

A Numerical Study of Seismic Response of Shallow Square Tunnels in Two-Layered Ground

Mahmoud Hassanlourad, Mehran Naghizadehrokni, Vahid Molaei

Abstract—In this study, the seismic behavior of a shallow tunnel with square cross section is investigated in a two layered and elastic heterogeneous environment using numerical method. To do so, FLAC finite difference software was used. Behavioral model of the ground and tunnel structure was assumed linear elastic. Dynamic load was applied to the model for 0.2 seconds from the bottom in form of a square pulse with maximum acceleration of 1 m/s^2 . The interface between the two layers was considered at three different levels of crest, middle, and bottom of the tunnel. The stiffness of the two upper and lower layers was considered to be varied from 10 MPa to 1000 MPa. Deformation of cross section of the tunnel due to dynamic load propagation, as well as the values of axial force and bending moment created in the tunnel structure, were examined in the three states mentioned above. The results of analyses show that heterogeneity of the environment, its stratification, and positioning of the interface of the two layers with respect to tunnel height and the stiffness ratio of the two layers have significant effects on the value of bending moment, axial force, and distortion of tunnel cross-section.

Keywords—Dynamic analysis, shallow-buried tunnel, two-layered ground.

I. INTRODUCTION

IN a world where the population is growing, using underground spaces is, for sure, one of the most useful ways to preserve and improve the quality of life [1]. Moreover, design, calculation, and execution of underground structures are among the most difficult, and also the most important issues in geotechnical engineering [2]. Analysis and seismic design of this kind of structures has gained greater attention in recent years due to increased frequency of seismic damages of underground structures [3]. In the great earthquake of Osaka – Kobe in 1995, for example, a number of subway stations and tunnels were severely damaged. The underground structures built in areas affected by the earthquake activities should resist against static and earthquake loading. Increased knowledge about the risk of earthquake for underground structures has also made the engineers increase their perception of factors affecting seismic behavior of underground structures [4], [5].

In 1974, the American Society of Civil Engineers analyzed the damages caused by 1971 San Fernando earthquake in Los Angeles. Japanese Society of Civil Engineers [3] investigated implementation of several underground structures, namely

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tunnels with prefabricated rings, under seismic loads.

Dowding and Rozan studied 71 different cases to understand the effect of earthquake on the behavior of tunnels. The damages of these 71 cases vary from creation of crack to complete blockage of the tunnel. According to these studies, there was no report of rock fall in the tunnel up to horizontal acceleration of 0.19 g, even in the tunnels without cover, and no report was registered regarding creation of crack in covered tunnels. There was only a few number of cracks observed in the tunnel covers up to acceleration of 0.25 g (with maximum ground velocity of 92 cm/s) [6].

Gomes at al. investigated the seismic response of shallow circular tunnels embedded in a two-layered ground using numerical method. They investigated the effect of classification of the soil around the tunnel on seismic response of shallow circular tunnels by applying a single short impact as dynamic input (acceleration of 1 m/s^2 for 0.2 seconds), the deformations created at tunnel cross section due to propagation of seismic waves, and the amount of internal forces created at tunnel cover [7], [8].

Owen and Scholl examined 127 tunnels, including rectangular tunnels implemented using cut and cover 3 methods [9]. The focus was on shallow-buried rectangular tunnels. The result of their studies is in accordance with that of Dowding and Rozan [6]. Furthermore, breakdown of the tunnels drilled is caused by increased lateral pressure of the soil behind the wall. Duration of the earthquake is also an important factor causing tunnel breakdown.

According to the damages caused by earthquake, Hori concluded that safety of tunnels during earthquake depends on the ground conditions, but poor conditions of the environment around the tunnel do not improve just by increasing cover thickness [8].

Sharma and Joud studied 192 cases from the reports of underground structures regarding 85 earthquakes around the world. They created a database from the data collected to determine the effect of different factors on stability of underground spaces. Finally, they proposed a relationship between maximum acceleration at ground surface and depth of overburden, and the amount of damage, which can be used for initial estimation of tunnel stability before dynamic analysis [11].

In this study, the behavior of a shallow square tunnel embedded in a single layer and two-layered ground under an impact load propagated from the bottom is investigated. Deformation, axial force, and bending moment created in the tunnel structure is also studied at the interface of the two layers in different sections of the tunnel.

II. MODELING AND VALIDATION

The process of modeling and accuracy of calculations was first compared with the model proposed by [8]. FLAC finite difference software was used for 2D numerical modeling.

A. Assumptions Governing the Model Geometry

Validation model includes a circular tunnel with 5 m diameter, whose center is located 15 m from the ground surface. The tunnel structure is modeled as a circular ring with linear elastic behavior. Behavioral model of the soil is also considered linear elastic. Geometry of the validation model in meshed form is shown in Fig. 1. It has to be noted that the dimensions and shapes of the meshes and boundary conditions are considered similar to those of the model proposed by Gomes et al. [8]. Ground model is considered as a two-layered ground in three states of boundary layer located in the crest, middle, and bottom of the tunnel. The values of elasticity modulus for the upper layer and lower layer are assumed to be 50-500 MPa and 10-50-100-250-500-1000 Mpa, respectively.

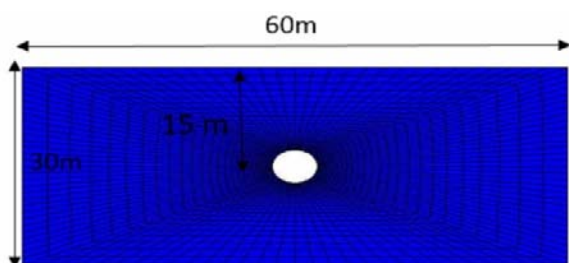


Fig. 1 Model geometry to perform validation

B. Comparing the Results

The values of internal forces obtained by the present dynamic analysis and by the results of Gomes et al. are presented in Tables I and II, respectively [8]. The parameters Eu and El in these tables represent the elasticity modulus of upper layer and lower layer, respectively. According to the values presented in these tables, it is clear that the maximum and minimum relative error of the results of the present numerical model is 15% and 5%, respectively. The comparisons were also made for other states and similar results were obtained.

TABLE I

THE VALUES OF INTERNAL FORCES OBTAINED BY THE PRESENT NUMERICAL MODEL IN THE BOUNDARY LAYER LOCATED AT THE TUNNEL CREST [8]

Eu	50	500
EL=10 M(KN.m)	23.57	24.75
EL=10 N(KN)	34.25	34.94
EL=50 M(KN.m)	40.27	40.11
EL=50 N(KN)	94.6	108.6
EL=100 M(KN.m)	48.51	45.18
EL=100 N(KN)	174.41	193.8
EL=250 M(KN.m)	81.16	51
EL=250 N(KN)	418.23	394.17
EL=500 M(KN.m)	114	54.09
EL=500 N(KN)	846	682
EL=1000 M(KN.m)	189.1	74.86
EL=1000 N(KN)	1600	1300

TABLE II

THE VALUES OF INTERNAL FORCES OBTAINED BY THE MODELING OF GOMES ET AL. IN THE BOUNDARY LAYER LOCATED AT THE TUNNEL CREST [8]

Eu	50	500
EL=10 M(KN.m)	20	20
EL=10 N(KN)	30	30
EL=50 M(KN.m)	37	37
EL=50 N(KN)	85	80
EL=100 M(KN.m)	48	45
EL=100 N(KN)	180	170
EL=250 M(KN.m)	70	48
EL=250 N(KN)	400	370
EL=500 M(KN.m)	105	50
EL=500 N(KN)	800	600
EL=1000 M(KN.m)	170	70
EL=1000 N(KN)	1500	1200

III. MODELING THE SQUARE TUNNEL

A. Model Geometry

Fig. 2 shows the meshed geometry of a soil mass with 30 m thickness and 60 m width, within which a 5x5 square cross section tunnel is drilled. The tunnel center is located 15 m from the ground surface. The thickness of the concrete tunnel cover is 0.25 m.

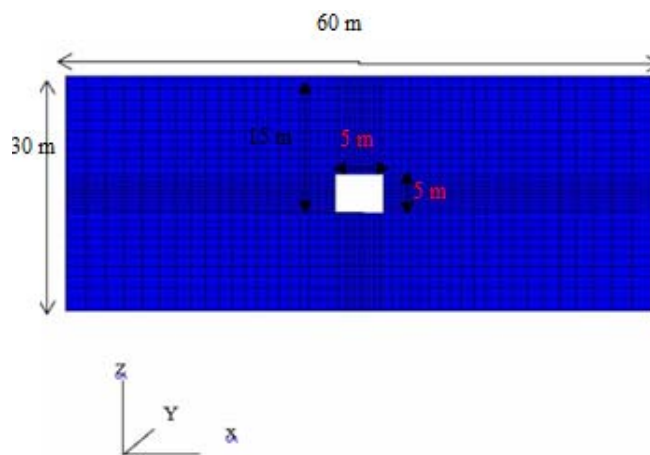


Fig. 2 Model geometry

B. Behavioral Model of the Materials

Linear elastic model is used to model soil behavior and tunnel cover structure. Material properties, including elasticity modulus and Poisson's ratio, are given in Table III.

TABLE III

MATERIAL PROPERTIES REQUIRED TO PRESENT ELASTIC MODEL [8]

Parameters	Ground	Tunnel structure
Young's modulus	10-50-100-250-500-1000	24800
Poisson's ratio	0.3	0.2
Density (ton/m ³)	2	2.5

IV. DYNAMIC ANALYSIS

To perform the dynamic analysis, a short impact was applied to the model as the input move (acceleration of 1 m/s² for 0.2 seconds) from the bottom, so that the amount of shear

strain created in the boundary was 0.001. Rayleigh damping was used for the model with damping factor equal to 5%. The model vibrated in the longitudinal direction under the input wave, following which the wave propagated from the lower

boundary towards the upper boundary, normal to the tunnel cross section. The displacement contours created in the horizontal direction as a result of upward shear wave propagation are shown in Fig. 3 [12].

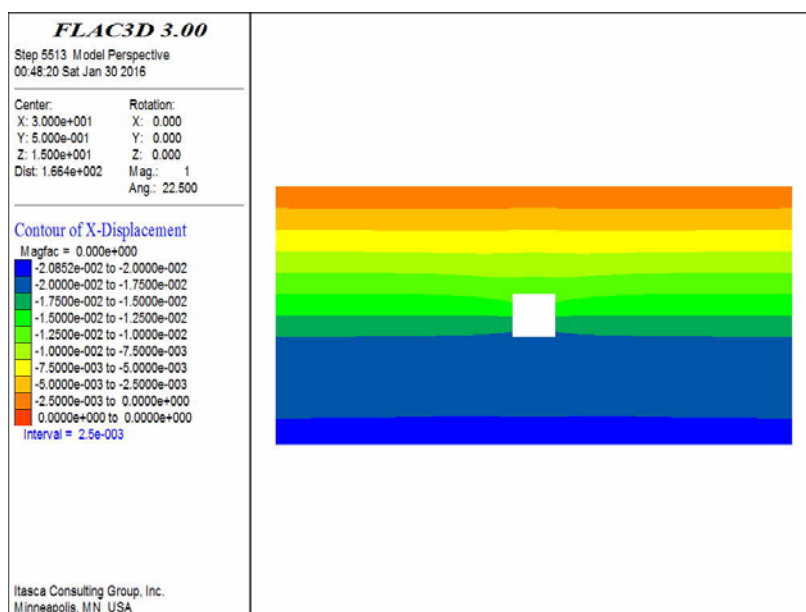


Fig. 3 Displacement contours created in the horizontal direction because of shear wave propagation towards the top of the model

Deformation of the tunnel structure and the amounts of axial force and bending moment created in the tunnel structure due to collision of the tension impact to the tunnel structure are investigated in the two following states: Single layer soil state, in which tunnel cross section is located in a homogeneous layer of the soil. Two-layered soil state, in which the interface of the two layers is positioned in three states relative to tunnel cross section: The interface of the two layers is located in the tunnel crest level (12.5 m depth). The interface of the two layers is located in the middle of the tunnel height (15 m depth). The interface of the two layers is located in the tunnel floor level (17.5 m depth).

V. ANALYSIS RESULTS

A. Two-Layered Boundary at Tunnel Crest Level

The values of bending moment and axial force of the tunnel structure for the state in which the interface of two layers is located at the tunnel crest level are shown in Figs. 4 and 5, respectively. It is seen that in case of homogeneous soil, the internal forces of tunnel structure increase by increasing environment stiffness. It is clear in Fig. 5 that in case of homogeneous environment, the minimum axial forces are created in the tunnel structure. The same is almost true for the values of bending moment, but this trend changes by higher stiffness of the upper layer (1000 MPa and $E_u=500$, for example). On the other hand, if the tunnel cross section is located between two soil layers in a heterogeneous environment, stiffness of the lower layer will not have any effect on the bending moment and axial force in the case in which the elasticity modulus of the upper layer is 10 MPa (the

minimum value). However, both bending moment and axial force of the tunnel structure increase by increasing the stiffness of the upper layer and decreasing the stiffness of the lower layer, even if their rate of changes are different.

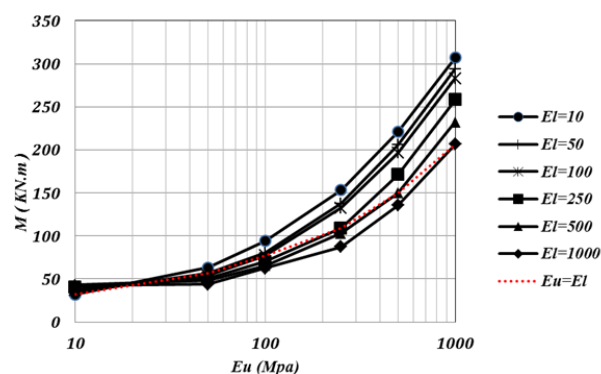


Fig. 4 Bending moment of the tunnel structure in case the interface of the two layers is located at tunnel crest level

The maximum normalized values (the ratio of corresponding values for heterogeneous to homogeneous environment) of bending moment and axial force versus upper/lower layer stiffness ratio are shown in Figs. 6 and 7, respectively. It is clear from Fig. 6 that, except the case in which elasticity modulus of the upper layer is 10 MPa, the ratio of bending moment of heterogeneous to homogeneous layer increases with increasing upper/lower layer stiffness ratio. Furthermore, the ratio of heterogeneous/homogeneous bending moment is less than 1 for the case in which upper/

lower layer stiffness ratio is less than 1, while it is greater than 1 for upper/lower layer stiffness ratios greater than 1, i.e., the softer the upper layer in a layered ground, the less the bending moment of the tunnel structure. However, according to the analysis results, another trend is observed in case the upper layer is very soft ($E=10$ MPa).

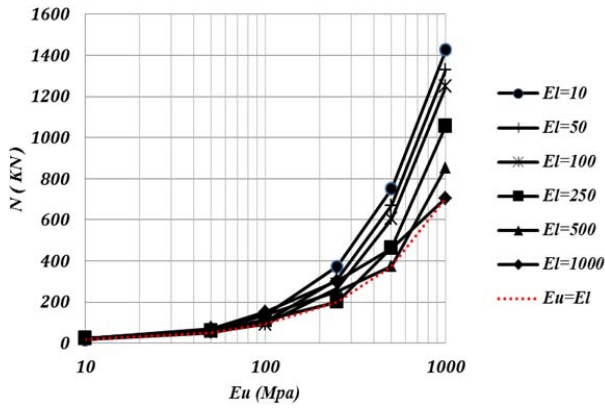


Fig. 5 Axial force of the tunnel structure in case the interface of the two layers is located at tunnel crest level

Contrary to Fig. 6, Fig. 7 shows that the more homogeneous the environment is, less axial forces will be created in the tunnel structure. In other words, ground heterogeneity increases the tunnel structure axial force and the probability of breakdown of the buried structure.

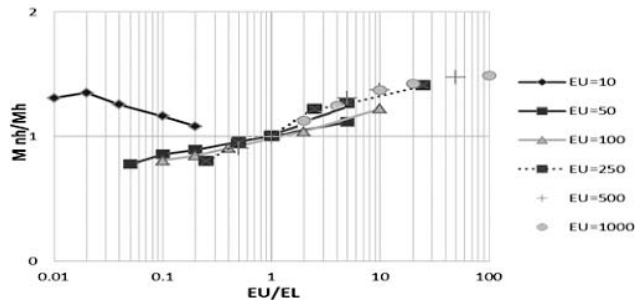


Fig. 6 The ratio of maximum bending moment in heterogeneous soil to homogeneous soil in case the interface of the two layers is located at tunnel crest level

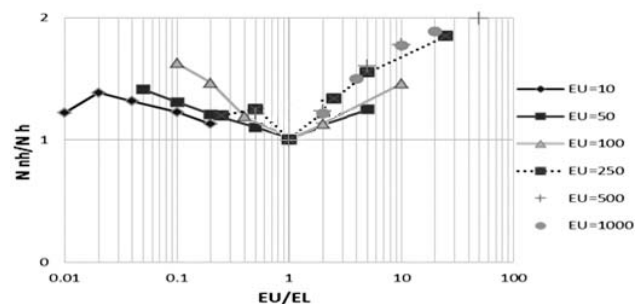


Fig. 7 The ratio of maximum tunnel structure axial force in heterogeneous soil to homogeneous soil in case the interface of the two layers is located at tunnel crest level

B. Two-Layered Boundary Located at the Middle of Tunnel Height

Maximum bending moment and axial force of the tunnel structure for the case in which the interface of the two layers is located at the middle of tunnel height are shown in Figs. 8 and 9, respectively. Both figures show that bending moment and axial force of the tunnel structure increase by increasing the stiffness of upper layer, as well as the lower layer. Environment heterogeneity has different effects on the internal forces of the tunnel structure; it might reduce the internal forces of the tunnel structure (e.g., below the dashed line) or on the contrary, increase them (upper section of the dashed line).

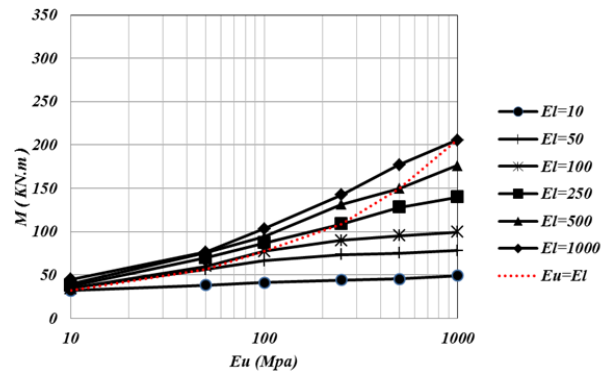


Fig. 8 Maximum bending moment of the tunnel structure when the interface of the two layers is located at the middle of tunnel height

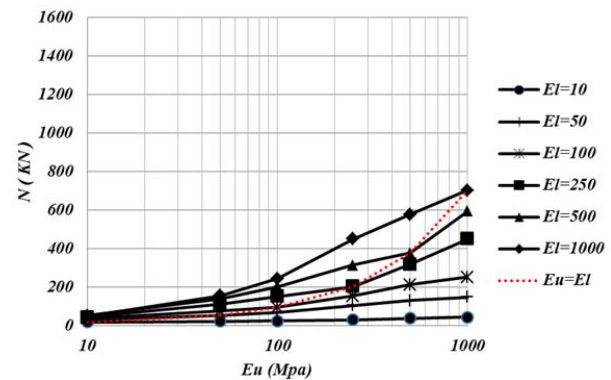


Fig. 9 Maximum axial force of the tunnel structure when the interface of the two layers is located at the middle of tunnel height

The ratio of maximum bending moment and axial force of the tunnel structure between heterogeneous and homogeneous environments for the case in which the interface of the two layers is located at the middle of tunnel height are shown in Figs. 10 and 11, respectively. Both figures indicate that increasing the ratio of upper/lower layer stiffness (Eu/EI) results in decreasing the bending moment and axial force of the tunnel structure. Both M_{nh}/M_h and N_{nh}/N_h ratios are greater than 1 when Eu/EI is greater than 1, while both of them are less than one when Eu/EI is less than 1.

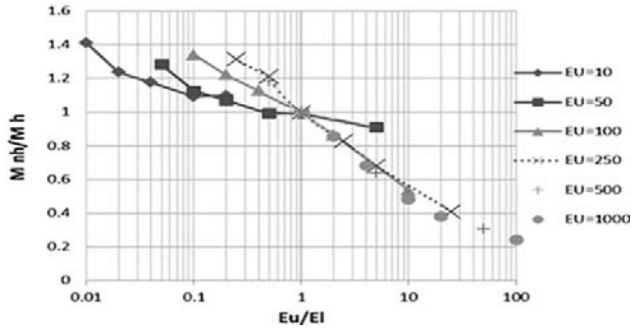


Fig. 10 The ratio of maximum bending moment of the tunnel structure between heterogeneous and homogeneous soils, when the interface of the two layers is located at the middle of tunnel height

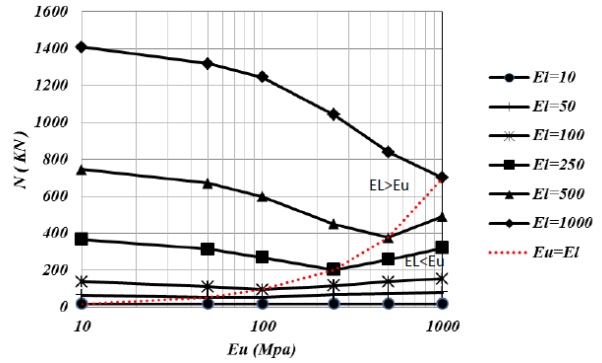


Fig. 13 Maximum axial force of the tunnel structure when the interface of the two layers is located at the tunnel bottom level

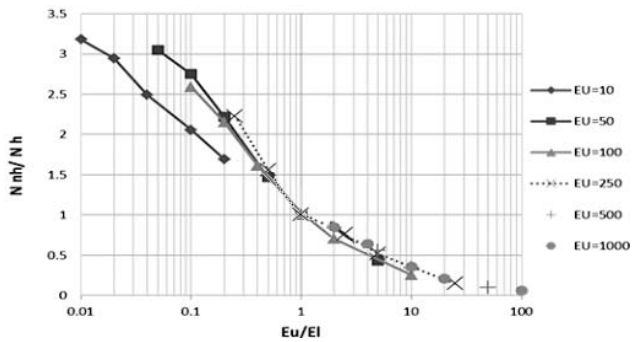


Fig. 11 The ratio of maximum axial of the tunnel structure between heterogeneous and homogeneous soils, when the interface of the two layers is located at the middle of tunnel height

C. Two-Layered Boundary Located at the Tunnel Bottom Level

Maximum values of bending moment and axial force against the stiffness of the upper layer for different values of elasticity modulus of the lower layer, when the interface of the two layers is located at the tunnel bottom level, are shown in Figs. 12 and 13, respectively. According to these figures, by increasing the stiffness of the upper layer and decreasing the stiffness of the lower layer, the bending moment and axial force of the tunnel structure decreases under dynamic load (except a few cases in axial force). Moreover, if $EI > Eu$, greater internal forces will be created than the homogeneous state, and vice versa.

Figs. 14 and 15 indicate that when the interface of the two layers is located at the foundation bottom level, the M_{nh}/M_h and N_{nh}/N_h ratios decrease non-linearly. Even in high ratios of Eu/EI (e.g., greater than 10 and 2 for bending moment and axial force, respectively), they approach zero. This implies that when the whole tunnel is located in a stiff layer whose lower layer stiffness is much lower, the effect of dynamic load gradually disappears. This can create much greater forces in the tunnel structure (up to 10 times greater) when the upper layer is much softer than the lower layer (e.g., $Eu/EI=0.01$) and the interface of the two layers is located at the tunnel bottom level, which leads to greater breakdown of the tunnel.

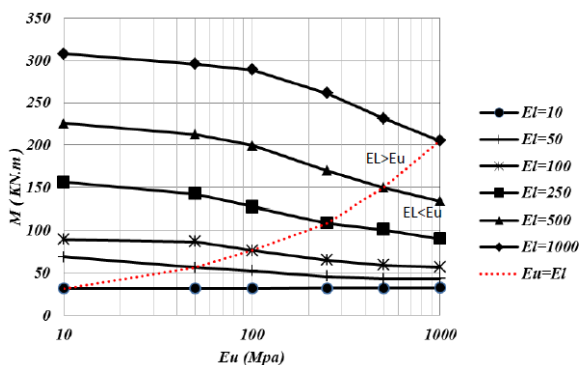


Fig. 12 Maximum bending moment of the tunnel structure when the interface of the two layers is located at the tunnel bottom level

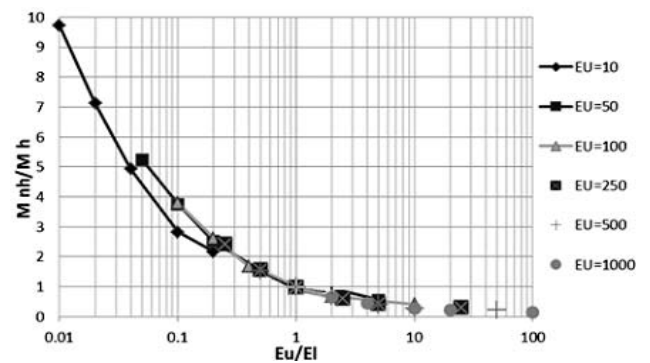


Fig. 14 The ratio of maximum bending moment of the tunnel structure between heterogeneous and homogeneous soils (interface of the two layers located at the tunnel bottom level)

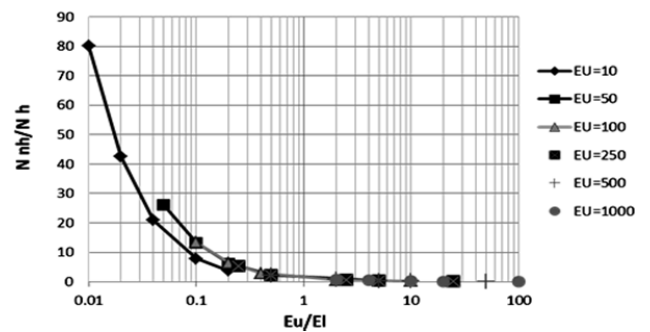


Fig. 15 The ratio of maximum axial force of the tunnel structure between heterogeneous and homogeneous soils (interface of the two layers located at the tunnel bottom level)

VI. DEFORMATION OF THE TUNNEL STRUCTURE

Figs related to deformation of the tunnel structure for two values of elasticity modulus of the upper layer (50 and 500 MPa) and different values of elasticity modulus of the lower layer with an amplification factor equal to 300, when the interface of the two layers is located at the tunnel crest level, are presented in Fig. 16. Replication factor (R), as the difference between horizontal displacement of the tunnel crest and bottom divided by tunnel height, is used to investigate the amount of deformation of the tunnel structure.

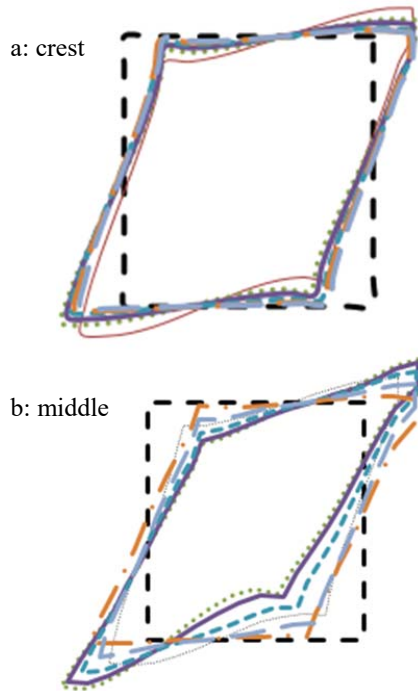


Fig. 16 Deformation of the tunnel structure when the interface of the layers is located at the tunnel crest level
 a: crest
 b: middle
 c: bottom

According to Fig. 17, when tunnel is located in a homogeneous layer of soil ($E_l = E_u$), Replication factor (R) increases with increasing elasticity modulus of the soil. For example, when the tunnel cross section is located in a homogeneous environment and very soft soil with elasticity modulus of 10 MPa, “ R ” would be equal to 1.01, and if the soil around the tunnel is very stiff with elasticity modulus of 1000 MPa, “ R ” would be 1.7. This trend is a bit thought provoking, but is consistent with the changing trend of internal forces.

When the tunnel is located between two layers of soil in a heterogeneous environment, i.e., if the upper layer soil is a highly stiff soil with elasticity modulus of 1000 MPa, and the elasticity modulus of the lower layer soil is 10 MPa (i.e., very soft soil), the deformation resulted would be maximum (2.5).

Fig. 17 (b) shows that when the interface of the two layers is located at the middle of the tunnel height, and the upper layer is stiffer, tunnel structure replication would be less than that of a homogeneous environment. In other words, the tunnel would be less deformed in this case. According to Fig. 17 (c),

when the interface of the two layers is located at the tunnel bottom level, an intermediate state occurs. So that the greater stiffness of the lower layer might result in increasing the tunnel structure replication, depending on the amount of layer stiffness and relative stiffness of the two layers (e.g., the right-hand side of the figure). Therefore, it can be generally argued that the effect of environment stratification can lead to increasing or decreasing the amount of tunnel structure replication, depending on the positioning of the interface of the two layers and their relative stiffness.

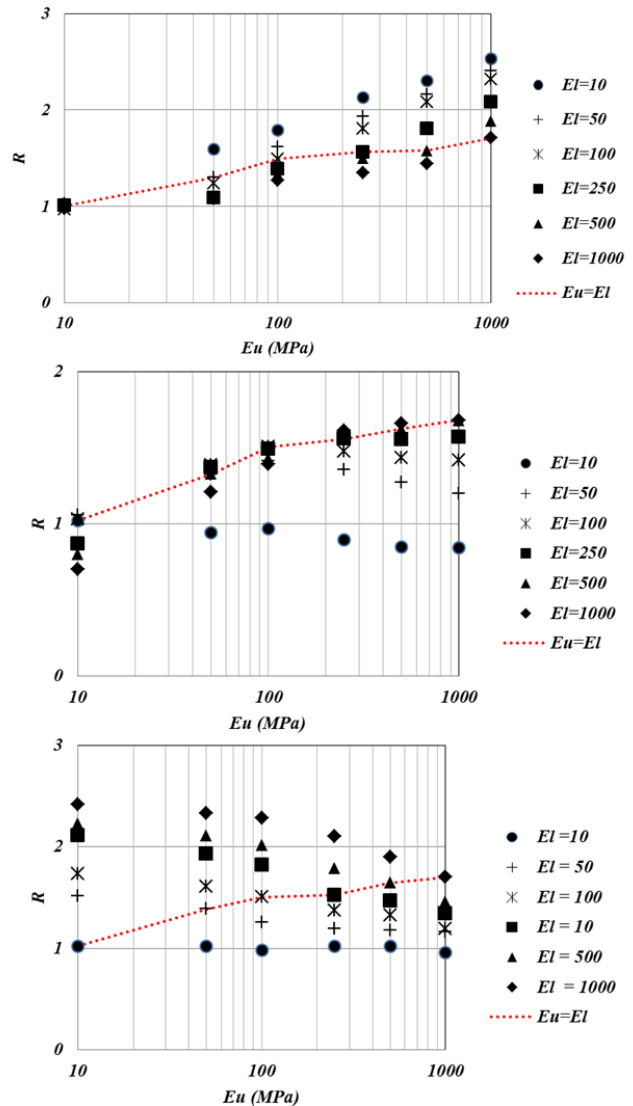


Fig. 17 Tunnel structure replication factor versus stiffness of the upper layer

VII. BENDING MOMENT AND TUNNEL STRUCTURE DEFORMATION RELATIONSHIP

The internal forces created in the tunnel structure are definitely proportional to its deformation. Therefore, the ratio between bending moment against increased diameter in two-layered and single layer ground, when the interface of the two layers is located at the crest, middle and height of tunnel

height are shown in Figs. 18-20, respectively.

According to Fig. 18, when the interface of the two layers is located at the tunnel crest level, the ratio between bending moment of heterogeneous and homogeneous environments increases linearly by the impact of the stress wave (except for $E_u=10$ MPa), regardless of the elasticity modulus. Fig. 19 shows that when the interface of the two layers is located in the middle of the tunnel height, the ratio between bending moment of two-layered and single layer soil increases by increasing the diameter ratio (except for the two cases of $E_u=10, 50$ MPa), but the increasing trend is non-linear. Fig. 20 shows the same process for the state in which the interface of the two layers is located at the bottom of the model and implies that the amount of increased diameter ratio is greater than the two previous stated (up to 40%). Since the ground was assumed elastic, no failure and breakdown is seen in the structure. In such cases and with this much increase in tunnel diameter (40%) and bending moment (80 times), tunnel structure will definitely be destroyed. Investigating shows that although the ground and tunnel structure are assumed linear elastic, their interaction creates a non-linear behavior.

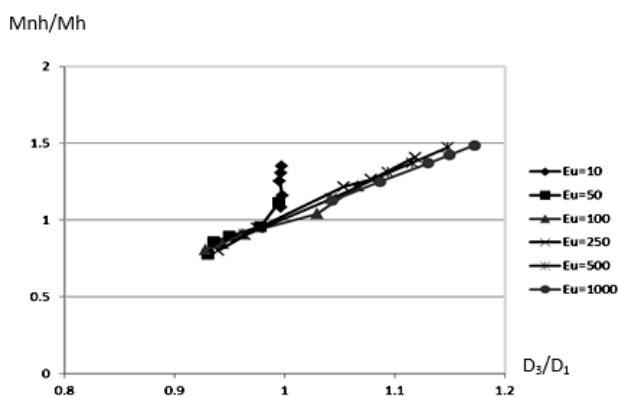


Fig. 18 The ratio between bending moment against increased diameter in two-layered and single layer ground, when the interface of the two layers is located at the tunnel crest level

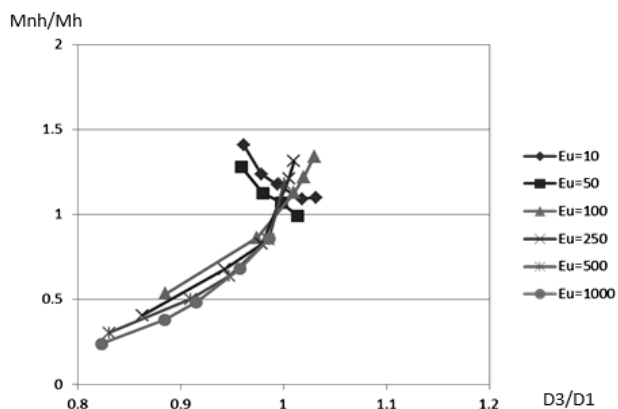


Fig. 19 The ratio between bending moment against increased diameter in two-layered and single layer ground, when the interface of the two layers is located at the middle of tunnel height

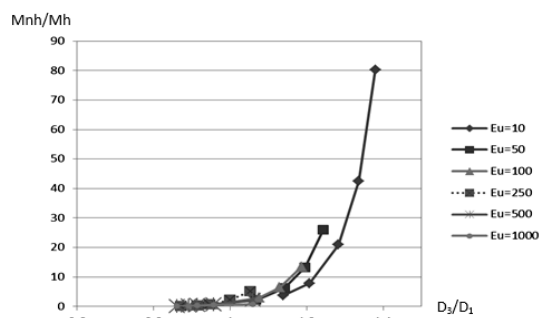


Fig. 20 The ratio between bending moment against increased diameter in two-layered and single layer ground, when the interface of the two layers is located at the tunnel bottom level

In this study, the behavior of a shallow-buried square tunnel, including tunnel cross section deformation caused by seismic wave propagation, as well as the amount of the axial force and bending moment applied to the tunnel structure positioned in two-layered environment in three states of interface located at the crest, middle, and bottom level of the tunnel were numerically studied. The elasticity modulus of the soil layers was considered 10-1000 MPa, and the elasticity modulus of the tunnel structure was considered to be 24.8 GPa. Analysis results indicate that: When tunnel (with greater stiffness than the environment) is located in a homogeneous layer of soil, the amount of deformation (replication) in the tunnel cross section increases by increasing the soil elasticity modulus. Furthermore, greater stiffness of the environment in this case results in creation of greater bending moment and axial force in the tunnel structure, which is in accordance with the trend of deformations.

The changing trend of axial forces and bending moment created at the tunnel cross section in a heterogeneous environment is rather complicated, so that no explicit rule can be observed for all cases.

The ratio between bending moment and axial force in a two-layered and homogeneous environment, when the interface of the two layers is located at the tunnel bottom, decreases by increasing the ratio of upper/lower stiffness ratio, and finally approaches zero at a ratio around 10. The same is true when the interface of the two layers is located at the middle of the tunnel height, except that the rate of decline and its trend is a little different. When the interface of the two layers is located at the tunnel crest level, the trend is rather sporadic. However, the ratio between bending moment and axial force in two-layered and single layer ground generally decreases up to stiffness ratio of 1, and then increases.

Regarding the maximum axial force and bending moment created at the tunnel cross section in a layered environment, it was observed that, except in very soft environments, as the tunnel cross section is more replicated due to dynamic load, a greater bending moment would be created at the tunnel cross section. Depending on the positioning of the interface of the two layers with respect to the tunnel height, the relationship between replication and maximum bending moment can be linear or non-linear.

VIII. CONCLUSION

In this study, the behavior of a shallow-buried square tunnel, including tunnel cross section deformation caused by seismic wave propagation, as well as the amount of the axial force and bending moment applied to the tunnel structure positioned in two-layered environment in three states of interface located at the crest, middle, and bottom level of the tunnel were numerically studied. The elasticity modulus of the soil layers was considered 10-1000 MPa, and the elasticity modulus of the tunnel structure was considered to be 24.8 GPa. The observations proved that locating a tunnel with greater stiffness than the soil in homogeneous soil can lead to increasing the soil elasticity modulus. Increasing the bending moment and axial force in the tunnel structure can be a result of increasing the stiffness.

Increasing the ratio of upper/lower stiffness ratio can lead to decreasing the ratio between bending moment and axial force in a soil. The same phenomenon will happen when the interface of the two layers is located at the middle of the tunnel height. However, the ratio between bending moment and axial force has a gradual reduction up to stiffness 1 in two and single layered soil.

Concerning the maximum axial force and bending moment, the results proved that replicating the tunnel cross section due to dynamic load could lead to increasing the bending moment at the tunnel cross-section. The relationship between replication and maximum bending moment can be rather linear or non-linear, which is based on the position of the interface of the two layers with respect to the tunnel height.

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