) = (2i + 2t - 2, 4n - 2t, 4n - 2t - 3) \Rightarrow s = 2t, \quad s = 2t, \quad s = 2t - 1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i + s - 2, 4n - s + 1, 4n - s - 4) = (2i + 2t - 2, 4n - 2t, 4n - 2t - 3) \Rightarrow s = 2t, \quad s = 2t + 1, \quad s = 2t - 1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i + s - 2, 4n - s + 2, 4n - s - 4) = (2i + 2t - 2, 4n - 2t + 3, 4n - 2t - 3) \Rightarrow s = 2t, \quad s = 2t - 1, \quad s = 2t - 1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s-1, 4n-s-4) = (2t, 4n-2t, 4n-2t-2i) \Rightarrow s = 2[t - i + 1], \quad s = 2t - 1, \quad s = 2[t + i - 2]$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (s, 4n-i-s, 4n-i-s-1) = (2t, 4n-2t, 4n-2t-2i) \Rightarrow s = 2t, \quad s = 2t-i, \quad s = 2t+i-1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s, 4n-s-4) = (2t, 4n-2t, 4n-2t-2i) \Rightarrow s = 2[t - i + 1], \quad s = 2t, \quad s = 2[t + i - 2]$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s+1, 4n-s-4) = (2t, 4n-2t, 4n-2t-2i) \Rightarrow s = 2[t - i + 2], \quad s = 2t + 1, \quad s = 2[t + i - 2]$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s+2, 4n-s-4) = (2t, 4n-2t, 4n-2s-2i) \Rightarrow s = 2[t + 1 - i], \quad s = 2t + 2, \quad s = 2[t + i - 2]$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i + s - 2, 4n - s - 1, 4n - s - 4) = (2i + 2t - 2, 4n - 2t, 4n - 2t - 3) \Rightarrow s = 2t, \quad s = 2t - 1, \quad s = 2t - 1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (s, 4n - i - s, 4n - i - s - 1) = (2i + 2t - 2, 4n - 2t + 1, 4n - 2t - 3) \Rightarrow s = 2i + 2t - 2, \quad s = 2t - 1 - i, \quad s = 2t + 2 - i$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i + s - 2, 4n - s, 4n - s - 4) = (2i + 2t - 2, 4n - 2t + 1, 4n - 2t - 3) \Rightarrow s = 2t, \quad s = 2t - 1, \quad s = 2t - 1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i + s - 2, 4n - s + 1, 4n - s - 4) = (2i + 2t - 2, 4n - 2t + 1, 4n - 2t - 3) \Rightarrow s = 2t, \quad s = 2t, \quad s = 2t - 1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s+2, 4n-s-4) = (2i+2t-2, 4n-2t+1, 4n-2t-3) \Rightarrow s = 2t+1$, $s = 2t-1$, $s = 2t-1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s-1, 4n-s-4) = (2i+2t-2, 4n-2t+3, 4n-2t-3) \Rightarrow s = 2t$, $s = 2t-4$, $s = 2t-1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (s, 4n-i-s, 4n-i-s-1) = (2i+2t-2, 4n-2t+3, 4n-2t-3) \Rightarrow s = 2i+2t-2$, $s = 2t-3-i$, $s = 2t-i+2$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s, 4n-s-4) = (2i+2t-2, 4n-2t+3, 4n-2t-3) \Rightarrow s = 2t$, $s = 2t+3$, $s = 2t-1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s+1, 4n-s-4) = (2i+2t-2, 4n-2t+3, 4n-2t-3) \Rightarrow s = 2t$, $s = 2t-2$, $s = 2t-1$ which is contradiction. Let v_s^i and a_t^i are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(a_t^i|H)$$

$\Rightarrow (2i+s-2, 4n-s+2, 4n-s-4) = (2i+2t-2, 4n-2t+3, 4n-2t-3) \Rightarrow s = 2t$, $s = 2t-1$, $s = 2t-1$ which is contradiction. Let v_s^i and c_j are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(c_j|H)$$

$\Rightarrow (2i+s-2, 4n-s-1, 4n-s-4) = (2t-1, 4n-2t, 4n-2t) \Rightarrow s = 2t-2i+1$, $s = 2t-1$, $s = 2t-4$ which is contradiction. Let v_s^i and c_j are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(c_j|H)$$

$\Rightarrow (s, 4n-i-s, 4n-i-s-1) = (2t-1, 4n-2t, 4n-2t) \Rightarrow s = 2t-1$, $s = 2t-i$, $s = 2t-1-i$ which is contradiction. Let v_s^i and c_j are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(c_j|H)$$

$\Rightarrow (2i+s-2, 4n-s, 4n-s-4) = (2t-1, 4n-2t, 4n-2t) \Rightarrow s = 2t-2i+1$, $s = 2t$, $s = 2t-4$ which is contradiction. Let v_s^i and c_j are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(c_j|H)$$

$\Rightarrow (2i+s-2, 4n-s+1, 4n-s-4) = (2t-1, 4n-2t, 4n-2t) \Rightarrow s = 2t-2i+1$, $s = 2t+1$, $s = 2t-4$ which is contradiction. Let v_s^i and c_j are two distinct vertices from $V(L(HC(n))$ then

$$r(v_s^i|H) = r(c_j|H)$$

$\Rightarrow (2i+s-2, 4n-s+2, 4n-s-4) = (2t-1, 4n-2t, 4n-2t) \Rightarrow s = 2t-2i+1$, $s = 2t+2$, $s = 2t-4$ which is contradiction. Let a_s^i and c_j are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^i|H) = r(c_j|H)$$

$\Rightarrow (2i+2s-2, 4n-2s, 4n-2s-3) = (2t-1, 4n-2t, 4n-2t) \Rightarrow s = \frac{2t-2i+1}{2}$, $s = t$, $s = \frac{2t-3}{2}$ which is contradiction. Let a_s^i and c_j are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^i|H) = r(c_j|H)$$

$\Rightarrow (2s, 4n-2s, 4n-2s-2i) = (2t-1, 4n-2t, 4n-2t) \Rightarrow s = \frac{2t-1}{2}$, $s = t$, $s = \frac{2t-2i}{2}$ which is contradiction. Let a_s^i and c_j are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^i|H) = r(c_j|H)$$

$\Rightarrow (2i+2s-2, 4n-2s+1, 4n-2s-3) = (2t-1, 4n-2t, 4n-2t) \Rightarrow s = \frac{2t-2i+1}{2}$, $s = \frac{2t-1}{2}$, $s = \frac{2t-3}{2}$ which is contradiction. Let a_s^i and c_j are two distinct vertices from $V(L(HC(n))$ then

$$r(a_s^i|H) = r(c_j|H)$$

$\Rightarrow (2i+2s-2, 4n-2s+3, 4n-2s-3) = (2t-1, 4n-2t, 4n-2t) \Rightarrow s = \frac{2t-2i+1}{2}$, $s = \frac{2t-1}{2}$, $s = \frac{2t-3}{2}$ which is contradiction.

II. Conclusion

There are various designs in which hexagons tenders to build honeycomb network. If $HC(1)$ is a hexagonic honeycomb network, we add six hexagons to the exterior edges of $HC(1)$ in order to obtain the honeycomb network

$HC(2)$. Leading on, the honeycomb network is obtained from $HC(n-1)$, when a layer of hexagon on the boundary of honeycomb network $HC(n-1)$, honeycomb network $HC(n)$ is obtained. The hexagons between the centre and boundary of $HC(n)$ firm about the parameter n of $HC(n)$. A metric dimension of G is the mininum cardinality of a resolving set of G. We have computed metric dimensions of line graph of honeycomb,derived from honeycomb network. Further, we calculated metric dimensions of line graph of Aztec Diamond.

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