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Simulation-Based Optimization of a Non-Uniform Piezoelectric Energy Harvester with Stack Boundary

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***Abstract*—**This research presents an analytical model for the development of an energy harvester with piezoelectric rings stacked at the boundary of the structure based on the Adomian decomposition method. The model is applied to geometrically non-uniform beams to derive the steady-state dynamic response of the structure subjected to base motion excitation and efficiently harvest the subsequent vibrational energy. The in-plane polarization of the piezoelectric rings is employed to enhance the electrical power output. A parametric study for the proposed energy harvester with various design parameters is done to prepare the dataset required for optimization. Finally, simulation-based optimization technique helps to find the optimum structural design with maximum efficiency. To solve the optimization problem, an artificial neural network is first trained to replace the simulation model, and then, a genetic algorithm is employed to find the optimized design variables. Higher geometrical non-uniformity and length of the beam lowers the structure natural frequency and generates a larger power output.

***Keywords*—**Piezoelectricity, energy harvesting, simulation-based optimization, artificial neural network, genetic algorithm.

I. Introduction

MECHANICAL energy harvesters are being designed to capture the ambient mechanical energy and convert it into electrical energy. Conversion of ambient energy into usable electrical energy is achieved through electromagnetic induction, electrostatic generation, and piezoelectricity [1]. Numerous studies comparing the performance of these energy conversion approaches are published [2]-[5]. While each method delivers a certain amount of energy, piezoelectric materials have received the most attention because of small size, custom shape fabrication ability, ease of integration, and high power conversion potential [6], [7].

One of the most attractive research areas with significant potential in the energy harvesting filed is piezoelectric energy harvesting from vibrations [8]. Vibration energy is renewable energy with comparatively low power output but several superior advantages over other conventional energy sources. Power harvesting from mechanical vibrations is an important step toward providing self-powered microelectronic systems.

A huge number of research and review papers have examined piezoelectric energy harvesting from various points of views [1], [9]-[11]. Most of the current researches are focused on efficiency improvement through physical and geometrical configuration, circuitry design, and energy removal methods. Generally, the design of a piezoelectric energy harvester contains numerous variables that can be controlled and manipulated in order to obtain higher electrical energy. These physical system parameters have a substantial influence on the maximum obtainable electrical energy and the overall system performance. As a result, employing optimization techniques to improve the efficiency and effectiveness of the energy harvesters could be the next step toward self-powered microelectromechanical systems.

Depending on the design requirements, certain objects and factors should be considered in the optimization process. However, the optimization procedure for a mechanical system can result in a complicated objective function with a large number of design variables. The optimum values of an engineering objective function have been calculated by using various analytical and numerical approaches. Although these classical methods perform well in some circumstances, they typically fail in practical complex design scenarios because of many design parameters and their nonlinear effect. Therefore, advanced optimization algorithms are employed to solve the optimization problem within a feasible time and computational cost [12].

There are numerous optimization methods offered in the literature. A comprehensive review of simulation-based optimization presented by [13] explains some of the currently available optimization techniques. Normally, simulation-based optimization involves multiple evaluations of the objective function. However, the accurate simulation model process for piezoelectric energy harvesters is time-consuming and also computationally complex and expensive [14]. As a result, a combination of statistical machine learning and simulation-based optimization can be used to solve the optimization problem more efficiently.

Here, the in-plane polarization of the piezoelectric stacks is used to maximize the energy output. The steady state vibration response of the harvester subjected to harmonic base motion is obtained and electrical outputs are analytically derived. Additionally, a parametric study for the energy harvester with different design parameters is done. Lastly, simulation-based optimization technique is used to solve the optimization problem. Then, an artificial neural network (ANN) is trained to replace the computationally expensive simulation model and generate a bigger dataset for the genetic algorithm (GA) that finds the optimized values of the design variables.

This paper presents a design and analytical model for energy harvesting applications. In addition, a simulation-based optimization technique is utilized to find the optimum

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mechanical design. This is just a preliminary overview of the simulation-based optimization for energy harvesters that could pave the way for more advanced optimization models and studies in the future.

II. Theoretical Model

The proposed piezoelectric energy harvester includes a nonlinearly tapered beam of length L having one free end at $x=L$. The other end of the non-uniform beam at $x=0$ is attached to a piezoelectric stack fixture embedded in a solid platform. Since the piezoelectric polarization direction is along the bending stress in the axial direction, piezoelectric stacks are using the in-plane polarization with higher piezoelectric coefficient [15]. The structure is transversely fixed, and the piezoelectric stacks at the beam joint acts like a torsional spring, like shown in Fig. 1 [24].

The non-uniform geometry of the structure is defined by an exponential function $R_0e^{mx/L}$, where R_0 is the radius of the circular cross-section of the beam at $x=0$, m is the geometry taper ratio, and L is the length. Moreover, r_o, r_i, and h_p are outer radius, inner radius, and thickness of the piezoelectric ring, respectively. Lastly, it is assumed that the electrode layer is made of a polyvinylidene thin film, which means the electrode thickness is negligible [24].

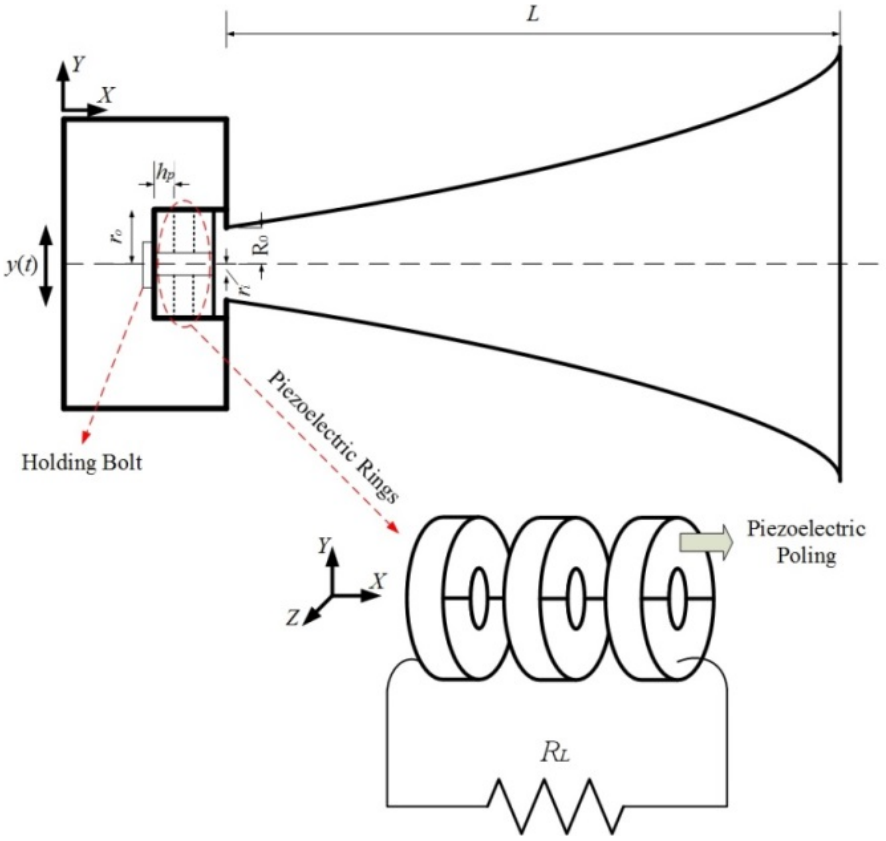


Fig. 1 Piezoelectric energy harvester with piezoelectric stacks

The energy harvesting efficiency of the harvester can be investigated by setting the mechanical vibration input for the system in the form of $y(t)=Y\sin\varphi t$, where t is the time variable, Y is the amplitude of the base displacement, and φ is the angular frequency of the harmonic excitation. Therefore, the forced lateral vibration of the non-uniform structure can be expressed by,

$$\rho_s A(x)\frac{\partial^2 w(x,t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[E_s I(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right]=-\rho_s A(x)\varphi^2 Y\sin\varphi t \quad (1)$$

where x is the position variable along length of the beam, ρ_s is the density of the beam, $A(x)$ is the cross-sectional area, $w(x,t)$ is the deflection function, E_s is the elasticity modulus of the beam, and $I(x)$ is the second moment of area [17].

Boundary conditions are defined as,

$$[w(x,t)]_{x=0}=\left[E_s I(x)\frac{\partial^2 w(x,t)}{\partial x^2}-K_T\frac{\partial w(x,t)}{\partial x}\right]_{x=0}=0 \quad (2)$$

$$\left[E_s I(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right]_{x=L}=\left[\frac{\partial}{\partial x}\left(E_s I(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right)\right]_{x=L}=0 \quad (3)$$

where K_T is the equivalent torsional stiffness of the piezoelectric stack rings. By assuming that the rotation of the ring cross-section remains in its plane, the equivalent torsional stiffness of the stack can be expressed by $K_T=[EI/r_o]_{\text{ring}}$ [18].

Following the analytical process presented by [19], [20], mass normalized normal mode eigenfunctions in the recurrence format are derived as,

$$X_0(x) = C_1 + C_2 x + C_3 \frac{x^2}{2!} + C_4 \frac{x^3}{3!},$$
$$X_{k+1}(x) = -O_x^{-1} \begin{bmatrix} 2\left(\frac{\frac{dI(x)}{dx}}{I(x)}\right)\frac{\mathrm{d}^3 X_k(x)}{\mathrm{d}x^3} + \left(\frac{\frac{d^2 I(x)}{dx^2}}{I(x)}\right)\frac{\mathrm{d}^2 X_k(x)}{\mathrm{d}x^2} \\ -\omega^2\left(\frac{\rho_s A(x)}{E_s I(x)}\right) X_k(x) \end{bmatrix} \quad k \geq 0. \quad (4)$$

The modal participation coefficients $T_i(t)$ are the solution of the ordinary differential equation,

$$\frac{\mathrm{d}^2 T_i(t)}{\mathrm{d}t^2} + {\omega_i}^2 T_i(t) = -\rho_s \varphi^2 Y\left(\int_0^L X_i(x) A(x) \mathrm{d}x\right) \sin \varphi t \quad (5)$$

The steady state solution can be expressed by,

$$T_i(t) = \frac{\rho_s \varphi^2 Y\left(\int_0^L X_i(x) A(x) \mathrm{d}x\right)}{\omega_i^2 - \varphi^2} \sin \varphi t \quad (6)$$

As a result, normal mode eigenfunctions in (4) and modal participation coefficients in (6) are used to derive the dynamic steady state response of the non-uniform structure relative to its base as,

$$w(x,t) = \sum_{i=1}^{\infty} X_i(x) T_i(t) = \sum_{i=1}^{\infty} X_i(x) \frac{\rho_s \varphi^2 Y\left(\int_0^L X_i(x) A(x) \mathrm{d}x\right)}{\omega_i^2 - \varphi^2} \sin \varphi t \quad (7)$$

The piezoelectric base is subjected to an external stress field that generates electrical charge in response [17]. Following the distributed model presented by [21], the sensor constitutive relation reduces to,

$$D_3(x,t) = d_{33}\sigma_3(x,t) + \varepsilon_{33}^{\sigma} E_3 \quad (8)$$

where D_3 is the electrical displacement, d_{33} is the in-plane piezoelectric coefficient, σ_3 is the stress acted on the piezoelectric stacks, $e_{33}{}^{\sigma}$ is the permittivity at constant stress, and E_3 is the electrical field along 3-axis. It is noted that 3-axis for piezoelectric patches is aligned with x direction along the length of the beam.

The axial stress acting on the piezoelectric stacks along their thickness direction can be expressed in terms of bending strain at the root of the beam. Thus, the electric displacement in (8) can be modified as,

$$D_3(t) = d_{33} E_s \frac{r_o + r_i}{2}\left(\frac{\partial^2 w(x,t)}{\partial x^2}\Big|_{x=0}\right) - \varepsilon_{33}\frac{V(t)}{N_p h_p} \quad (9)$$

where N_p is the number of piezoelectric stacks, h_p is the piezoelectric stack thickness, ε_{33} is the permittivity component at constant strain, and $V(t)$ is the voltage across the piezoelectric rings.

It is assumed that the electrode area covers the surface of each stack and they are connected in series as shown in Fig. 1. Therefore, the total electric charge on the electrode of piezoelectric rings considering only half of the area that is under the compression load can be estimated as,

$$q(t) = d_{33} E_s \frac{r_o + r_i}{2}\frac{\pi\left(r_o^2 - r_i^2\right)}{2}\left(\frac{\partial^2 w(x,t)}{\partial x^2}\Big|_{x=0}\right) - \varepsilon_{33}\frac{V(t)}{N_p h_p} \quad (10)$$

The generated electric current can be expressed as,

$$i(t) = \frac{dq(t)}{dt} = d_{33} E_s \frac{r_o + r_i}{2}\frac{\pi\left(r_o^2 - r_i^2\right)}{2}\left[\frac{d}{dt}\left(\frac{\partial^2 w(x,t)}{\partial x^2}\Big|_{x=0}\right)\right] - \frac{\varepsilon_{33}\pi\left(r_o^2 - r_i^2\right)}{2N_p h_p}\frac{dV(t)}{dt} \quad (11)$$

Based on [21], the calculated electric current in (11) is a function of the non-uniform beam vibration and the voltage across piezoelectric rings. The piezoelectric rings with the capacitance of $\frac{\varepsilon_{33}\pi\left(r_o^2 - r_i^2\right)}{2N_p h_p}$ are directly connected to the resistive load R_L as a current source. Therefore, the generated voltage across the resistive load is given by,

$$V(t) = R_L i(t) = R_L d_{33} E_s \frac{r_o + r_i}{2}\frac{\pi\left(r_o^2 - r_i^2\right)}{2}\left[\frac{d}{dt}\left(\frac{\partial^2 w(x,t)}{\partial x^2}\Big|_{x=0}\right)\right] - \frac{R_L \varepsilon_{33}\pi\left(r_o^2 - r_i^2\right)}{2N_p h_p}\frac{dV(t)}{dt} \quad (12)$$

In the other words, the electrical differential equation can be rewritten as,

$$\frac{R_L \varepsilon_{33}\pi\left(r_o^2 - r_i^2\right)}{2N_p h_p}\frac{d}{dt}V(t) + \frac{1}{R_L}V(t) = d_{33} E_s \frac{r_o + r_i}{2}\frac{\pi\left(r_o^2 - r_i^2\right)}{2}\left[\frac{d}{dt}\left(\frac{\partial^2 w(x,t)}{\partial x^2}\Big|_{x=0}\right)\right] \quad (13)$$

By using (7) in (13) and solving the first order differential equation of the electrical circuit, the steady state voltage and the electric current can be found. Lastly, the output electrical power of the energy harvester is calculated.

The presented analytical model can be used to study the effects of various parameters on the efficiency of the energy harvesting system. Here, the efficiency factor η is defined in (14) as the ratio of the root mean square (RMS) of the output electrical power $P_{rms}{}^{e}$ to the RMS of the input mechanical power $P_{rms}{}^{m}$ when the piezoelectric capacitor is charged for the time period of T.

$$\eta = \frac{P_{rms}^{e}}{P_{rms}^{m}} = \frac{\sqrt{\frac{1}{T}\int_0^T [V(t) i(t)]^2 dt}}{\sqrt{\frac{1}{T}\int_0^T \left\{\left[\int_0^L \rho_s A(x) dx\right]\varphi^3 Y^2 \sin \varphi t \cos \varphi t\right\}^2 dt}} \quad (14)$$

III. Optimization Model

In order to find the optimized design of the proposed harvester, a simulation-based optimization technique is used. The simulation model is the mathematical model of the electromechanical response of the harvesting system under base motion. One way to find the optimized design is to run the simulation for all possible input parameters. This approach is not practical in the case of complex simulation models or timely and computationally expensive ones. The procedure without explicitly calculating all of the possibilities is referred to as computer-aided optimization [22]. In order to efficiently find the optimal solution, the mathematical model is resolved

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by iterative methods that generate sequences of progressively improved approximations, which eventually satisfies the optimal condition [23].

For the presented harvester design, the simulation model process is time-consuming and computationally expensive. Therefore, it is not sensible and effective to use the physical simulation model to find the optimized configuration of the harvester. As a result, an ANN that is trained based on the dataset obtained from the physical simulation model replaces the simulation model in the optimization process.

First, a small training dataset of the design parameters is produced using the original mathematical model described in section II. This training dataset is randomly shuffled and is then used to train the ANN. A smaller test dataset with known input/output relationships is then used to evaluate and further quantify the training performance. Then, the properly trained ANN that is far more computationally efficient compared with the physical simulation model is used to evaluate the objective function. Last of all, the GA solves the optimization problem and delivers optimal design parameters.

IV. Simulation & Discussion

Based on the developed model, the energy efficiency factor is analytically obtained to examine the effect of various parameters. As an illustration, a beam like Fig. 1 with PIC252 piezoelectric stacks embedded in the base is considered with dimensions, material properties, and electromechanical coefficients listed in Table I. Besides, the upper and lower bounds for each optimization parameter are defined.

The excitation of the piezoelectric energy harvester is due to the harmonic base motion and the steady state dynamic response of the system is of interest. Based on the convergence study for vibration analysis of non-uniform beams, 20 terms of mode shape function series (n=20) are used to accurately present the final vibration mode shape function for the first three vibration modes [19], [20].

As can be seen in Table I, there are huge number of possible arrangements for optimization parameters. Because of the large number of design variables and their nonlinear impact on the complicated objective function, using the mathematical model to evaluate the objective function is problematic and expensive in terms of time and computation power. Therefore, it is not even practical to use simulation-based optimization to find the optimized design. In order to remedy this problem, a well-trained ANN replaces the simulation model. Then, the GA solves the optimization problem. The objective function in this design is the efficiency factor η presented in (14).

TABLE I
Geometry & Material Properties

Non-uniform beam						
taper ratio		Length (m)		radius (mm)	elasticity (GPa)	density (kg/m^3)
$0\leq m\leq 0.6$		$0.45\leq L\leq 0.55$		$3\leq R_0\leq 5$	E_s=68.9	ρ_s=2700
Piezoelectric stack ring						
inner outer radius (mm)	thickness (mm)	No.	elasticity (GPa)	density (kg/m^3)	Piezo coeff. (pm/V)	Permittivity coeff. (nF/m)
$1\leq r_i\leq 2$ $3\leq r_o\leq 5$	h_p=0.1	N=4	E_p=48.3	ρ_p=7800	d_{33}= 400	ε_{33}= 15.494
External load			Excitation			
Electrical resistance (kΩ)			Amplitude (m)		Frequency (rad/s)	
R_L=10			Y=0.001		φ_1=100	

First, a small dataset of the input parameters and the corresponding efficiency output is produced, randomly shuffled, and loaded to train the ANN. A training set is defined which contains 75% of the dataset and is used to train the ANN. The remained 25% of the dataset is used as the test set. The test set evaluates the precision and accuracy of the trained neural network. The measure for performance evaluation of the ANN regression model is calculated by R^2 which is defined as,

$$R^2 = 1 - \frac{\sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \quad (15)$$

where $e_i = y_i - \hat{y}_i$ is the residual prediction error, y_i is the test set output obtained from the physical simulation model, $\hat{y}_i$ is the predicted output by ANN, n is the total number of test data, and $\bar{y}$ is the mean of the test data outputs.

With the ANN structure including two hidden layers and ten neurons in each layer, the network training resulted in R^2=0.9081. The performance comparison between ANN predictions and numerical simulation results is represented in Fig. 2. As can be seen, the trained neural network greatly predicts the test set outputs.

Once the ANN is successfully trained, it replaces the simulation model to generate a large simulation dataset for the GA optimization procedure. The simulation model substitution simply changes the optimization running time from weeks to seconds. In addition, there is no doubt that the required computational power is much less.

For the optimization problem, four parameters consisting of beam taper ratio, beam length, piezoelectric ring inner radius, and piezoelectric ring outer radius are considered. These parameters are chosen because they have a greater influence on energy harvester performance. In order to prevent the convergence to a local solution and increase the chance of convergence to a global solution, a 5% portion of the population is mutated to search for new traits. Finally, the algorithm returns the optimal design parameters when either the predefined maximum number of iterations is reached, or

the preset objective function value is obtained. The GA convergence graph toward a global solution is plotted in Fig. 3. It is noted that multiple optimizations have been done to confirm that the obtained solution is, in fact, a global solution.

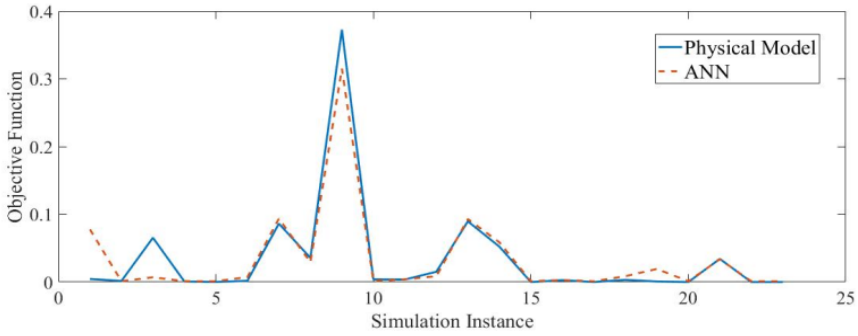


Fig. 2 ANN and numerical simulation performance comparison

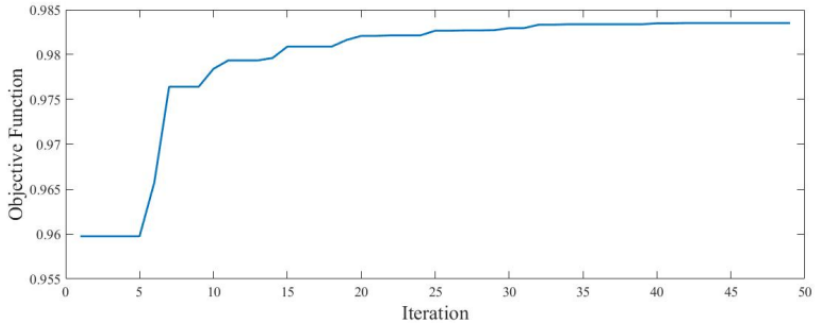


Fig. 3 GA convergence graph

The optimized design parameters evaluated from the GA are tabulated in Table II. As can be seen, for low excitation frequencies (φ≈100rad/s), high nonlinearity in the design delivers better energy harvesting efficiency. The higher geometrical non-uniformity and length of the beam lowers the structure natural frequency and generates a larger bending moment that directly applies to the piezoelectric stacks and produces a higher power output.

TABLE II
OPTIMIZATION SETTING & RESULTS

ANN parameters			GA parameters		
dataset	hidden layers	neurons	initial population	mutation rate	maximum iteration
100	2	10	1000	5%	50
Optimized design values					
taper ratio	length (m)	inner radius (m)	outer radius (m)	power RMS (μW)	efficiency
0.3	0.55	0.002	0.005	4.8395	0.000072

V. CONCLUSION

An analytical model and optimized design of a new energy harvesting system were presented. The steady state vibration response of the harvester subjected to a harmonic base motion was obtained and electrical outputs were analytically derived. Moreover, the in-plane polarization of the piezoelectric materials was used to maximize energy harvesting efficiency. Finally, machine-learning models were developed to find the optimum mechanical design following the simulation-based optimization technique.

Since the presented simulation model was very complex and expensive in terms of time and computation power, a properly trained ANN replaced the simulation model. A training set was generated, randomly shuffled, and loaded to train the neural network. Then, a test set was used to evaluate the performance of the trained ANN. As the final point, the properly trained neural network replaced the physical model in the simulation-based optimization process. Finally, the GA was employed to find the optimized values of the design parameters.

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