

# Predominance of Teaching Models Used by Math Teachers in Secondary Education

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**Abstract**—This research examines the teaching models used by secondary math teachers when teaching logarithmic, quadratic and exponential functions. For this, descriptive case studies have been carried out on 5 secondary teachers. These teachers have been chosen from 3 scientific-humanistic and technical schools, in Chile. Data have been obtained through non-participant class observation and the application of a questionnaire and a rubric to teachers. According to the results, the didactic model that prevails is the one that starts with an interactive strategy, moves to a more content-based structure, and ends with a reinforcement stage. Nonetheless, there is always influence from teachers, their methods, and the group of students.

**Keywords**—Teaching models, math teachers, functions, secondary education.

## I. INTRODUCTION

THE instruction in mathematics is performed mainly in the frame of professional work. However, teaching math is a complex and demanding work. Though being a certified teacher is a basic condition for successful learning, it has not been sufficient. Low results have prompted to question the traditional methods of mathematics.

Difficulties are solved successfully when matching mathematics to proper teaching models. In this way, it is possible to obtain a process which is performed in harmony, within the frame of professional work, pedagogy, science, methodology and psychology, among others [1]. As harmony in education is not easy to achieve, there are some negative elements, such as weaknesses and failures in the teaching of mathematics that significantly influence the quality of mathematics education.

Absence of articulate work has negative repercussions on modern teaching objectives of mathematics which emphasizes participation of individual and research work, developing skills for problem resolutions, development of creative thoughts and skills. Modern teaching methodologies of mathematics offer different options to solve the problem previously mentioned.

A teacher may find several possibilities within pedagogical and methodological framework to finally make students learn mathematics, effectively, which requires students to reckon what they know and identify what they need to learn. Such motivation may allow them to learn even more [2]-[4]. In experience, teachers have not used highly articulated teaching-learning methods, plus, they are based on theories that allow

them to teach, yet not always successfully. According to [5], theories in practice are not fully articulated among each other and they may obey to multiple needs from different conceptual framework. Furthermore, the teaching principles cannot be understood as static dogmas, but rather as procedures that underlie the learning theories plus features from the student-teacher connections.

In the opinion of the Chilean Ministry of Education (Mineduc), it is absolutely necessary to rethink the school experience in the classroom to be in tune with the world in which students are immersed [6]. This statement is based on the results obtained from both, the Mathematics PISA test [7], in which a large number of students could not reach the minimum expected in the level of competences, and the SIMCE results which could not increase more than 14 points between 2006 and 2016 [6]. What is more, other national Mineduc tests have proved throughout statistic data that Mathematical Functions have not yet been successfully understood nor learnt by students, reflecting a dramatic educational failure, in this area of study.

Researchers [8] have provided a study about the basic knowledge that math teachers manage in class. These researches show that most teachers base their teaching on their own way of understanding the concept of “function” ignoring their students’, preventing them from free discussion by pushing the teaching of the “teachers’ knowledge”. According to [9] the failure of traditional teaching paradigm is based on lack of students’ meaningful learning. Once finished obligatory education, people tend to forget mathematics and so, they may try to avoid every problem or situation which may require it, for instance, when trying to understand the way the human body removes drugs from the body; how computer viruses are spread; or the cost and utilities of common financial loans. Yet, all of these skills and knowledge needed to solve them are mentioned and required by the national curriculum for secondary education. In this context, the research objective is to characterize the teaching models applied by secondary school mathematics teachers in the learning process of logarithmic, quadratic and exponential functions.

## II. THEORETICAL FRAMEWORK

Most educational institutions deal with mathematics through the teaching of content, followed by some examples where content is applied, and finally students solve a set of exercises by following steps previously shown to them, until they concrete successfully every exercise [10]. Additionally, [11] states that most teachers do not know in which manner

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their own points of view are affecting the students' meaningful learning process [12].

The main characteristics of the models used by the teacher, plus teachers' perspectives and ideals, are all linked to the strategies applied in the teaching-learning process, in order to make it simpler and easier for students. The most popular teaching strategies may teach mathematics through "lectures" (teachers provide the lesson for students); the use of technological devices such as: calculators, websites, interactive webpages); mathematical modelling; mathematical proofs; games and other ludic activities; individual tutoring; directed studies, and problem solving, among others. [13]. Even so, teachers may tend to return to lectures, sooner or later.

For the understanding of this article, the definition of Teaching Model will be understood, hereinafter, as the systematic way of organizing and managing the process of teaching mathematics [14], and Teaching Strategies, as the steps followed by the teacher in order to promote learning. It is all about the orientations given by the teachers so their students can acquire different capacities for the interpretation of the information found in a certain task. The authors [15] state that the teaching strategies are tools used by teachers during the learning process.

Several models have been designed in order to support teachers in the process of mathematics teaching, to provide a common approach, and so to be able to articulate all teaching elements used in class. Reference [14] compares both, the "Euclidians", who would be all teachers who focus on theory, on technical studies, and the almost empiric epistemologists, also reckoned as the general model of mathematical knowledge, and the Modernists, who would be all constructivist and/or modernist teachers.

This study considers as theoretical framework, [16]'s proposals of Teaching Models in which (s)he states that every single unit must be prepared considering not only mathematical knowledge according to level and age, but also its relevance and applications to real life contexts. Likewise, the complexity of mathematics teaching requires high standard knowledge in state-of-the-art methodology and strategies from teachers for the new era.

The teaching models present schemes of a diversity of actions, techniques and means used by teachers during the teaching process, while teaching strategies are approaches and methods which make teachers manage the learning process, successfully. Teaching strategies favor learning [17].

It is a great challenge for teachers to apply strategies or teaching proposals for all activities that must be performed in every heterogeneous group of students [18]. On the other hand, most teachers in service think it is hard to apply a variety of strategies in a mathematics course due to the considerably short number of hours provided by the programme. References [19]-[21] reckon that students must learn and assess the use of the strategy in the learning process in order to make learning more meaningful for them. Therefore, the use of teaching strategies has several intentions within, so the teacher manages to achieve meaningful

learning, in every task [15]. Nonetheless, knowledge and teaching strategies are strongly connected yet they are in constant improvement [22]. Teachers need deep and wide knowledge in mathematics to be effective in teaching [23], [24], specially to create learning environment based on standards [25] that promote discourse in the classroom and promote conceptual comprehension of mathematics.

Multiple efforts have been made to define accurate knowledge in mathematics necessary for teaching, and several researchers [26], [27], [24] have highlighted a Specialized Content Knowledge, understood as mathematical knowledge necessary to perform common tasks when teaching mathematics [28], [27].

With regard to the in general function, the comprehension of concepts of mathematical function, according to [29], may allow them to develop mathematical thinking skills, to students in second year of secondary schools, such as: arguing about variations that occur in the graphic representation of functions, modelling diverse situations through functions, and demonstrating properties and theorems. Particularly, the understanding of quadratic functions lies in the modelling of a diverse number of situations.

The design of teaching methodologies has been oriented to model diverse situations and work contexts, for both exponential and logarithmic functions, such as in Chemistry, Physics, Biology, Economy, and Engineering, among others [30].

When accounting both, the research problem and the theoretical reference, we shall set the question: Which is the predominant mathematics teaching model in secondary school teachers?

### III. RESEARCH METHODOLOGY

The present is a descriptive study, framed in a qualitative research approach [31], based on case studies [32] that allow deepening on the subject investigated.

The subjects, objects of study, are 5 mathematics teachers from 3 scientific-humanistic and technical secondary schools, from "Región de Los Ríos" and "Región de Los Lagos", in Chile, through 6 pedagogical hours a week, for 2 months, during the first term of 2017.

Next, the data sources and procedures used are presented.

#### A. Questionnaire

A questionnaire is applied to all study cases, which must be answered in written. The objective of the questionnaire is to know about teachers' didactic planning through the actions, techniques and means declared in the planning, for which 4 dimensions are considered part of their teaching performance:

1. Class preparation: It is intended to understand the planning and preparation mechanism of class, through the following questions: Do you usually prepare the class before doing it? How? Which aspects stand out in the planning used? Do you consider it to be useful for learning objectives achievement?
2. Class performance: It is intended to know the way in which lessons are developed through three questions: do

you stimulate students at the beginning of the lesson? Which ones? Which tools or items do you use to make students apply what they have learned? Do you use to provide feedback at the end of the lesson? How do you do it?

3. Application of knowledge: It is intended to know the evaluation method and instruments used by the teacher, as well as the qualifications obtained. The following questions are asked for this purpose: do you use a specification table when developing your assessment instruments? How often do you apply summative assessment in each content? Which assessment contributes is significant to students' learning? Why?
4. Pedagogical leadership: it is intended to know the relation between teacher and environment through these questions: Which are the main boundaries set in your lesson? What is your problem-solving method used when there is an issue with a student? Do you think you have group management skills? Why?

### B. Rubric

The second instrument applied in this research is the rubric. The instrument includes seven items to be assessed, such as content knowledge, group management, team work, class organization, among others. Assessment criteria are expressed in a qualitative scale set from achieved, moderately achieved, and not achieved, based on quality standard. The items to be assessed through the rubric are the following: content knowledge; students' interest encouragement; comprehension, analysis and reasoning; Group management; Team work; Time provided for learning; Class structure.

### C. Non-Participant Observation

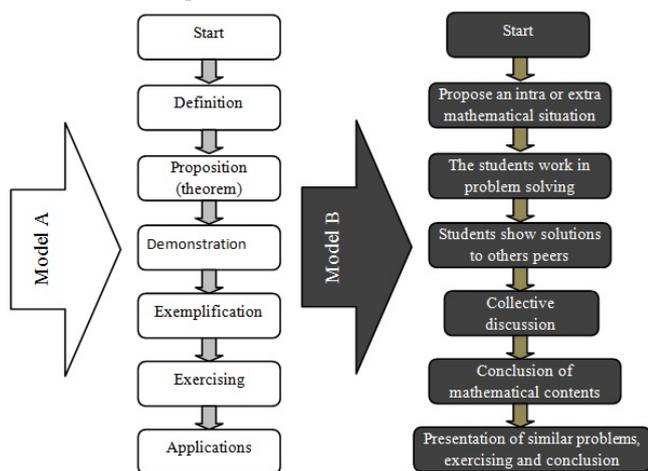


Fig.1 Didactic Models

It is used a non-participant observation, as method for systematic observation of people, behavior and interactions in the classroom. In class observations performed in the classroom, the teaching models are identified in five case studies for the teaching of logarithmic, quadratic and exponential functions.

The strategies studied correspond to didactic models A and

B, proposed by [16], and it is presented in Fig. 1.

## IV. ANALYSIS OF INFORMATION AND RESULTS

The application of the questionnaire, the rubric and the non-participant observation to case studies has made the comprehension of the didactic process easier.

### A. Questionnaire Answers

The results of the applied questionnaire before mathematics class observation are articulated with the 4 dimensions based on performance, such as: class preparation, teaching performance, knowledge application and pedagogical leadership.

Figs. 2-5 show an example of answer in each dimension assessed. Fig. 2 shows dimension.

<p style="text-align: center;"><u>Preparación de la clase</u></p> <p>Se, primero viendo la planificación, segundo analizando el orden y la secuencia que van a tener los contenidos a trabajar, tercero generar actividades pertinentes a la necesidades de los cursos y los estudiantes y por ultimo como general evaluaciones (formativas y Sumativas) acorde a lo que se esta trabajando.</p> <p>Los aspectos que destaco van el inicio, desarrollo y cierre. lógicamente conlleva un orden progresivo del desarrollo de una clase estableciendo los tiempos y la profundidad de los aprendizajes trabajados, con el fin de lograr el objetivo de aprendizaje previamente establecido.</p>	<p style="text-align: center;">Preparation of the class</p> <p>I go first to see the planning, seeing and analyzing the order and sequence of contents to be worked in class. I elaborate activities which are pertinent to the class and students' needs. Finally, I elaborate formative and summative assessments, based on what is being worked, in class.</p> <p>The aspects which are highlighted go at the beginning, development and closing. Logically, it is important to follow a progressive order, establishing length and depth in each learning task, to achieve the learning goal previously set.</p>
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Fig. 2 A teacher's answer to the question about Class preparation dimension

Before class development, the teacher declares to analyze the organization and sequence of contents, providing activities which meet the needs of the class to eventually assess accordingly. Beginning, development and closing of the class are highlighted. Fig. 3 corresponds to teaching performance dimension.

The teacher appreciates the relevance of the attitude as a motivation asset for students' learning. The teacher reckons the use of handouts to solve problems and practice, elaboration of concrete work material to articulate content and environment, to finish with the feedback through schemes and open or closed questions. Fig. 4 corresponds to use of knowledge dimension.

Se utilizan guías de resolución de ejercicios y/o problemas, construcción de material concreto, relación de los contenidos con el entorno.

Al finalizar todas las clases, se realiza la retroalimentación correspondiente. Esto puede ser por medio de esquemas (generalmente uso esquemas-resumen), preguntas abiertas o dirigidas.

Handouts are used to solve exercises and/or problems, elaboration of concrete material, relationship between content and environment.

At the end of every lesson, the feedback is provided. This can occur through schemes (I often use scheme-summary), open or directed questions.

Fig. 3 Teacher's answer to the question about Teaching performance dimension

Aplicación del conocimiento Cada evaluación tiene una tabla con objetivo de aprendizaje y Criterios de evaluación.

Las habilidades son aplicar, calcular, analizar

Las evaluaciones se realizan cada 2 o tres semanas

La evaluación que más genera aprendizaje es la formativa, ya que promueve al alumno diversos retroalimentaciones, por ser evaluada en las evaluaciones sumativas

Application of knowledge. Each assessment has a table with the learning objective and assessment criteria.

Skills are: apply, calculate, analyze.

Assessment are applied every two or three weeks

The assessment which is more significant for learning is the formative, as tests students in a more varied set of contexts, which will be assessed again in the summative test

Fig. 4 A teacher's answer to the Application of knowledge dimension

The teacher uses specification tables, and the skills assessed are to apply, calculate and analyze. The assessments are applied every two or three weeks, and it is believed that formative assessment is the most effective due to the possibility of testing students in multiple contexts. Fig. 5 corresponds to pedagogical leadership dimension.

With respect to pedagogical leadership, the teacher states that the main norm set in the classroom is to respect other peers, solving problems through communication. The teacher claims to have a strong relationship with the students, highlighting group management, endorsed by standardized test results.

Liderazgo pedagógico.

Mi relación con los estudiantes es cercana debido al vínculo que se genera en la relación entre profesor y estudiante. Desde el compromiso que toman los estudiantes a la asistencia y la entrega es debido a la relación creada.

Pedagogical leadership

My relationship with the students is tight due to a bond cultivated through the student-teacher relationship, where commitment taken by the students and the class subject is due to the bond.

Fig. 5 A teacher's answer about Pedagogical leadership dimension

### B. Study Based on Rubric

Fig. 6 provides the results of the statistical analysis of data obtained through the application of the rubric for teacher's assessment in the classroom. The results referred to content knowledge, as well as comprehension, analysis and reasoning, show a 100% achievement in all case studies, while other items show lower levels. Team work has not been achieved in 80% of the teachers.

In an analysis of each item in the rubric, plus what concerns to content knowledge, it is possible to state that all teachers show full understanding of logarithmic, quadratic and exponential functions. With respect to stimulation to increase the level of interest in students, 60% of teachers are able to make students understand purposes, as well as meaning of what is learnt. Most activities varied in format and form, as well as aligned to these purposes.

100% of case studies make students answer questions about the topic in study and promote students to inquire of solving the problem. Concerning to group management, 60% of teachers manage to make most students participate actively, in class.

80% of teachers fail to provide team work to students, and prefer individual work. Only 20% divides the class in groups and supervises full participation in class.

In respect to maximization of time for learning, 80% of teachers manage to make students feel engaged in the learning tasks. Finally, it is possible to state that 60% of teachers fully perform the structure of the class, which means activation, development and concluding. The remaining 40% perform only the first two stages: activation and development.

### C. Non-Participant Observation

Through non-participant observation, it is possible to identify the didactic strategy used by each case study teacher, according to Teaching-strategy models A and B, proposed by [16].

Fig. 7 shows the results of statistical analysis of data obtained from didactic models, observed in mathematics lessons when teaching functions.

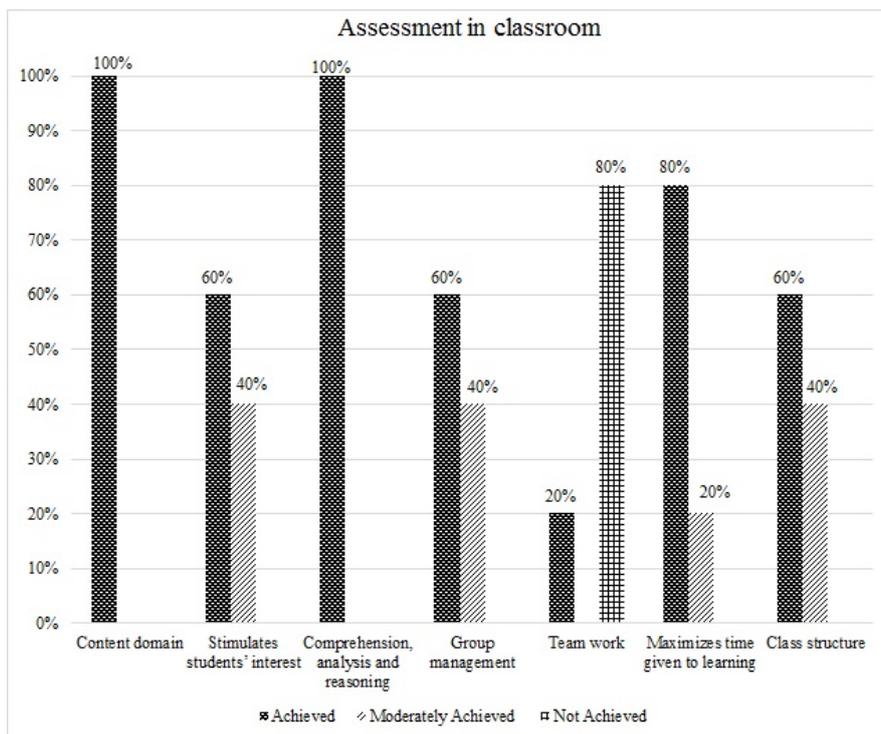


Fig. 6 Percentage of achievement levels based on class assessment

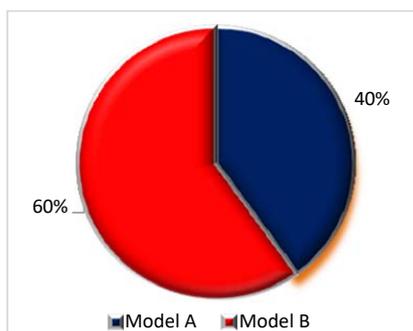


Fig. 7 Didactic models observed when teaching functions

Results show that 60% of teachers use model B as a strategy for teaching functions, which considers the beginning of the class, the proposal of an intra or extra mathematics, the students' work for problem solving, the presentation of solution to peers, a collective discussion, a conclusion of mathematical contents, the presentation of similar problems, exercising and conclusion.

The remaining 40% apply model A as a methodological strategy. The teacher starts the lesson, delivers some definitions, presents theorems about the topic in study, performs a demonstration of the exercise, provides examples, and finally applies the exercise.

## V. CONCLUSIONS AND DISCUSSION

The main objective of this research is to characterize the didactic models used by the secondary teachers [16], when teaching logarithmic, quadratic and exponential functions. Specifically, this study focused on 5 mathematics teachers and

their teaching schemes when teaching functions to get a grasp of what may occur in their lessons.

Results were obtained through three different sources. Firstly, based on answers from a questionnaire, all teachers prepare their lessons considering both, the type of students they have, and their school's standards. Overall, they declare to maintain good relationships with their students.

They present differences in terms of assessment. As such, three out of five cases consider summative assessment as the most significant for learning.

They concur in the need of considering the learning objectives of the class. This fact supports [33]'s claim about course design and the relevance of determining the objectives, the articulation of them with the activities in class, and the search of learning evidence in the light of the course. As a matter of fact, [34] claims that teaching is a designing activity, so instructors make decisions on the sequence of content and selection of strategies and materials to be used, in the light of the learning objectives. On the other hand, the reflexion and explanation about the use of learning strategies affect significantly the application of the strategy itself. Likewise, [35] reckons that the teacher's methodology plays a meaningful role when choosing the strategies to be used when developing any activity.

Secondly, according to the rubric used to assess teachers in the classroom, content knowledge is highlighted, as well as comprehension, analysis and reasoning, in all case study teachers, unlike teamwork, as only 80% of teachers encouraged it, in their lessons.

In the third place, it was possible to conclude based on all the results and analysis, that the predominant didactic model

in the teaching of quadratic, exponential and logarithmic functions, is model B, proposed by [16], which responds to a more interactive strategy which starts from an intra or extra mathematical situation, moving to development of content to finish in the conclusion of it. Unlike other studies conducted in Nicaragua, Venezuela, Bolivia and Germany, Model A prevails over Model B. Based on studies such as the TIMSS (Third International Science and Mathematics Study), PISA (Programme for International Student Assessment), PIRLS (Progress in International Reading Literacy Study) and LLECE (in Spanish "Laboratorio Latinoamericano para la Evaluación de la Calidad de la Educación) in the last ten years [36], [16], are also models the most common in mathematics lessons.

Results show that case study teachers tend to the use of Model B as predominant over Model A, nevertheless, they present some influence from their own profile as teachers, because of their level of competence and the type of students they work with. One of these modifications in the application of models is present in the sequence and combination of themselves. One group of teachers use only Model A or B, while others combine a bit of both, as strategies are in constant evolution, adapting them to content and the standard level to be worked, which concur with [37] as it is stated that teachers do not have a pure conception of work, and show some features which can be matched to one trend, over others [12].

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