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# Supersonic Flow around a Dihedral Airfoil: Modeling and Experimentation Investigation

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***Abstract*****—**Numerical modeling of fluid flows, whether compressible or incompressible, laminar or turbulent presents a considerable contribution in the scientific and industrial fields. However, the development of an approximate model of a supersonic flow requires the introduction of specific and more precise techniques and methods. For this purpose, the object of this paper is modeling a supersonic flow of inviscid fluid around a dihedral airfoil. Based on the thin airfoils theory and the non-dimensional stationary Steichen equation of a two-dimensional supersonic flow in isentropic evolution, we obtained a solution for the downstream velocity potential of the oblique shock at the second order of relative thickness that characterizes a perturbation parameter. This result has been dealt with by the asymptotic analysis and characteristics method. In order to validate our model, the results are discussed in comparison with theoretical and experimental results. Indeed, firstly, the comparison of the results of our model has shown that they are quantitatively acceptable compared to the existing theoretical results. Finally, an experimental study was conducted using the AF300 supersonic wind tunnel. In this experiment, we have considered the incident upstream Mach number over a symmetrical dihedral airfoil wing. The comparison of the different Mach number downstream results of our model with those of the existing theoretical data (relative margin between 0.07% and 4%) and with experimental results (concordance for a deflection angle between 1° and 11°) support the validation of our model with accuracy.

***Keywords*****—**Asymptotic modelling, dihedral airfoil, supersonic flow, supersonic wind tunnel.

## I. Introduction

THE supersonic flows are frequently encountered in many fields of application. Indeed, many aerospace applications are concerned regular by highly compressible flows (aircraft, spaceship, missile...). In general, all these flows are very complex, despite, for simple geometries involving straight and oblique shocks, detachments and attachments and strong interactions between shock waves and boundary layer. For these studies, several resolutions have been adopted based on numerical simulation or experimental measurements [1], [2]. The numerical modeling of supersonic flow around the airfoils has been the topic of wide research, in the engineering applications [3]. The combination of analytical and numerical methods is conceivable by study of chaotic motions [4].

Others studies interested to local existence and the uniqueness of weak shock solution in steady supersonic flow past a wedge [5]. An analytical solution for the generation of shock wave obtained a result of supersonic flow around a wedge [6]. Others methods are deployed to study the supersonic flow profile: the method of hydraulic analog simulation (the method of gas-hydraulic analogy) [7] and simulation using both continuum and particle approaches with inter-molecular collision modeling [8].

In this work, we use asymptotic methods to develop a model of a supersonic flow around thin wing airfoil. Then, we employ an application on the dihedral airfoil and an experimental study to validate the developed model.

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## II. Problem Formulation

Taking into account the Bernoulli integral and the slip-condition, for a compressible, isentropic, and irrotational Eulerian fluid flow, and in a two-dimensional steady-state case, we obtain the non-dimensional Steichen Equation, namely:

$$\begin{aligned}&(M_\infty^2-1)\frac{\partial^2\varphi}{\partial x^2}-\frac{\partial^2\varphi}{\partial z^2}\\&+\varepsilon M_\infty^2\left\{(\gamma+1)\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial x^2}+(\gamma-1)\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial z^2}+2\frac{\partial\varphi}{\partial z}\frac{\partial^2\varphi}{\partial x\partial z}\right\}\\&+\varepsilon^2 M_\infty^2\left\{[\frac{\gamma+1}{2}(\frac{\partial\varphi}{\partial x})^2+\frac{\gamma-1}{2}(\frac{\partial\varphi}{\partial z})^2]\frac{\partial^2\varphi}{\partial x^2}\right.\\&\left.+[\frac{\gamma+1}{2}(\frac{\partial\varphi}{\partial z})^2+\frac{\gamma-1}{2}(\frac{\partial\varphi}{\partial x})^2]\frac{\partial^2\varphi}{\partial z^2}+2\frac{\partial\varphi}{\partial x}\frac{\partial\varphi}{\partial z}\frac{\partial^2\varphi}{\partial x\partial z}\right\}=0\end{aligned} \tag{1}$$

where $M_\infty$ is the characteristic far upstream Mach number, $\varphi(x,z)$ is the velocity potential around the body, $\gamma$ is a constant with the value 1.40 for dry air and $\varepsilon$ characterizes a perturbation parameter.

We obtained (1) after the linearization of non-dimensional Steichen Equation about the particular solution, far upstream of an obstacle, as:

$$u=1+\varepsilon\frac{\partial\varphi}{\partial x}\quad;\quad w=\varepsilon\frac{\partial\varphi}{\partial z} \tag{2}$$

where $u(x,z)$ and $w(x,z)$ are the velocity components.

The steady-state Steichen (1) is a hyperbolic equation. But, the signal speed of disturbances is finite in compressible flow.

According to the *"Least Degeneration Principle"*, by keeping the maximum terms in (1) and consequently a lot of information, the dimensionless equation of fluid flow around a profile in the (x, y) plane is:

$$z = \varepsilon f(x) \quad ; \quad x \in [0,1] \tag{3}$$

The relation (1) shows that the flow perturbation is caused by the relative thickness of the obstacle.

In most applications, the bodies of interest are ***thin***, so that generally $\varepsilon$ is a small parameter. So, an interesting case, from the point of view of asymptotic methods, is the so-called *supersonic* case when:

$$M_\infty \succ 1 \qquad \text{and} \qquad \varepsilon \ll 1 \tag{4}$$

Thus, we suppose that the velocity potential $\varphi(x,z,\varepsilon)$ admits a generalized asymptotic expansion with respect to ε with the parameters $\gamma$ and $M_\infty$ fixed [9], [10], as:

$$\varphi(x,z,\varepsilon) = \varphi_0(x,z) + \varepsilon\ \varphi(x,z) + \mathrm{O}(\varepsilon^2) \tag{5}$$

Thus, using the Taylor expansion in the vicinity of $z = \varepsilon f(x)$ with the slip condition and the boundary conditions associated with the limits, the appropriate equations of our problem are written:

At order 0 in ε:

$$\begin{cases} (M_\infty^2 - 1)\dfrac{\partial^2 \varphi_0}{\partial x^2} - \dfrac{\partial^2 \varphi_0}{\partial z^2} = 0 \\ \dfrac{\partial \varphi_0}{\partial z}\Big|_{z=0} = \dfrac{df}{dx} \quad 0 \le x \le 1 \\ \dfrac{\partial \varphi_0}{\partial x} \to 0; \dfrac{\partial \varphi_0}{\partial z} \to 0 \ ; x \to \infty \end{cases} \tag{6}$$

At order 1 in ε:

$$\begin{cases} (M_\infty^2 - 1)\dfrac{\partial^2 \varphi_1}{\partial x^2} - \dfrac{\partial^2 \varphi_1}{\partial z^2} = \\ -M_\infty^2 \left\{ (\gamma+1)\dfrac{\partial \varphi_0}{\partial x}\dfrac{\partial^2 \varphi_0}{\partial x^2} + (\gamma-1)\dfrac{\partial \varphi_0}{\partial x}\dfrac{\partial^2 \varphi_0}{\partial z^2} + 2\dfrac{\partial \varphi_0}{\partial z}\dfrac{\partial^2 \varphi_0}{\partial x \partial z} \right\} \\ \dfrac{\partial \varphi_1}{\partial z}\Big|_{z=0} = \dfrac{\partial \varphi_0}{\partial x}\Big|_{z=0}\dfrac{df}{dx} - f(x)\dfrac{\partial^2 \varphi_0}{\partial z^2}\Big|_{z=0} ; \ 0 \le x \le 1 \\ \dfrac{\partial \varphi_1}{\partial x} \to 0; \dfrac{\partial \varphi_1}{\partial z} \to 0 \ ; x \to \infty \end{cases} \tag{7}$$

The systems (6) and (7) show that if the solution *at order 0 in ε* is known then we can deduce the solution at order 1 in ε.

## III. Resolution Process

In order to avoid detachment of the leading edge shock wave, the leading edge and the trailing edge of the supersonic airfoils should be sharp (or only slightly rounded), and the section should be relatively thin. Otherwise, the shock wave will be detached and relatively strong. Moreover, for thin airfoils, the thickness, the camber and the angle of attack of the section are such that they weakly perturbe the upstream flow. This is due to the fact that the compression changes in the direction of the flow are sufficiently small for the inviscid flow to be everywhere isentropic.

In reality, when the supersonic flow encounters the two-dimensional double-wedge airfoil, an attached shock wave is formed. Thus, since the shock wave is attached to the leading edge and is planar, the downstream flow is isentropic.

If we restrict our attention to supersonic flow, the first equation of the system (6), at order 0 in ε, is the 2-D wave equation and has solutions of hyperbolic type, namely:

$$\begin{cases} \varphi_0^+(\eta,\xi) = -\dfrac{1}{n} f^+(\eta) \\ \varphi_0^-(\eta,\xi) = \dfrac{1}{n} f^-(\xi) \end{cases} \tag{8}$$

Where $n = \sqrt{M_\infty^2 - 1}$, $\eta = x - nz$ and $\xi = x + nz$.

Along the characteristics all properties of the flow, velocity, pressure, temperature are constant . So the supersonic flow is analyzed using the fact that the properties of the flow are constant along the characteristic lines $\boldsymbol{x \pm nz}$ = constant.

Fig. 1 illustrates supersonic flow past a thin airfoil with several characteristics lines shown. Notice that in the linear approximation the characteristics lines are all parallel to one another and lie at the Mach angle $\mu_\infty$ of the free stream. Information about the flow is carried in the value of the potential assigned to a given line characteristic and in the spacing between lines characteristics for a given flow change.

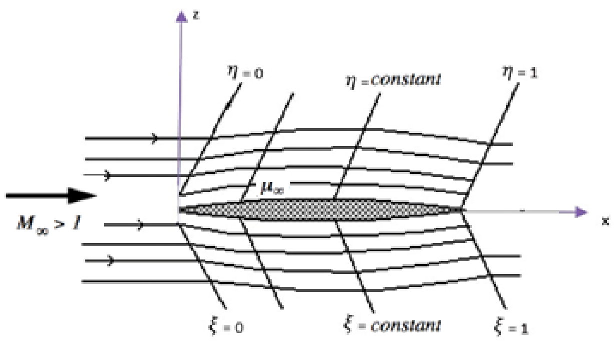



Fig. 1 Right and left leaning characteristics on a thin 2D airfoil.

Since disturbances only propagate along downstream running characteristics we can deduce the velocity potential for the upper and lower surfaces.

The solution, at order 1 in ε, is obtained by integrating twice the (8) and taking into account the slip condition (2nd equation of the system (7)). Indeed, on the upper side, we obtain:

$$\varphi_1^+(\eta,\xi) = -\frac{M_\infty^4}{8n^4}(\gamma+1)\xi\left(\frac{df^+}{d\eta}\right)^2 + G^+(\eta) \tag{9}$$

where

$$\frac{dG^{+}(\eta)}{d\eta}=(\frac{1}{n^{2}}-\frac{M_{\infty}^{4}}{8n^{4}}(\gamma+1))(\frac{df^{+}}{d\eta})^{2}-f^{+}\frac{d^{2}f^{+}}{d\eta^{2}} +\frac{M_{\infty}^{4}}{8n^{4}}(\gamma+1)\eta\frac{d}{d\eta}[(\frac{df^{+}}{d\eta})^{2}] \quad (10)$$

Similarly, on the lower side, we obtain $\varphi_1^{-}(\eta,\xi)$.

Relations (8)-(10) allow to determine the velocity field and then the Mach number at each point along the upper and lower profile surfaces. Indeed, the Mach number along the profile, at order 2 in ε, is defined as:

$$M^{\pm}=M_{\infty}\frac{\left\|\overrightarrow{u^{\pm}}\right\|}{\sqrt{1+\frac{\gamma-1}{2}M_{\infty}^{2}(1-\left\|\overrightarrow{u^{\pm}}\right\|^{2})+\mathrm{O}(\varepsilon^{3})}} \quad (11)$$

In relation (11), the denominator is defined at order $\varepsilon^3$ (i.e. all terms proportional to $\varepsilon^3$ will be neglected). We note that the model developed depends on several physical parameters as the upstream Mach number, the relative thickness and equation of the profile, and the nature of the gas.

## IV. Applications and Validation

In a wide variety of physical situations, a compression shock wave occurs which is inclined at an angle to the flow. Such a wave is called an oblique shock. This type, either straight or curved, can occur in such varied examples as supersonic flow over a thin airfoil or the presence of a wedge in a supersonic stream or during a supersonic compression in a corner (see Fig. 2).

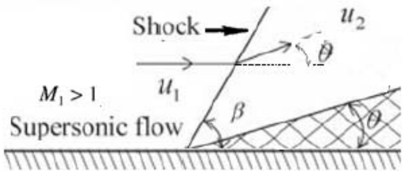



Fig. 2 Attached an oblique shock for a corner flow

We consider a supersonic flow passes over a slender semi-wedge of $\theta$ angle, as shown in Fig. 2, the plane shock wave is formed and is inclined by an angle of $\beta$ with respect to the incoming flow direction. When the upstream supersonic stream encounters a compression corner, the downstream flow is deflected by an angle $\theta$.

In order to validate our model, we compare the results of our model with those of the theory. Indeed, we refer to the existing theoretical data to extract different Mach number downstream. The values for downstream Mach number of oblique shock, as a function of the deflection angle and the Mach number upstream, are shown in Fig. 3.

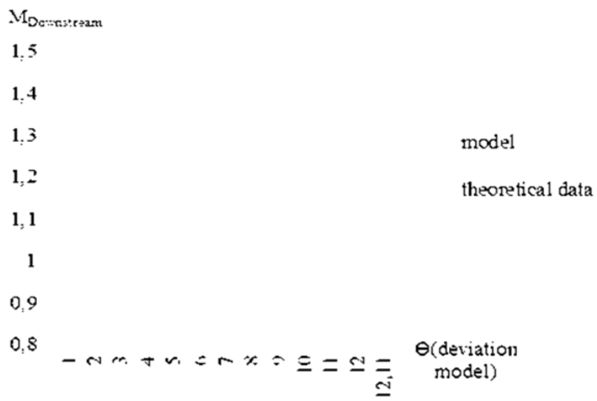



Fig. 3 Weak oblique shock for $M_{upstream}$=1,5

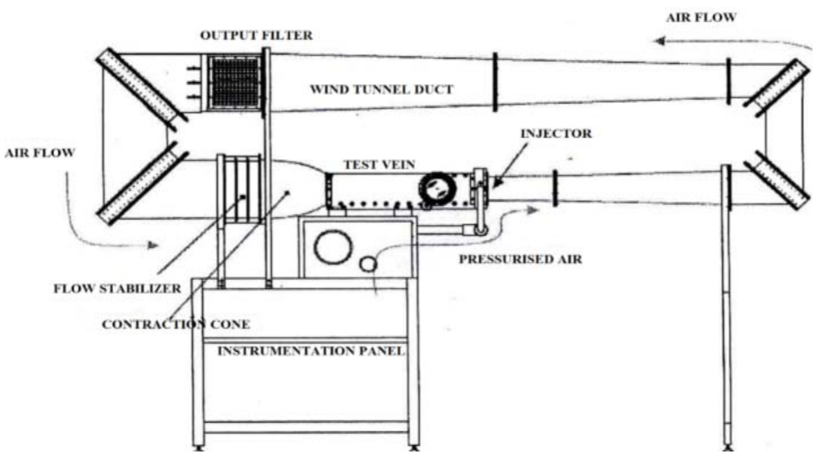



Fig. 4 AF300 supersonic wind tunnel TecQuipment Company

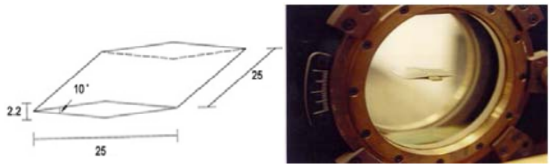
In Fig. 3, for the Mach number ($M_{upstream}$=1.5), we remark that the results of our model are in good agreement with theoretical data for a deflection angle between 1 ° and 11 °. But beyond, a divergence between our model and the theoretical data is observed when the downstream flow becomes subsonic.

Finally, we note that in the weak shock solution, M2 is supersonic, except for a small region near θmax.

## V. Experimental Study

Experimentation was carried out in the AF300 supersonic wind tunnel. The test section has a rectangular shape, the top wall has the convergent-divergent profile and the bottom wall plate has 25 pressure taps, see Fig. 4.

The double-wedge airfoil dimensions are 25 by 25 mm with the angle of 10°. The double-wedge airfoil is showed in Fig. 5 (a).




(a) (b)

Fig. 5 The double-wedge airfoil: (a) experimental setup; (b) dimensions (mm)

In Fig. 5 (b), circle containing double wedge airfoil represents the+

Schlieren window. On the other hand, with the aim to know the flow behavior in the double-wedge airfoil, an experiment was realized at 1.4 Mach number and the static pressure data were registered at this condition. The static pressure used to compute the Mach number, temperature and density in the wind tunnel test section. These results are shown in Fig. 6, for upstream pressure of 0.391 bar.

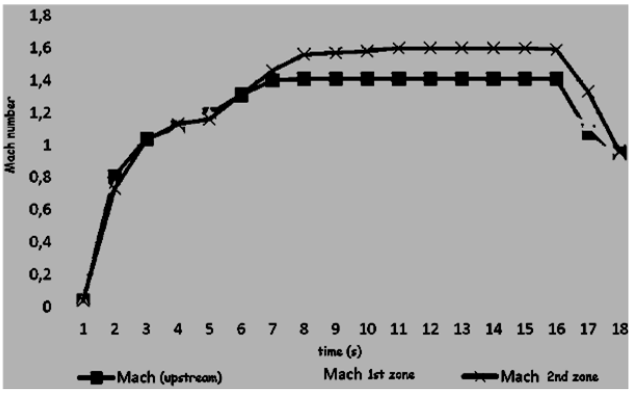
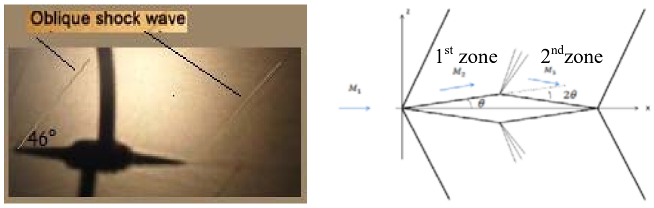



Fig. 6 Evolution of downstream Mach number on the dihedral airfoil.

Fig. 7 presents the experimental development that includes the procedure of oblique shock. Also shows, in Fig. 7 (b), the compression area (1st zone), and the Mach number decreases to 1.22 due at change of flow direction, while expansion area (2nd zone) the flow is accelerating to reach a 1.61 Mach number.

Fig. 7 (a), wave visualization characteristics in the leading edge and trailing edge on the double-wedge airfoil, at 1.4 Mach number, by means of the Schlieren method. The Mach angle in oblique shock wave in the leading edge is 46˚. The shock wave is visible only on the top side of the profile.




(a) (b)

Fig. 7 Oblique shock waves on double-wedge airfoil: (a) visualization characteristics; (b) schematic of compression and expansion zones

In order to validate our model, we compare the model results with the experimental data above. Indeed, we consider a 10° angle dihedral airfoil (ϴ=5°), in which equation profile is split in two zones (Fig. 7 (b)), as:

$$\begin{cases} f^{+}_{1st\ zone} = x & 0 \le x \le 1/2 \\ f^{+}_{2\mathrm{nd}\ zone} = 1 - x & 1/2 \le x \le 1 \end{cases} \tag{12}$$

Then, using the relation (11), we finds according to our model that, for an upstream Mach number $M_1$= 1.4, in the compression zone $M_2$=1.220724175 and the $M_3$ =1.569034156 in expansion area. The estimated differences between model results and experience are, respectively, of order 0.07% in the 1st region and 4% in the 2nd region.

## VI. Conclusion

In this work, the two-dimensional isentropic and inviscid supersonic flow, and around a wedge has been modeled using the asymptotic analysis and characteristics method. Various parameters of our model (Mach number, deviation) were found in good agreement with the results of the theory. The results achieved demonstrate a very high accuracy: the errors in the proposed model are estimated at about $10^{-5}$. These solutions allow us to estimate the flow parameters downstream the shock.

The exploitation of the results of the experimental study, indicates that on the first zone of the dihedral airfoil is a compression area and the second is an expansion area. The estimated differences between model results and experience are, respectively, of order 0.07% in the 1st region and 4% in the 2nd region. The acquisition of Mach number values shows a good agreement. The Schlieren photographs of the shock waves were not satisfactory for quantitative comparisons with the theoretical shapes. However, definite qualitative agreement was observed.

As it is evident from the comparison with the experimental data shows, our model is capable of predicting physically realistic distributions of Mach numbers on the airfoil.

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