Abstract—Many modern synchronous generators in power systems are extremely weakly damped. The reasons are cost optimization of the machine building and introduction of the additional control equipment into power systems. Oscillations of the synchronous generators and related stability problems of the power systems are harmful and can lead to failures in operation and to damages. The only useful solution to increase damping of the unwanted oscillations represents the implementation of the power system stabilizers. Power system stabilizers generate the additional control signal which changes synchronous generator field excitation voltage. Modern power system stabilizers are integrated into static excitation systems of the synchronous generators. Available commercial power system stabilizers are based on linear control theory. Due to the nonlinear dynamics of the synchronous generator, current stabilizers do not assure optimal damping of the synchronous generator’s oscillations in the entire operating range. For that reason the use of the robust power system stabilizers which are convenient for the entire operating range is reasonable. There are numerous robust techniques applicable for the power system stabilizers. In this paper the use of sliding mode control for synchronous generator stability improvement is studied. On the basis of the sliding mode theory, the robust power system stabilizer was developed. The main advantages of the sliding mode controller are simple realization of the control algorithm, robustness to parameter variations and elimination of disturbances. The advantage of the proposed sliding mode controller against conventional linear controller was tested for damping of the synchronous generator oscillations in the entire operating range. Obtained results show the improved damping in the entire operating range of the synchronous generator and the increase of the power system stability. The proposed study contributes to the progress in the development of the advanced stabilizer, which will replace conventional linear stabilizers and improve damping of the synchronous generators.

Keywords—Control theory, power system stabilizer, robust control, sliding mode control, stability, synchronous generator.

I. INTRODUCTION

Synchronous generators (SG) produce most of the electricity. In the past decade, we have witnessed significant increase of stability problems of many SG [1]. There are two main reasons for that:
- changes in construction of SG, which come from requirements of cost optimization for production and transport of the generators. As a consequence, a damping coefficient of rotor’s damping windings and a rotor’s inertia moment are reduced;
- introduction of additional control systems into power systems. The introduction is inevitable due to altered modes of operation of the power systems, e.g. operation of facilities at their peak performance, reaching of novel, strict requirements for transmission system operators, open transmission access, environmental constraints and increased competitiveness. The additional control systems improve operation in a particular operation regime. In some other operation conditions, the additional control systems could reduce damping and can lead to instability of the power system.

Consequences of elevated stability problems are clearly observed in an increasing number of major power system blackouts in recent years. The examples include the India blackout of July 30th and 31st, 2012 (totaling 1 billion people affected), the Southern Brasil blackout of March 11th, 1999 (97 millions of people affected), the Northeast USA-Canada blackout of August 14th, 2003 (55 millions of people affected) and the Italian blackout of September 28th, 2003 (55 millions of people affected) [2]. The key causing factors for the blackouts are numerous.

In this article, we have limited ourselves on treatment of a problem of small signal stability [3]. The oscillations covered in this article are not the main reason for the blackouts. However, it is safe to say that the considered instability could be a causing co-factor for the blackout and also that the oscillations which do not cause the instability of the system are harmful. The oscillations increase mechanical loading and shorten an operational life span, increase losses and reduce peak performance in production and transmission.

The only useful solution for damping of small signal oscillations of SG and prevention of instability represents usage of the additional damping control system. A core element of the system is a power system stabilizer (PSS). Based on the measurement of oscillatory variables, PSS generates the signal which changes field excitation voltage, hence increases damping of SG. Such the control system for the stabilization can be integrated into a static excitation system, which represents the most common method for the excitation of SG. A conventional variant of PSS is based on the linear control theory [4]. This PSS is simple to realize, but its application shows non-optimal damping throughout the entire operating range. The dynamics of the SG depends on the operation conditions. Therefore the conventional linear PSS stabilizer which is determined for the nominal operating point does not assure the optimal damping in the entire operating range.
In recent 45 years, we are witnessing intense development of control theories. Due to increased interests for improved solutions for stabilization of SG, almost all new control theories were tried to be utilized. The problem incorporates various specifics, such as limited access to measured quantities, availability of only one generator’s input for control, saturation of a control actuator, soft and hard non-linearities in a control plant and highly strict requirements for safe operation of the control system. Due to these specifics, till date, none of the advanced control theories was able to replace the conventional linear concept of PSS, which is still in use in majority of commercial static excitation systems. Following wide actualization of the stability problems of the power systems, development of an improved stabilization system becomes highly intense in recent years. There is considerable cooperation between PSS producers, academic research institutions and power system facilities, which results into effective transfer of theories into prototype systems. It is challenging at the moment to predict which of the PSS concepts will prevail in the future.

The theory of robust control represents an appropriate theoretical basis for ensuring optimum damping of a SG in the entire operating range. There are numerous techniques available for the implementation in PSS. One of an emerging area of robust methods is a sliding mode one. Its various applications were realized primarily due to its simplicity in design as well as its robustness property towards matched uncertainty. Due to its advantages and its relative novelty in industry, the implementation of the robust sliding mode control for PSS was selected for further exploration and elucidation within this article.

II. MATHEMATICAL MODELS OF SG

For simulation and analysis of SG and for design and synthesis of the PSS control system, an adequate mathematical model of SG is needed. The model has to accurately describe dynamics of oscillations. At the same time, the model has to be sufficiently simplified for design of a control system and transparent enough for an analysis. To study a small signal stability phenomenon, the model of SG connected to a power transparent enough for an analysis. To study a small signal dynamics of oscillations. At the same time, the model has to model of SG is needed. The model has to accurately describe synthesis of the PSS control system, an adequate mathematical

\[
\begin{bmatrix}
\delta_x(t) \\
\omega_y(t) \\
E^q_{eq}(t)
\end{bmatrix} = 
\begin{bmatrix}
0 & \omega_y & 0 \\
-\frac{K_1}{2H} & -\frac{D}{2H} & -\frac{K_2}{2H} \\
K_1 & 0 & -\frac{K_2}{T_{d0}}
\end{bmatrix}
\begin{bmatrix}
\delta_x(t) \\
\omega_y(t) \\
E^q_{eq}(t)
\end{bmatrix} + 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{1}{2H} & 0 & \frac{1}{T_{d0}}
\end{bmatrix}
\begin{bmatrix}
T_{m0}(t) \\
0 \\
E_{dih}(t)
\end{bmatrix}
\]

(1)

\[
\begin{bmatrix}
P_{eq}(t) \\
F_{eq}(t)
\end{bmatrix} = 
\begin{bmatrix}
K_1 & 0 & K_2 \\
K_1 & 0 & K_2
\end{bmatrix}
\begin{bmatrix}
\delta_x(t) \\
\omega_y(t) \\
E^q_{eq}(t)
\end{bmatrix} + 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
T_{m0}(t) \\
0 \\
E_{dih}(t)
\end{bmatrix}
\]

(2)

where \(T_{m0}(t)\) represents mechanical torque deviation (p.u.), \(P_{eq}(t)\) is electrical power deviation (p.u.), \(\omega_y(t)\) is rotor speed deviation (rad s\(^{-1}\)), \(\delta_x(t)\) is rotor angle deviation (rad), \(E^q_{eq}(t)\) is a terminal voltage (p.u.), \(H\) is an inertia constant (s), \(D\) is a damping coefficient representing total lumped damping effects from damper windings (p.u./p.u.), \(\omega_y\) is rotor angular velocity (rad s\(^{-1}\)), and \(T_{d0}\) is a d-axis transient open circuit time constant (s), \(K_i\) through \(K_a\) are linearization parameters.

Using a benchmark SG [3], linearization parameters for the whole operating range were calculated. Values of the linearized parameters are changing significantly with a changing operating point. The range of values for the linearized parameters of the considered SG in the entire operating range is:

\[
\begin{align*}
K_1 &= 0.3...2.0 \\
K_2 &= 0.2...1.9 \\
K_3 &= 0.307 \\
K_4 &= 0.1...2.0 \\
K_5 &= -0.15...0.10 \\
K_6 &= 0.3...0.7
\end{align*}
\]

(3)

Inertia constant \(H\), damping coefficient \(D\) representing total lumped damping effects from damper windings, rated
synchronous speed $\omega_R$, and $d$-axis transient open circuit time constant $T_{do}$ are:

$$H = 2.37\;$\text{s} \quad D = 2.0\;\text{pu} \quad \omega_R = 377\;\text{rad}\;\text{s}^{-1} \quad T_{do} = 5.9\;\text{pu} \quad (4)$$

Effectiveness of the studied stabilizers was tested with simulations using the 7th order non-linear model in the entire operating range. In this article, only results of the two most distinctive operating points will be presented:
- the nominal operating point in which a SG mostly operates is $P = 1.0\;\text{pu}$ and $\cos \varphi = 0.85$, and
- the operating point with low active power and high reactive power. This point represents the weakly damped real operating point and therefore the worst case for testing of the PSS, i.e. $P = 0.2\;\text{pu}$ and $Q = 1.0\;\text{pu}$.

For the nominal operating point, $P = 1.0\;\text{pu}$ and $\cos \varphi = 0.85$, the following linearization parameters of the simplified linearized model are calculated:

$$K_1 = 1.448 \quad K_2 = 1.317 \quad K_3 = 0.307 \quad K_4 = 1.805 \quad K_5 = 0.029 \quad K_6 = 0.526 \quad (5)$$

The following eigenvalues belong to these parameters:

$$\lambda_1 = -0.273 \quad \lambda_2 = -0.350 + j\;10.727 \quad \lambda_3 = -0.350 - j\;10.727 \quad (6)$$

Damping of oscillatory eigenmodes is considerably reduced by decreasing of power factor $\cos \varphi$. The following parameters of the simplified linearized model are calculated for operating point $P = 0.2\;\text{pu}$ and $Q = 1.0\;\text{pu}$:

$$K_1 = 1.276 \quad K_2 = 0.286 \quad K_3 = 0.307 \quad K_4 = 0.300 \quad K_5 = 0.027 \quad K_6 = 0.620 \quad (7)$$

The following eigenvalues belong to these parameters:

$$\lambda_1 = -0.540 \quad \lambda_2 = -0.217 + j\;10.074 \quad (8)$$

Fig. 1 shows SG inputs: mechanical torque $T_m$ and field excitation voltage $E_{fd}$. Fig. 2 displays outputs of the 7th order non-linear model of the SG connected to the infinite bus: electrical power $P_e$, rotor angle $\delta$, rotor speed $\omega$ and terminal voltage $V_t$ for the nominal operating point without PSS. Figs. 1 and 2 display inputs and outputs of the 7th order non-linear model of SG by simulation in the vicinity of the nominal operating point. From Fig. 2, oscillatory behavior of SG can be recognized. A need for applications of PSS is visible.

From the analysis of the effects of the different operating points on the linearization parameters of the simplified linearized model, dynamic characteristics of SG can be established. Variations in the dynamics of the SG are considerable. Therefore, implementation of a robust PSS is a logic solution to this problem.

### III. CONVENTIONAL LINEAR PSS

A conventional linear PSS approach is based on utilization of the static excitation system. Through this system, the PSS is changing field excitation voltage of SG. As a consequence, an additional component of an electrical torque is generated. This torque must be in phase with rotor speed and thus increases damping of SG [4]. Fig. 3 presents a block diagram of the simplified linearized model of the SG with the 1st order model of the excitation system and PSS. Symbols on Fig. 3: $k_{es}$ and $T_{es}$ are the excitation system gain (p.u.) and the time constant (s), respectively; $E_{fd,\text{ref}}$ is the reference for field excitation voltage $E_{fd}$ (the both in (p.u.)), while $u_{PSS}$, $y_{PSS}$ and $G_{PSS}(s)$ are the PSS input, the output (the both in (p.u.)) and the transfer function, respectively. As in (1) and (2), subscript $\Delta$ denotes the deviation of the variables from the steady-state operating point and $s$ is the Laplace complex variable.
For PSS input $u_{\text{PSS}}$, the variables which contain information about oscillations must be used. These variables are electrical power, rotor angle, rotor speed, frequency, terminal voltage and acceleration torque. The electrical power is commonly selected as the input to the PSS. The output of the PSS is the control signal for the excitation system. A transfer function of conventional linear PSS is represented as:

$$G_{\text{PSS}}(s) = \frac{y_{\text{PSS}}(s)}{u_{\text{PSS}}(s)} = \frac{k_{\text{PSS}}}{1 + T_1 s + T_2 s + T_3 s + T_4 s} G_{\text{paf}}(s) \quad (9)$$

where $k_{\text{PSS}}$ denotes the stabilizer gain (p.u.), $T_1$, $T_2$, $T_3$, $T_4$ are time constants of the stabilizers lead-lag compensators (s), $T_w$ is the time constant of the high-pass (washout) filter (s) and $G_{\text{paf}}(s)$ is the transfer function of the low-pass (antialiasing) filter.

Based on the block diagram in Fig. 3 and the transfer function in (9), the IEEE association established the IEEE standard for the PSS studies [8]. The standard enables unification of commercial applications of PSS. The standard sets out four basic types of PSS, which differ mainly in regards to an available input and a degree of a transfer function. Most of the commercial PSS are realized on the standardized proposals.

For a synthesis of PSS, knowledge of a mathematical model of SG with an excitation system is required. A required model is calculated from a known data of SG or by means of identification. For considered SG, transfer function $G_{\text{PSS}}(s)$ of PSS with an electrical power input, for nominal operation point $P_\text{n}=1.0 \text{ p.u.}$ and $\cos \phi = 0.85$ was calculated. The transfer function $G_{\text{PSS}}(s)$ was calculated by means of the root locus diagram. Transfer function of $G_{\text{PSS}}(s)$ is:

$$G_{\text{PSS}}(s) = -4.2 \left( \frac{s^2 + 1}{s^{0.2} + 1} \right) \left( \frac{s^{20} + 1}{s^{0.2} \cdot s^{20} + 1} \right) \quad (10)$$

Behavior of the stabilized system on the same input perturbations as in Fig. 2 calculated with the 7th order non-linear model is displayed in Fig. 4.

The actuator saturation was included in the simulations. Hard type saturation of the PSS’s output with a limited value of ±35% of a value of a nominal rotor excitation voltage was utilized. The set value represents a limitation in real excitation systems.

Calculated PSS assures effective damping for the nominal operating point and stable operation across the entire operating range, though with a significantly decreased damping for some non-nominal operating points.

Usage of systematic methods for tuning parameters of conventional PSS gives satisfactory results. The main disadvantages of these methods represent the requirement of the SG mathematical model and the time consuming tuning. Therefore, in practice, the systematic methods are rarely implemented. Hence, neither optimal damping in the nominal operating point nor stable operation in the entire operating range is secured.

Due to not available mathematical model of SG, sophisticated and time consuming synthesis of the conventional linear PSS and its proven non-optimum damping in the entire operating range of SG, the robust control becomes meaningful techniques for the PSS implementation.
IV. ROBUST PSS

Among many robust control approaches, the sliding mode control is the one of the most interesting. The main advantages of this control are insensitiveness to parameter variations, rejection of disturbances, a decoupling design procedure and simple implementations by means of power converters [9].

The fundamentals of the sliding mode control theory date back in the late 1950s years. Since that time, new research directions emerged due to the appearance of the new classes of control problems, new mathematical methods and new prospects of implementation [10], [11].

For the proposed PSS design, the modification of the sliding mode control based on the decoupling principle will be used. The mathematical model of the controlled plant must be transformed to the regular form:

\[ \begin{align*}
\dot{x}_{RF_1}(t) &= A_{RF_1}x_{RF_1}(t) + A_{RF_2}x_{RF_2}(t) \\
\dot{x}_{RF_2}(t) &= A_{RF_2}x_{RF_1}(t) + A_{RF_22}x_{RF_2}(t) + B_{RF_2}u_p(t)
\end{align*} \] (11)

where \( A_{RF_1} \) and \( B_{RF_2} \) are constant matrices of relevant dimensions, \( x_{RF_1}(t) \in \mathbb{R}^{n-m} \) and \( x_{RF_2}(t) \in \mathbb{R}^m \) are state-space vectors and \( u_p(t) \) is controlled plant input vector. Matrix \( B_{RF_2} \) must be nonsingular.

For the PSS design being based on the simplified linearized model of SG, state-space vector in regular form \( x_{RF}(t) \) where \( n = 3 \) and \( m = 1 \) could be selected as:

\[ x_{RF}(t) = \begin{bmatrix} x_{RF_1}(t) \\ x_{RF_2}(t) \end{bmatrix} \] (13)

where:

\[ x_{RF_1}(t) = \begin{bmatrix} \delta_s(t) \\ \delta_{sa}(t) \end{bmatrix} \quad \text{and} \quad x_{RF_2}(t) = \begin{bmatrix} \delta_a(t) \\ \delta_{sa}(t) \end{bmatrix} \] (14)

Sliding mode control for the implementation in PSS requires knowledge of all state-space variables of the SG’s regular form model. Measurements of electrical power, rotor speed and terminal voltage are feasible only at SG. For the sliding mode control, the state-space variables for the regular form model need to be calculated from the measured variables. To calculate regular form state-space variables firstly the variables \( \delta_a(t) \) and \( E'_{\phi s}(t) \) can be calculated by inverting (2), such as:

\[ \begin{bmatrix} \delta_s(t) \\ E'_{\phi s}(t) \end{bmatrix} = \begin{bmatrix} K_a & -K_s \\ K_s & K_5 \end{bmatrix}^{-1} \begin{bmatrix} K_4 & -K_s \\ K_s & K_5 \end{bmatrix} \begin{bmatrix} \delta_a(t) \\ \delta_{sa}(t) \end{bmatrix} \] (15)

Finally, state-space variables \( x_{RF_1}(t) \) and \( x_{RF_2}(t) \) can be calculated with transformation:

\[ \begin{bmatrix} \delta_s(t) \\ \delta_{sa}(t) \\ \delta_a(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega_r & 0 \\ -K_\omega & 2H & -K_\omega \end{bmatrix} \begin{bmatrix} \delta_s(t) \\ \omega_r(t) \\ E'_{\phi s}(t) \end{bmatrix} \] (16)

In such a way, the state-space variables could be obtained without explicit differentiation.

A sliding surface was selected such that the rotor’s angle deviation and rotor’s speed deviation converge exponentially to zero. For this aim, a linear equation of the sliding surface was selected:

\[ s(t) = Dx_{RF_1}(t) + x_{RF_2}(t), \quad s(t) \in \mathbb{R}^n \] (17)

When the sliding mode appears on manifold \( s(t) = 0 \), the system behavior is governed by \((n-m)\)th-order equation

\[ \dot{s}_{RF_1} = (A_{RF_1} - A_{RF_2}D)x_{RF_1} \] (18)

To obtain the required dynamic properties of the control system, we assigned eigenvalues of a closed-loop system with a linear feedback. For controllable system described with (10) and (11) there exists matrix \( D \), which ensures the desired eigenvalues of the system in (16).

In the first stage of design of the sliding mode, we chose the desired eigenvalues of the system described with (16). From the desired eigenvalues we determine matrix \( D \) as the solution to the \((n-m)\)th-order eigenvalue task. Matrix \( D \) determines the equation of discontinuous surfaces (15).

The second stage of the design procedure represents the selection of the discontinuous control law, such that the sliding mode always arises at manifold \( s(t) = 0 \), which is equivalent to stability of the origin in \( m \)-dimensional space \( s(t) \). The dynamics on the \( s(t) \) space is described by equation:

\[ \dot{s}(t) = \left[ DA_{RF_1} + A_{RF_2} \right] x_{RF_1}(t) + \left[ DA_{RF_1} + A_{RF_22} \right] x_{RF_2}(t) + B_{RF_2}u(t) \] (19)

An appropriate choice of the control law represents the discontinuous control described with

\[ u_p(t) = -g[x_{RF}(t)]B_{RF_2}^+ \text{sgn}[s(t)] \] (20)

where \( [x_{RF}(t)] \) is the sum of vector \( x_{RF}(t) \) component moduli and \( g \) is the constant.

The selected discontinuous control leads to:

\[ \dot{s}(t) = Ex_{RF}(t) - g[x_{RF}(t)]\text{sgn}[s(t)] \] (21)

There exists such positive value of \( g \) that the functions \( s(t) \)
and $s(t)$ have different signs. It means that the sliding mode will occur on a discontinuity surface. The influence of discontinuity of the control signal is reduced with a varying amplitude of the control signal.

For SG with the data [3], we selected desired eigenvalues $\lambda_{1,2} = -2$ for the system in (16). The following control law parameters were calculated:

$$D = [4 \ 4] \quad \mathbf{B}^{-1}_{RF} = -0.06 \quad g = 350 \quad (22)$$

The robustness of the presented sliding mode controller was tested with the simulations of the 7th order non-linear model of the SG in the entire operating range numerically, as well as experimentally in the laboratory. During the operation in the entire operating range the simplified linearized model parameters vary between the limits described in (3). The sliding mode controller with the parameters (20) was stable and robust and displays the effective damping in all operating conditions. The theoretical analysis of the invariance of the proposed control system to the disturbances and the variation of the plant parameter are described in details in [9].

![Fig. 5 Electrical power $P_e$ and rotor speed $\omega$ in nomininal operating point $P = 1.0$ p.u. and $\cos \phi = 0.85$ with robust PSS](image)

![Fig. 6 Electrical power $P_e$ and rotor speed $\omega$ in operating point $P = 0.2$ p.u. and $Q = 1.0$ p.u. with robust PSS](image)

In the article only results of the two most distinct operating points are presented: the nominal operating point and the worst case – the least damped operating point. Figs. 5 and 6 show the responses of the 7th order non-linear model of considered 160 MVA SG in nominal operating point $P = 1.0$ p.u. and $\cos \phi = 0.85$ and in weakly damped operating point $P = 0.2$ p.u. and $Q = 1.0$ to the step changes of the mechanical torque and the field excitation voltage, as shown in Fig. 1. The same model of the actuator saturation as in Section III was included in the simulations.

V. CONCLUSION

Changes in construction of SG and tightened operating conditions of contemporary power systems require development of more capable PSS. Namely, conventional linear PSS do not fulfill the requirements. In this paper, the theoretical basis and the utilization of the sliding mode robust control for the implementation in PSS are presented.

The comparison of the developed robust PSS with conventional linear PSS shows the distinct advantage of the modern concepts in all the ranges of the operation considered. In our evaluation, the robust control emerged as the most prospective concepts for the implementation in PSS.

Due to the actuality and importance of the issues tackled, the development of more effective methods for the PSS design is inevitable. It is our estimation that intensity of research in this field will increase in the future. The research will lead to development of a vanquishing stabilizer, which will replace the to-date used conventional linear PSS.

REFERENCES