Performance Analysis of MATLAB Solvers in the Case of a Quadratic Programming Generation Scheduling Optimization Problem

Dávid Csercsik, Péter Kádár

Abstract—In the case of the proposed method, the problem is parallelized by considering multiple possible mode of operation profiles, which determine the range in which the generators operate in each period. For each of these profiles, the optimization is carried out independently, and the best resulting dispatch is chosen. For each such profile, the resulting problem is a quadratic programming (QP) problem with a potentially negative definite Q quadratic term, and constraints depending on the actual operation profile. In this paper we analyze the performance of available MATLAB optimization methods and solvers for the corresponding QP.

Keywords—Economic dispatch, optimization, quadratic programming, MATLAB.

I. Introduction

OPTIMAL scheduling of generators or economic dispatch [1]–[3] is a central problem of power systems research. In [4] a framework has been defined, in which the dispatch optimization of generators with potentially concave production characteristics may be carried out via a distributed manner. Since we assume that the production characteristics are described with piecewise linear production price/MW curves, which may be also decreasing, the optimization task results in a potentially non-convex QP programming problem. The paper [4] defines so called modes of operation and MoO-profiles, which determine the ranges in which the generators operate through the analyzed time frame. The algorithm proposed in [4] also allocates reserves to the generators via the optimization procedure in order to make it capable of optimizing the output of a reserve market bidding.

Joint energy and reserve markets [5] are recently born energy-economical constructions which allocate energy and reserve production in the same time (for an explicit example see eg. [6], [7]). As detailed in [8], joint auction models did not receive much attention so far, on the one hand because of the complexity of the underlying algorithms, and on the other hand, because the current impact of ancillary services onto the total end user tariff is quite small. However these frameworks which allocate energy and reserves simultaneously may facilitate reserve-production capable power plants to an increased vindication of their potential: In these frameworks it is immediately taken into account that as they produce energy, they create the potential of reserve allocation in the same time as well. However, as mentioned before, in this article we do not consider explicit market models, but a prior given zero-elasticty demand of power and reserves, similar to [4]. This approach is important in the topic of joint energy and reserve markets [8]. The increasing market share of renewable technologies [9] underline the importance of reserves and co-allocation methods.

In this paper, we analyze the freely available Matlab solvers for the optimization of a potentially non-convex quadratic co-allocation problem, and compare their performance on various scales as success rate, optimality and computational time.

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II. Materials and Methods

In this paper we analyze a two-level time-frame. First, we consider macro-periods, to which the MoOs of the generators are assigned, and second, we consider micro-periods inside the macro-periods. The power and reserve demands are defined on the level of micro-periods. We consider positive/negative secondary and tertiary reserves, denoted by $s+,\ s-,\ t+,\ t-$. We assume n generators, T macro-periods, and H micro-periods in each macro-period. In the following the variable t without subscript refers to macro-periods, and with subscript, as t_k, refers to the k-th micro-period of the t-th macro-period ($t \in [1, ..., T]$, $k \in [1, ...H]$).

The generation levels, and the amounts of allocated reserves of each type has to be defined for every plant for each micro-period. Thus, the variable vector to be optimized is

$$x = \begin{pmatrix} x_P \\ x_R \end{pmatrix} \qquad (1)$$

where

$$x_P = \begin{pmatrix} p^1(1_1) \\ \vdots \\ p^1(1_H) \\ p^1(2_1) \\ \vdots \\ p^1(2_H) \\ \vdots \\ p^n(T_H) \end{pmatrix} \qquad (2)$$

and

$$x_R = \begin{pmatrix} r^1_{s+}(1_1) \\ \vdots \\ r^1_{s+}(1_H) \\ r^1_{t+}(1_1) \\ \vdots \\ r^1_{t+}(1_H) \\ r^1_{s-}(1_1) \\ \vdots \\ r^1_{s-}(1_H) \\ r^1_{t-}(1_1) \\ \vdots \\ r^1_{t-}(1_H) \\ r^1_{s+}(2_1) \\ \vdots \\ r^1_{t-}(T_H) \\ r^2_{s+}(1_1) \\ \vdots \\ r^n_{t-}(T_H) \end{pmatrix} \quad (3)$$

x_P holds the power generation values, $p^i(j_k)$ denotes the power production value of generator i in the kth micro-period of macro-period j. Similarly, the value $r^i_{s+}(j_k)$ denotes the amount of secondary positive reserve allocated to plant i in the kth micro-period of macro-period j. The notations are straightforward for other reserve types.

x_P has the dimension nTH, while x_R is of dimension $4nTH$. While the prior given power and reserve demands define equality type constraints (eg. the power production in each micro-period has to be equal to the power demand, and similar equations for reserves), the MoO profiles define inequality constraints by the definition of operating intervals.

A. Analyzed Solvers

The following freely available solvers were compared in this study:

- CLP solves LPs using a Primal Simplex method, and QPs using a Dual Simplex method [10]
- SCIP is basically a MILP solver, which also solves non-convex quadratic (and quadratically constrained) problems to global optimality [11]
- OOQP [12] solves QPs using a Gondzio Predictor-Corrector method [13].
- IPOPT [14] is a general nonlinear solver for smooth, twice differentiable, nonlinear programs. While the objective need not be convex, IPOPT will only find local solutions.
- Finally, as a global alternative, the *fmincon* function of Matlab [15] was applied.

CLP SCIP, OOQP, IPOPTm, NLOPT were obtained from the OPTI toolbox [16], while fmincon was called via YALMIP [17].

B. Comparison Metrics

We compare the performance of the above solvers regarding the following points.

- *Successful solution rate*. Since, as detailed in [4], an MoO may be regarded as top-level-feasible or low-level-feasible. Top-level-feasible MoO profiles only ensure that eg. the maximal available power generation level, determined by the MoO does not exceed the power demands corresponding to the micro-periods of the macro-periods. On the other hand, low level-feasibility of an MoO profile means, that an actual dispatch may be generated in the case of the constraints implied by the MoO. Only a subset of top-level-feasible MoO profiles is low-level-feasibile. Considering this, we may analyze the success ratio of the solvers, considering various demand and MoO profiles. This ratio has a maximum not equal to 1, since not MoO profiles are low-level-feasibile.
- *Optimality*. It is very straightforward to calculate the price of the allocation returned by the solver. We compare this to the price of the most optimal case among all solutions.
- *Computational demand*. We measure the computational time of the optimization as well in the case of various solvers.

III. Results

We demonstrate our results on the following example set of plants. We consider 5 power plants, all with 4 MoOs ($n_m = 4$). Furthermore we are considering $T = 4$ and $H = 6$.

- PP 1 - Coal with the following operation intervals

$$p_1^{min} = [0 \quad 400 \quad 600 \quad 750]$$
$$p_1^{max} = [0 \quad 600 \quad 750 \quad 900]$$

- PP 2 - CCGT 1

$$p_1^{min} = [0 \quad 150 \quad 250 \quad 350]$$
$$p_1^{max} = [0 \quad 230 \quad 350 \quad 400]$$

- PP 2 - CCGT 2 with the same operation ranges as CCGT 1. We assume that PP 3 differs only in the offer price from PP 2 ($c^3_{max} > c^2_{max}$ elementwise). The prices demanded for reserves by this generator are also higher.
- PP 4 represents a virtual power plant, which is assumed to not being able to produce reserves.

$$p_1^{min} = [0 \quad 40 \quad 70 \quad 120]$$
$$p_1^{max} = [0 \quad 70 \quad 120 \quad 140]$$

- PP 5 represents a biomass power plant.

$$p_1^{min} = [0 \quad 120 \quad 145 \quad 180]$$
$$p_1^{max} = [0 \quad 145 \quad 180 \quad 200]$$

As mentioned above, we suppose that the plants linearly approximate their real production characteristics in the production intervals corresponding to MoOs. Here, we do not detail all the real and submitted production characteristics, the reserve characteristics and reserve prices. But to give an

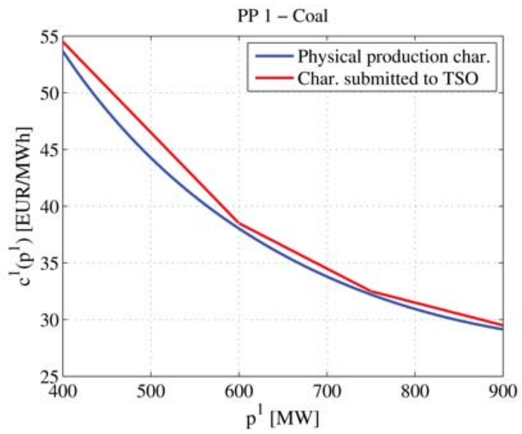

Fig. 1 Production characteristics of PP 1

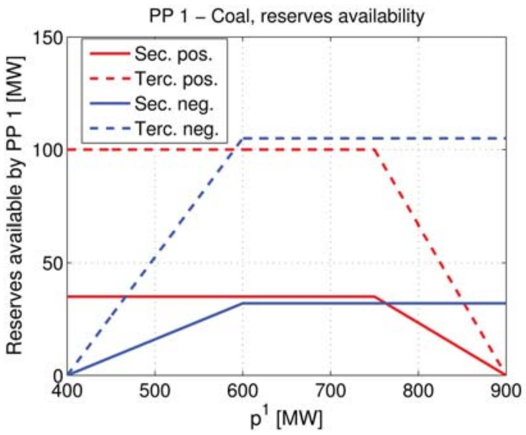

Fig. 2 Reserve characteristics of PP 1. The available tertiary reserves depict the case when no secondary reserve is allocated to the unit

example, we depict the production and reserve characteristics of PP 1 in Figs. 1 and 2 respectively.

the nominal power demand pattern was the following (in MW).

$$\begin{aligned} d(1_1) - d(1_6) &= [987 \quad 905 \quad 851 \quad 749 \quad 921 \quad 991] \\ d(2_1) - d(2_6) &= [1136 \quad 1271 \quad 1329 \quad 1391 \quad 1324 \quad 1311] \end{aligned} \tag{4}$$

4 further power demand vectors were generated as fluctuations of the nominal demand vector with $\pm$ 50 MW fluctuations for each micro-period (uniform distribution). We assumed that the secondary and tertiary reserve demands are 5 and 10 percent of the power demand in each micro-period.

A. Dimensions of the Problem

We considered $T = 2$ macro-periods, and $H = 6$ micro-periods each. This resulted in the dimension of 300 for x, 60 equality type constraints and 360 inequality type constraints.

B. An Example Solution

To give an impression how a possible solution for the above optimization problem looks like, we depict one possible allocation for the nominal power demand vector.

The solution was calculated for the MoO profile

$$MoO = \begin{pmatrix} 2 & 3 \\ 3 & 3 \\ 1 & 2 \\ 2 & 2 \\ 1 & 1 \end{pmatrix}$$

where the rows correspond to plants and the columns correspond to macro-periods. MoO(2,2)=3 refers to the fact that plant 2 is in its third operation interval in macro-period 2.

Figs. 3, 4, 5, 6 and 7 depict the allocation of power generation, secondary positiva, tertiaty positive, secondary negative and tertiary negative reserve allocation respectively. it is easy to recognize in Figs. 3, 4, 5, 6 and 7, that the sum of the amount of power/reserves allocated to various plants is always equal to the demand, thus the equality type constraints hold. With the help of the operation intervals defined in Section III, one may also easily check the validity of inequality type constraints regarding power production values.

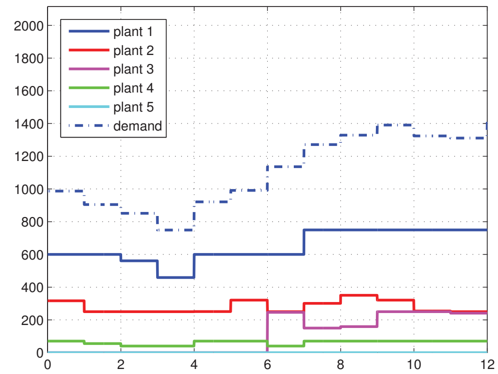

Fig. 3 Example of power allocation among the plants

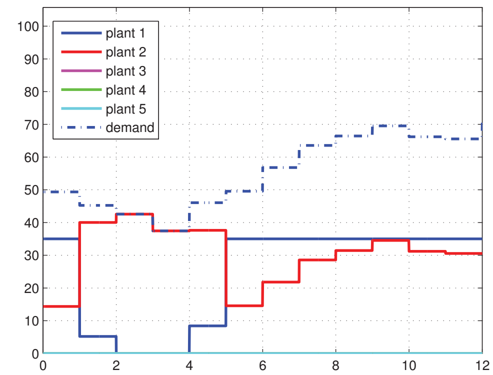

Fig. 4 Example of secondary positive reserve allocation among the plants

C. Results

The solvers were tested in 5 different cases of demand profiles. The numbers of top-level-feasible profiles for each demand curve are summarized in Table I.

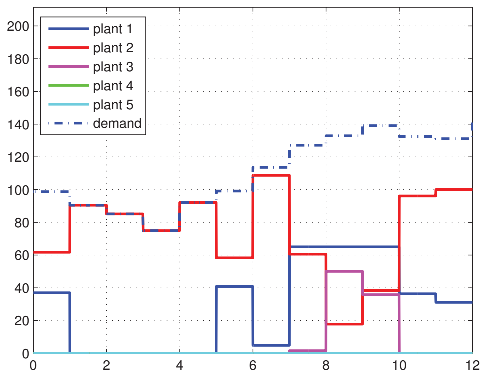

Fig. 5 Example of tertiary positive reserve allocation among the plants

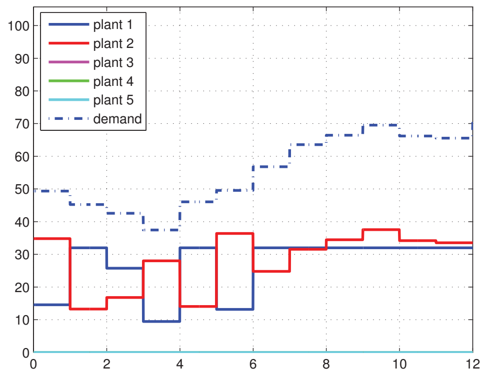

Fig. 6 Example of secondary negative reserve allocation among the plants

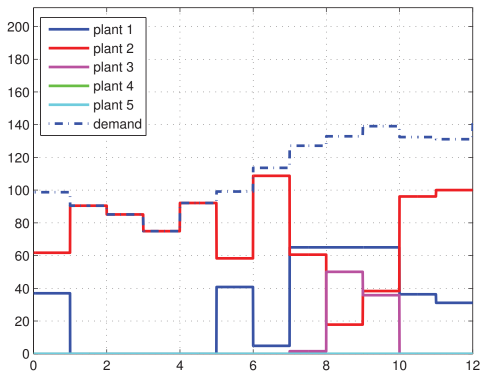

Fig. 7 Example of tertiary negative reserve allocation among the plants

TABLE I
NUMBER OF TOP-LEVEL-FEASIBLE PROFILES IN THE CASE OF VARIOUS DEMAND CURVES

1	2	3	4	5
81	61	41	49	58

No.1 is the nominal demand profile.

1) Successful solution rate: The solution rates of the various solvers are summarized in Table II.

TABLE II
SUCCES RATES OF THE SOLVERS

CLP	SCIP	OOQP	IPOPT	fmincon
1	1	1	1	0.5

We can see in Table II that all solvers except fmincon are able to solve all low-level-feasible profiles.

2) Optimality: Optimality of the various solvers are summarized in Table III.

TABLE III
OPTIMALITY OF THE SOLVERS

CLP	SCIP	OOQP	IPOPT	fmincon
0.1846	0	0.9739	0.0741	0.1431

The values in in Table III refer to the rate on which the cost of the returned solution exceeds the cost of the optimal solution (in percentage). As we can see, SCIP returns the optimal solution in the 100% of the cases. OOQP has the largest deviations from the optimum, while IPOPT performs also well.

3) Computational demands: Average and maximal solution times of the various solvers are summarized in Table IV in seconds.

TABLE IV
COMPUTATIONAL DEMAND OF THE SOLVERS

Solver	CLP	SCIP	OOQP	IPOPT	fmincon
avg.	0.0705	3.8799	0.5244	0.2470	2.4677
max.	0.0845	9.6256	1.0303	0.3039	3.5746

The computational time of fmincon was very high thus the number of iteration was limited to 100 in its case. This affected the results in Table III ansd possibly in Table II as well. We have to note that the computational demands of solvers may be similar in the case of infeasible problems as well. This means that if we evaluate a large number of possible MoO profiles, from which only a small subset is low-level-feasible, the solvers with high average execution times result in high total time.

The optimization tests were run on a standard desktop PC with an intel Core i3-2120 CPU 3.3 GHZ and 4 GB RAM.

IV. CONCLUSION

If one is dealing with a large number of QP problems, like the framework presented in [4], the choice of the appropriate solver is very important in order to get results of acceptable quality in a tractable time frame.

The results show that while SCIP is the most reliable solver regarding the optimality of the solutions, its computational demand is quite high. CLP represents a very fast alternative and the calculations show that it returns suboptimal results relatively close to the desired optimum, while its computational demands are in average only about 2 percent of SCIP. OOQP is also much faster than SCIP, but still almost ten times slower than CLP. The second fastest method is IPOPT, which represents a possible compromise between speed and optimality.

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