

# Performances Analysis of the Pressure and Production of an Oil Zone by Simulation of the Flow of a Fluid through the Porous Media

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**Abstract**—This work is the modeling and simulation of fluid flow (liquid) through porous media. This type of flow occurs in many situations of interest in applied sciences and engineering, fluid (oil) consists of several individual substances in pure, single-phase flow is incompressible and isothermal. The porous medium is isotropic, homogeneous optionally, with the rectangular format and the flow is two-dimensional. Modeling of hydrodynamic phenomena incorporates Darcy's law and the equation of mass conservation. Correlations are used to model the density and viscosity of the fluid. A finite volume code is used in the discretization of differential equations. The nonlinearity is treated by Newton's method with relaxation coefficient. The results of the simulation of the pressure and the mobility of liquid flowing through porous media are presented, analyzed, and illustrated.

**Keywords**—Darcy equation, middle porous, continuity equation, Peng Robinson equation, mobility.

## I. INTRODUCTION

THE important aspect in the study of porous media flow is the nature of the fluids in the pores, which determines the mathematical formulas of the model used to describe mechanical and thermodynamic phenomena related to flow. The simplest case is monophasic flow, where porous media are completely filled with single-phase fluid. Monophasic flows in porous media are of great interest in many situations to science and applied technology. For example, in some recently discovered oil or gas reservoirs, which are in the exploration state, the flow is essentially monophasic. In reservoir engineering, production is mainly the pressure that enters directly or indirectly into all phases of reservoir studies. It is used to characterize a reservoir, calculate reserves in place, and predict the future behavior of the reservoir so the prediction with the results of the numerical simulations, and the porous media have a lot of interest for the oil industry and from these results it is possible to understand some aspects of the flow, as a result, to determine the best schemes and exploration techniques. The present work aims at the modeling and the numerical simulation of fluxes in porous medium.

The flux is considered isothermal, monophasic and can compressible. The fluid (oil) studied is a mixture, the porous

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media is rectangular, and the flow is in two dimensions (2-D). The models used to describe the viscosity and density of the fluid are available. The finite volume method is used to solve numerically the differential equations that represent the transient motion in a porous medium. The results of pressure simulation and mobility of butane liquid and oil are presented, analyzed, and illustrated. Several works have addressed the problem of the flow of monophasic fluids through porous media, but with an experimental setup [1], [2].

## II. MATHEMATICAL MODEL

### A. Flow Equation

The oil mass conservation equations supplemented by the generalized Darcy law and the Neumann type boundary conditions allow us to write:

$$\begin{cases} \frac{\partial(\phi\rho)}{\partial t} + \nabla \cdot (\rho u) = q \\ u = -\frac{k}{\mu} \nabla P \\ P(x, y, 0) = P_0, \forall (x, y) \in \Omega \\ \rho u \cdot \hat{n} = 0, \forall (x, y) \in \partial\Omega \text{ et } t \in [0, t_f] \end{cases} \quad (1)$$

### B. Thermodynamic Model Density

This section describes the thermodynamic model used to calculate fluid density as a function of pressure and temperature.

The model used is the state equation of Peng and Robinson [3], which can be written in the form of a cubic polynomial as a function of  $v$ , as follows:

$$v^3 - \left( \frac{RT}{P} - b \right) v^2 + \left( \frac{a}{P} - \frac{2bRT}{P} - 3b^2 \right) v - b \left( \frac{a}{P} - \frac{bRT}{P} - b^2 \right) = 0 \quad (2)$$

$P$  is the pressure (atm),  $T$  the temperature (K) and,  $v$  ( $cm^3 mol$ ) is the molar volume,  $R = 82.053$  (atm.cm<sup>3</sup> mol.K) and the universal constant of the gases, the molar mass will be determined by the following formula  $\rho = M v$  where  $M$  (g mol),  $v$  is calculated by (2), the repulsion and attraction parameters ( $a$ ), ( $b$ ) are obtained generally by applying critical constraints or by fitting on experimental data, the terms  $a$  and  $b$  of the Peng - Robinson equation are given:

$$b = 0.077796 RT_c / P_c = a_c \alpha$$

$$a_c = 0.457235 (RT_c)^2 / P_c$$

$$m = 0.37464 + 1.54226\omega - 0.26992\omega^2$$

$P_c$ ,  $T_c$  are the critical parameters and  $\omega$  is the acentric factor,  $T_r = T/T_c$  ( $T_r$  reduced temperature) [4].

### C. Extension of the Peng-Robinson

"The Classic Mix Rule" gives the following expressions for the repulsion and mix attraction parameters

$$a_m = \sum_i x_i a_i = a_{mc} \alpha_m \quad (3)$$

$$b_m = \sum_i x_i b_i = 0.077796 \frac{RT_{mc}}{P_{mc}} \quad (4)$$

where  $x_i$  are the mole fractions for the constituents (i),  $a_i$  and  $b_i$  represent the repulsion and attraction parameters corresponding to the constituents (i),  $T_{mc}$  is the critical temperature of mixture,  $P_{mc}$  is the critical pressure of mixture,  $\alpha_{mc}$  is the parameter of repulsion critical of mixture,  $m$  is the constant alpha for mixing [5].

### D. The Crude Oil of Hassi-Messaoud (HMD)

The oil which is located in Hassi-Messaoud (southern Algeria) and very light is contained in the conditions of temperature and pressure of the deposit more than 25% of 7 C, 30% to 40 % of methane and 0.13 % of sulfur.

The critical pressure of the oil is  $P_c = 2.051$  MPa and the critical temperature is  $T_c = 661.714$  K. The rule of pseudocritical coordinates  $w = \sum x_i w_i$  will give us directly the value of the acentric factor  $w = 0.597$ . Critical molar is  $v_c = 6259.018$  (cm<sup>3</sup> /g mol). The molar mass is  $M = 211.939$  (g mol).

### E. Correlation for the Viscosity

For the calculation of the viscosity of the fluid, the formulation used is the Jossi correlation [6], given by the following equation:

$$\left[ (\mu - \mu^*) \xi + 1 \right]^{1/4} = 1.0230 + 0.23364 \rho_r$$

$$+ 0.58533 \rho_r^2 - 0.40758 \rho_r^3 + 0.093324 \rho_r^4 \quad (5)$$

The reduced density is defined by:

$$\rho_r = \frac{\rho}{\rho_c} = \frac{v_c}{v}, \text{ Or } \xi = T^{-1/6} M^{1/2} P^{2/3}.$$

is the value of the viscosity at low pressure,

$$\mu^* = \frac{34(10^{-1}) T_r^{0.94}}{\xi}$$

### F. The Domain Geometry

Here, we consider that the geometry of the porous medium is essentially 2-D, (Fig. 1).

The initial conditions can be written in the initial time ( $t=0$ ) as follows:

$$P(x, y, 0) = P_0(x, y) \quad \forall 0 \leq x \leq L_x \text{ et } 0 \leq y \leq L_y,$$

and the boundary conditions are of types Neumann which describes the absence of mass flux across the boundaries of the porous medium, mathematically represented by  $\rho u \cdot \hat{n} = 0$ ,  $\forall (x, y) \in \partial\Omega$ .

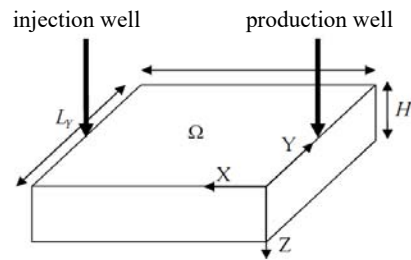


Fig. 1 Geometry of the Porous Medium

### G. Dimension of the Tank

Here, we simulate a case with "real" data reported from Hassi-Messaoud.

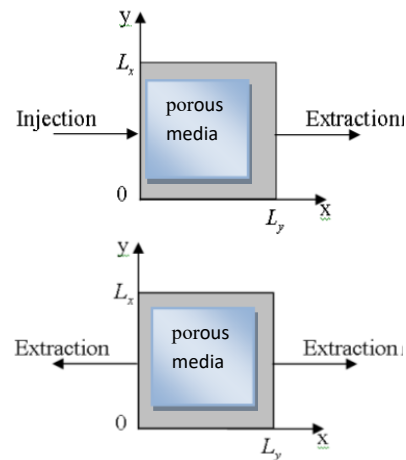


Fig. 2 Pressure variation along the tank

## III. FINITE VOLUME METHODS

The finite volume scheme consists of assigning an unknown at the center of each cell centered mesh. The goal is to obtain a numerical solution of mathematical model, summarized as:

$$\begin{cases} \frac{\partial(\phi\rho)}{\partial t} + \nabla \cdot (\rho u) = q \\ u = -\frac{k}{\mu} \nabla P \\ P(x, y, 0) = P_0, \forall (x, y) \in \Omega \\ \rho u \cdot \hat{n} = 0, \forall (x, y) \in \partial\Omega \text{ et } t \in [0, t_f] \end{cases} \quad (6)$$

By the Gauss-Seidel method with relaxation coefficient, the knowledge of the pressure field makes it possible to calculate the components of mobility density and speed.

$$\begin{aligned} (P_{I,J}^{t+\Delta t})^{k+1} &= (1-w) \cdot (P_{I,J}^{t+\Delta t})^k \\ &+ w \cdot \left[ a_e^k \cdot (P_{I,J+1}^{t+\Delta t})^k + a_w^k \cdot (P_{I,J-1}^{t+\Delta t})^k + a_n^k \cdot (P_{I+1,J}^{t+\Delta t})^k \right. \\ &\left. + a_s^k \cdot (P_{I-1,J}^{t+\Delta t})^k + (S_u^k - S_t^k) \cdot \Delta x \cdot \Delta y \right] / a_p^k \end{aligned} \quad (7)$$

#### IV. RESULTS AND DISCUSSION

a) Cas N ° 1: The emptying is simulated under modes of injection and extraction of fluid where the values (pressure, mobility) always vary with time, according to the model of the wells described by (6). For this, we consider in the injection well a constant pressure equal to  $P_{w(0,L/2)}=372.1619$  (atm) and the point of extraction  $P_{w(L,L/2)}=134.25$  (atm). Simulations have been made for several distinct times (each 3600 s), and it is verified that the model reaches the unsteady state before 10800 s. Attempts at higher times have been made and the behavior does not change. Fig. 2 shows the variation of the pressure along the tank after 10800 s of injection, so that the pressure in the blocks, which contains the wells, tends to approach respectively the pressures in the bottom of the tank.

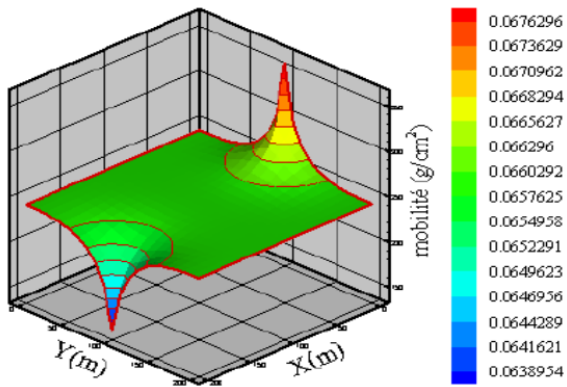


Fig. 3 Density distribution after 10800 injection seconds

This extraction process has a much similar distribution varying between values of 345.745 atm to 155.426 atm.

Fig. 3 shows the density distributions of (petroleum) oil by the Peng-Robinson model. It is observed that the oil is denser, whose variation along the porous medium changes until it reaches an approximate value ( $g/cm^3$ ) neighborhoods of the extraction well and  $0.0676531$  ( $g/cm^3$ ) in the block containing the injection well.

Fig. 5 shows the velocity vector field distribution of the oil (oil) and butane gas along the porous medium during (10800s), of which a pressure difference is imposed ( $p_2 > p_1$ ), or the velocity is in the opposite direction of the greatest slope.

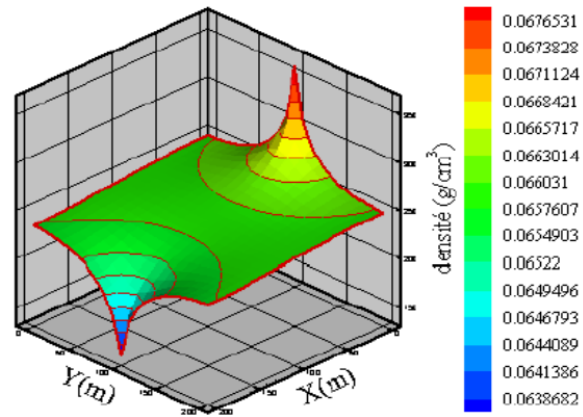


Fig. 4 Distribution of the density after 10 seconds of injection

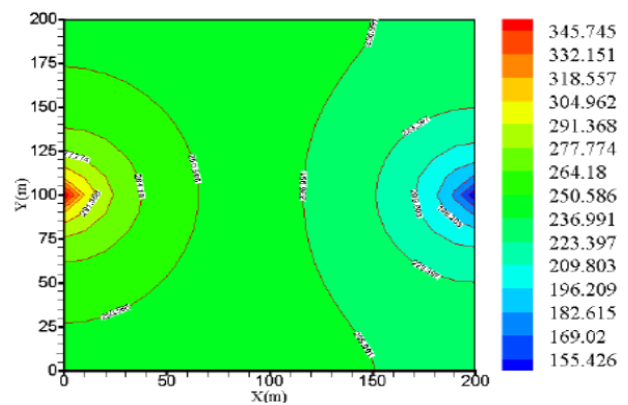


Fig. 5 Pressure field in a homogeneous porous injection medium

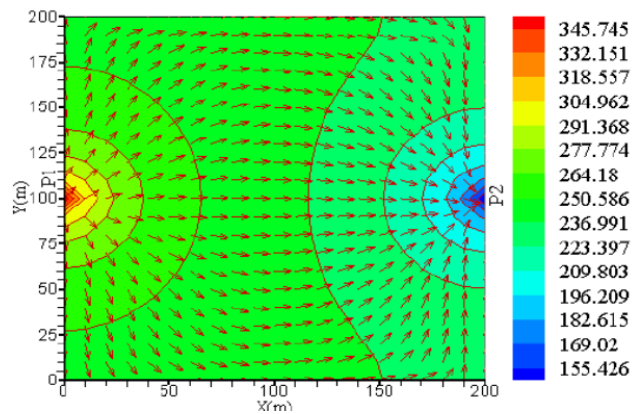


Fig. 6 Velocity vector field distribution after 10800 injection seconds

b) Cas N ° 2: In the second case one seeks to study the phenomenon of depression of a porous medium, which was initially found completely filled with the crude oil (crude oil) with an initial pressure of  $P_{int} = P_g = 600$  atm). The exploration regime is done by employing two localized extraction wells as described in Fig. 6. The process is performed at output pressure values that vary with time. The program stops in the case where the time  $t = 3672$  s second or the pressure  $P_{(1,10)} = P_{w(1,10)}$ . After that time, the extraction well (1, 10) will become a non-producing well.

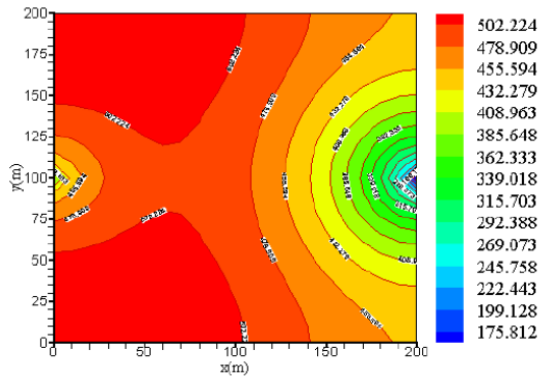


Fig. 7 Pressure field in a homogeneous porous medium after 100 seconds

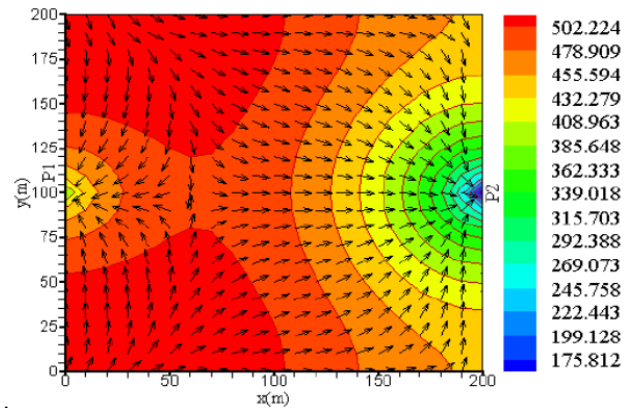


Fig. 11 Velocity vector field distribution after t = 100 seconds

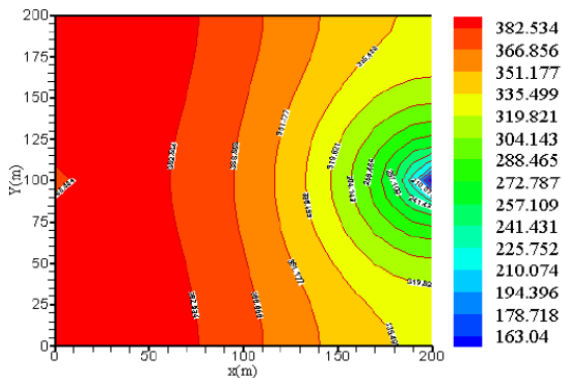


Fig. 8 Pressure field in a homogeneous porous medium in (3672)

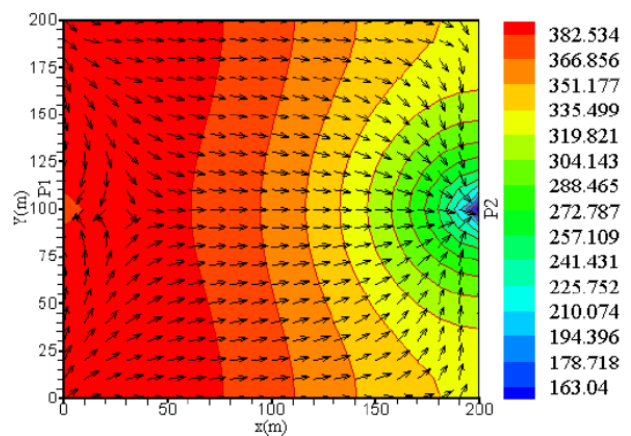


Fig. 12 Velocity vector field distribution after t = 3672 seconds

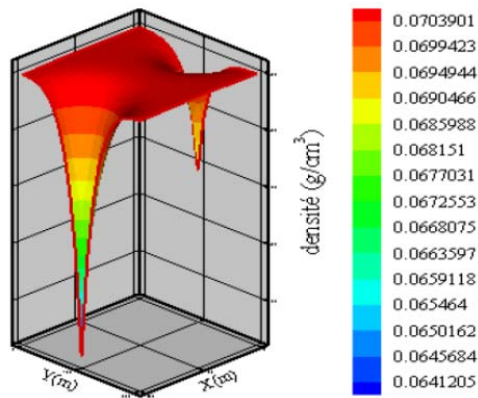


Fig. 9 Density distribution after t = 10 seconds

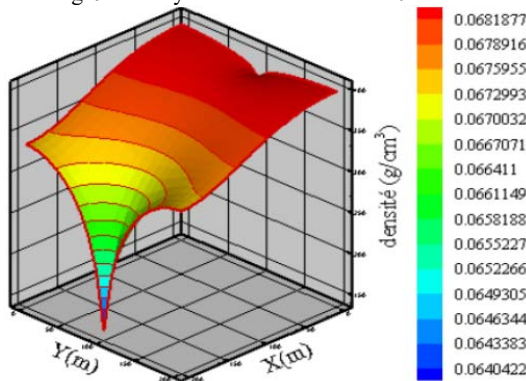


Fig. 10 Density distribution after t = 3672 seconds

In this exploration regime as can be seen in Figs. 7 and 8, before and after (3600 s), the pressure distribution has slightly lower values.

The pressure varies a lot with the time along the tank having values closer to the pressure at the bottom of the well  $P_w(1,10)$  and  $P_w(20,10)$ .

By observing Figs. 9 and 10, it can be noticed that the density of the oil, obtained by the Peng-Robinson model, varies slightly with time, from  $0.06412 \text{ g/cm}^3$  at  $t = 10$  seconds at  $0.06404 \text{ g/cm}^3$  at time  $t = 40$  minutes in the extraction well (20,10). On the other hand, in the well (1,10) there is a variation of  $0.0703901 \text{ (g/cm}^3)$  at  $t = 10$  seconds and  $0.0681 \text{ (g/cm}^3)$  at  $t = 3672 \text{ s}$ . The density which initially has relatively high values tends to decrease, due to continuous extraction with great mobility. The velocity vector field always shows the direction of flow due to the pressure gradient as shown in Fig. 11.

After  $t = 3672 \text{ s}$ , the extraction well at 1.20 turns to an injection well due to the decrease in reservoir pressure Fig. 11.

## V. CONCLUSION

A model for the emptying of a monophasic fluid has been formulated to study the properties of flows in porous media. The Peng-Robinson equation is used to give the density. The viscosity is considered subordinate to the pressure, it is

described through approximate correlations. Darcy's law and the mass conservation equation are combined to describe the variation of pressure values, with the variation of time, the along the porous medium. Drainage properties, such as density and fluid mobility, are obtained from the pressure values. The present work uses the finite volume method with the iterative method (Gauss-Seidel method with relaxation coefficient  $w$ ), allowing us to solve the highly nonlinear differential equations that govern the emptying. This methodology provided a fully free digital algorithm for storage and implicit computation of the associated Jacobian matrix.

The results of the simulations involving oil as substance, has a practical interest in the production sector, presented in the three-dimensional graphic form by describing the mobility of the emptying, under several injection and extraction regimes. have been able to prove the basic characteristics of emptying in porous media: basic compressibility and high mobility.

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