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Geopotential Models Evaluation in Algeria Using Stochastic Method, GPS/Leveling and Topographic Data

M. A. Meslem

Abstract—For precise geoid determination, we use a reference field to subtract long and medium wavelength of the gravity field from observations data when we use the remove-compute-restore technique. Therefore, a comparison study between considered models should be made in order to select the optimal reference gravity field to be used. In this context, two recent global geopotential models have been selected to perform this comparison study over Northern Algeria. The Earth Gravitational Model (EGM2008) and the Global Gravity Model (GECO) conceived with a combination of the first model with anomalous potential derived from a GOCE satellite-only global model. Free air gravity anomalies in the area under study have been used to compute residual data using both gravity field models and a Digital Terrain Model (DTM) to subtract the residual terrain effect from the gravity observations. Residual data were used to generate local empirical covariance functions and their fitting to the closed form in order to compare their statistical behaviors according to both cases. Finally, height anomalies were computed from both geopotential models and compared to a set of GPS levelled points on benchmarks using least squares adjustment. The result described in details in this paper regarding these two models has pointed out a slight advantage of GECO global model globally through error degree variances comparison and ground-truth evaluation.

***Keywords*—**Quasigeoid, gravity anomalies, covariance, GGM.

I. Introduction

FOR precise local and regional geoid determination, it is crucial to use a reference field to subtract long and medium wavelength of the gravity field from observations data when we use the common remove-compute-restore technique, therefore a comparison study between considered models should be made in order to validate and select the optimal reference gravity field to be used over the region under study. In this context, the Earth Gravitational Model (EGM2008) and the Global Gravity Model by Locally Combining GOCE Data and EGM2008 (GECO), complete respectively until 2160 and 2190 degree and order, have been selected to perform this comparison study. Free air gravity anomalies in the area under study, (33.0 deg. 37.0 deg. in latitude and -1.5 deg. 8.5 deg. in longitude), have been used to compute residual data using both gravity field models and a DTM to subtract the residual terrain effect from the gravity data. The resulted residual data were used to generate covariance functions in order to compare their statistical behaviors according to both cases. Finally, height anomalies were computed from both geopotential models and compared to a set of GPS levelled points on benchmarks using linear regression through least squares adjustment. The result described in details in this paper regarding these two models has pointed out a slight advantage of GECO global model through the signal spectra and covariance functions evaluation and comparison to GPS/levelling height anomalies. This result has shown as well that the additional data from GOCE Satellite have improved the agreement between GECO model and the local gravity field, at least over the region under study. It is recommended to use this model in refining the geoid over Algeria.

M. A. Meslem was with Polytechnic University of Turin, Italy. He is now with the Department of Geodesy, National Institute of Mapping and Remote Sensing, Algiers, Algeria (e-mail: mohamedaissa.meslem@studenti.polito.it, meslem_ma@hotmail.com).

II. The Data

A. Earth Gravitational Models

The Earth gravitational models (EGM2008) [1] and the Global Gravity Model (GECO) [2] are used in this study. The first one is complete to degree and order 2160, the second conceived with a combination of the anomalous potential derived from a GOCE satellite-only global model. These two models are described in terms of gravity anomaly degree variances and error degree variances, the related spectra of the signal (a) and error (b) are shown in Fig. 1. The degree-variances are positive numbers and are related to the spherical power spectrum of the earth gravity field. The gravity anomaly degree variances are evaluated by the following expression [3]:

$$\sigma_n^2(\Delta g) = \gamma^2(n-1)^2 \sum_{m=0}^{n}\left(\bar{C}_{nm}^2 + \bar{S}_{nm}^2\right) \quad (1)$$

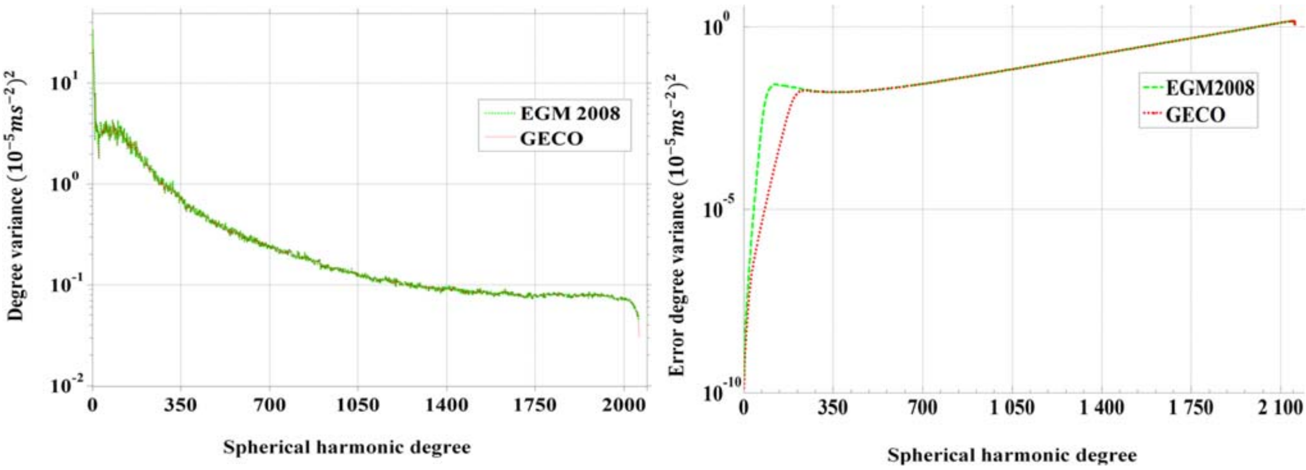
where γ is the mean value of normal gravity, $\bar{C}_{nm}^2$ and $\bar{S}_{nm}^2$ are the fully normalized spherical harmonic potential coefficient sets. The error (noise) anomaly degree variance of the erroneous potential coefficients with standard errors $e_{\bar{C}_{nm}}^2$ and $e_{\bar{S}_{nm}}^2$ derived from the GGMs is expressed as follows:

$$\delta\sigma_n^2(\Delta g) = \gamma^2(n-1)^2 \sum_{m=0}^{n}\left(e_{\bar{C}_{nm}}^2 + e_{\bar{S}_{nm}}^2\right) \quad (2)$$

In Fig. 1, it is shown that the signal degree variances of EGM2008 and GECO are very close. On the other hand, the differences between the error degree variances are significant and lower for GECO model, until the degree 200 corresponding to $10^{-2} \times (10^{-5}ms^{-2})^2$, the reason is probably due to the fact that only GECO from both GGMs is conceived using additional GOCE data, from this covariance value, both curves increase identically until to reach the $10^0 \times (10^{-5}ms^{-2})^2$ level.

B. Gravity Data

A set of 2359 selected free air gravity anomalies, provided by the International Gravimetric Bureau (BGI), referenced to the International Gravity Standardization Net (IGSN71), have been used with an estimated accuracy varying between 2 and 5 $\times$ $(10^{-5}ms^{-2})^2$. Furthermore, all these gravity anomalies have been transformed previously to the Geodetic Reference System (GRS80) and an atmospheric correction has been applied to the gravity anomalies, the indirect effect due to this correction has not been taken into account.



(a) (b)

Fig. 1 (a) degree variances of EGM 2008 (green), and GECO (red). (b) error degree variances of EGM 2008 (green) and GECO (red)

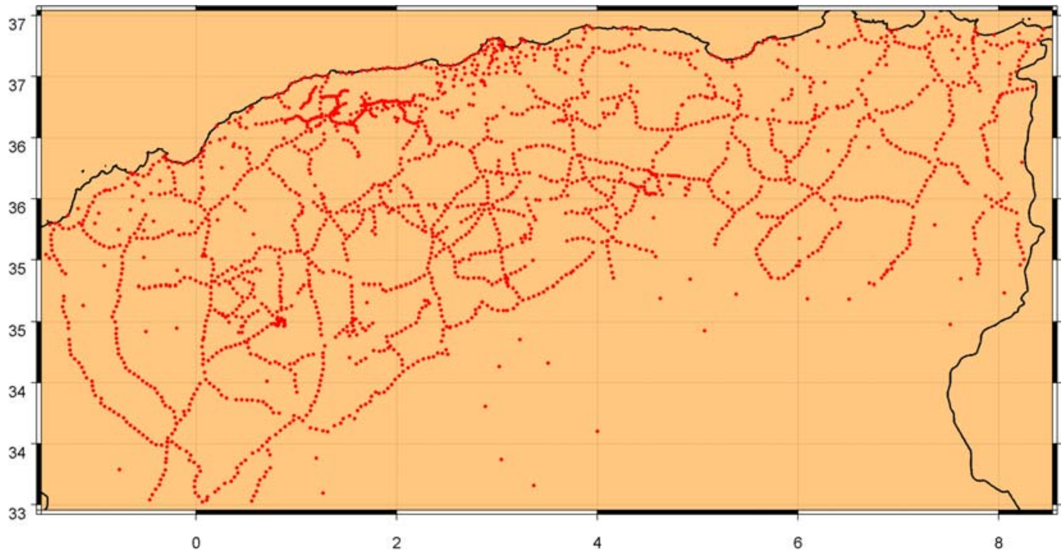


Fig. 2 Zone under experiment: gravity data distribution

C. Topographic Data

Relief morphology of Algeria is quite heterogeneous and can be roughly classified to mountainous, hilly and flat surface regions. For this study and in order to take into account as much as possible relief characteristic on the Algerian territory, we used various elevation data sources to generate a 30″ arc DTM covering all the area under study. The main data input were the scanned raster contour lines from 1/200000 scale maps, heights from fundamental and trigonometrical geodetic points and hydrographic features (lines of the streams and polygons of the lakes) – without height attributes. The fundamental geodetic points have a planimetric and altimetric accuracy down to 10 cm, and trigonometrical geodetic points with accuracy down to 20 cm. The contour lines planimetric accuracy is from 5 to 10 m and height accuracy about 10 m. For the quality control of the final DTM, a comparison has been done to orthometric heights from the gravity data used. The results of the comparison are summarized in Table I.

TABLE I
STATISTICS OF THE DIFFERENCES BETWEEN THE DTM AND ORTHOMETRIC HEIGHTS FROM GRAVITY DATA

DTM - Heights from BGI data	Values (m)
Min.	-26.000
Max.	36.000
E	0.086
1st Quartile	-1.200
3rd Quartile	1.000
Q3-Q1	2.200
Σ	4.901

D.GNSS/Levelling Data

A set of 23 GNSS levelled points, has been used to assess both Earth gravitational models, with 1 cm accuracy and good spatial distribution over the area under study. Furthermore, the 23 geoidal heights N from the data points have been converted to height anomalies ζ for a more rigorous assessment.

ζ is related to N by the following relation [3]:

$$\zeta = N - (\Delta g_{Bouguer} \cdot {}^{H}/_{\gamma}) \quad (3)$$

where H is the height of the computation point, $\Delta g_{Bouguer}$ is the Bouguer anomaly known on every point. The result of the corrections is shown in Table II.

TABLE II
STATISTICS OF THE TRANSFORMATION TO HEIGHT ANOMALIES OF THE 24 GPS LEVELLED POINTS

24 points	mean. (m)	std. (m)
Corrections	-.031	.040
ζ	39.272	7.762
N	39.241	7.749

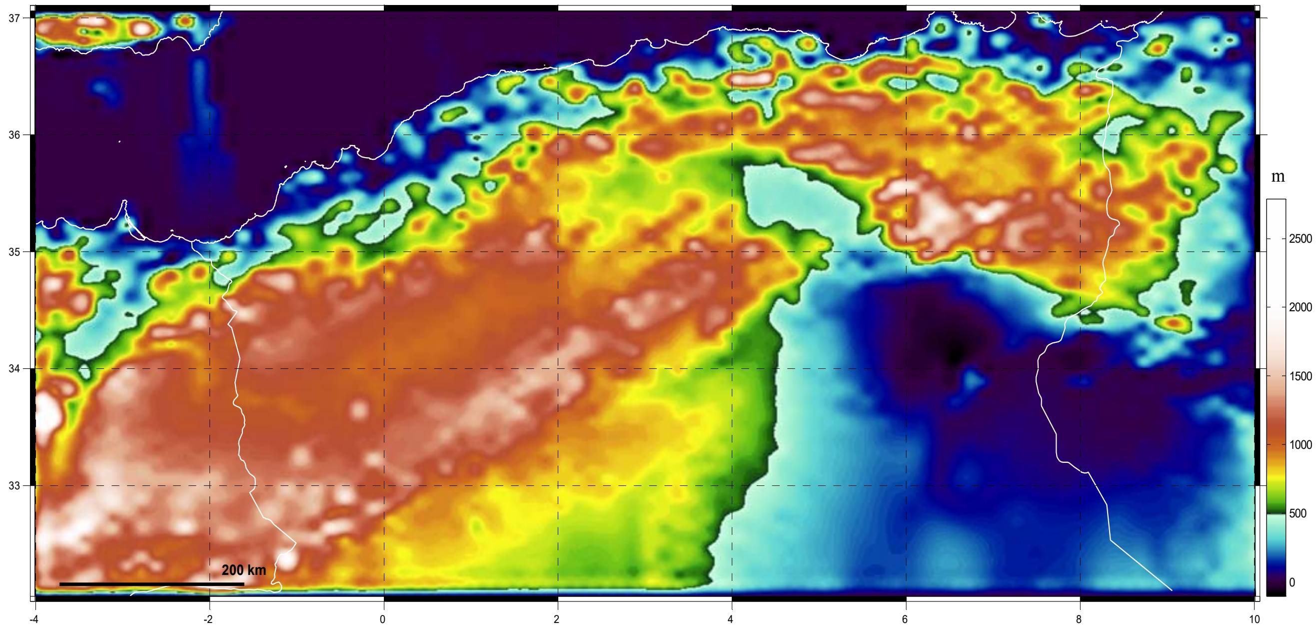


Fig. 3 DTM of the region under experiment (Northern Algeria)

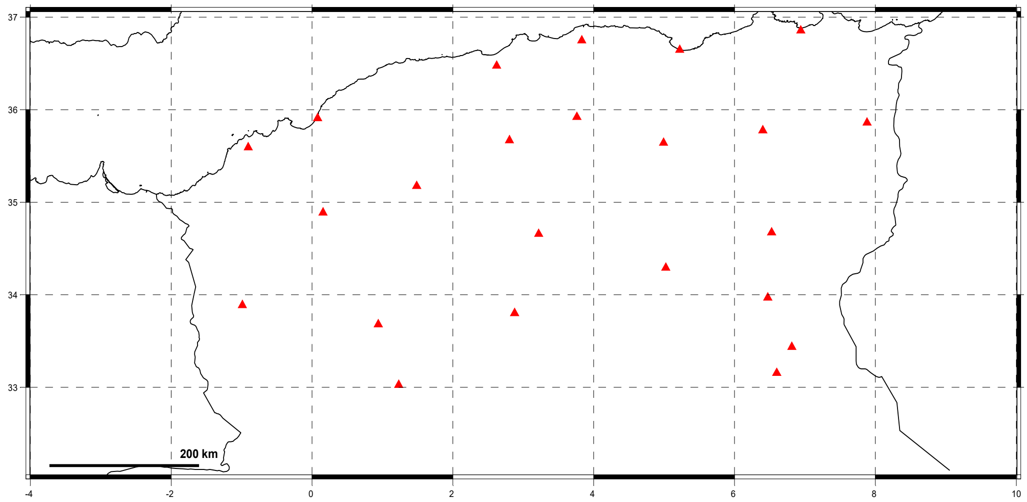


Fig. 4 Distribution of GNSS/levelled points over the region under experiment

III. Computations and Results

A. Remove the Effect of a Spherical Harmonic Expansion and the Topography (R T M) Effect

This step consists in subtracting from the observed gravity values the information of low and high frequency of the geopotential model and of the topography to get residual gravity anomalies from the following relation:

$$\Delta g_{res} = \Delta g_{obs} - \Delta g_{GM} - \Delta g_{RTM} \tag{4}$$

Δg_{GM} is computed throw spherical harmonic series of the geopotential model, Δg_{RTM} is the effect of RTM masses, including the attraction of the RTM masses on the computation point and the reduction of this point from the geoid to co-geoid [4], this quantity is calculated in planar approximation using the DTM. The R T M reduction approximated by a Bouguer reduction to the reference level is expressed by the relation:

$$\Delta g_{rtm} \approx 2\pi G\rho(h - h_{ref}) - c \tag{5}$$

where c is the classical terrain correction in planar approximation. The residual anomalies obtained have homogeneous statistical proprieties with a standard deviation considerably reduced, allowing to apply the prediction by the Least Squares Collocation method. In this step of the treatment, both Earth gravitational models EGM2008 and GECO, complete to their respective degree and order, have been used as reference model in order to substrate the information of long wavelengths from the free air gravity anomalies. The residual terrain effect has been computed by the use of three DTM, the detailed one of 30″ arc, the second for the external zone of 30′ arc and the third one as reference of 5′arc of resolution.

B. Covariance Function

As mentioned previously and for sake of comparison between both geopotential models, the residual gravity anomalies have been in first used for computing the empirical covariance.

Subsequently, the two empirical covariance functions corresponding to both models are adjusted to their closed expressions [5]. The covariance analytical model between two independent variables of points P_i and Q_j, with radial distances equal to r_i and r_j respectively and spherical distance psi (ψ_{ij}) is given by the following expression:

$$K(\psi_{ij}) = a\sum_{l=2}^{l_{\max}} \sigma_l^{\,e} \left(\frac{R}{r_i r_j}\right)^{l+1} P_l(Cos\,\psi_{ij}) + \sum_{k=l_{\max}+1}^{+\infty} \sigma_k \left(\frac{R_B}{r_i r_j}\right)^{k+1} P_k(Cos\,\psi_{ij}) \tag{6}$$

where P_l are Legendre polynomials, R is the mean radius of the Earth, R_B is the Bjerhammar sphere radius (Convergence radius), ($R_B < R$), a is the scale factor, σ_l^e are the errors variances of the geopotential coefficients and σ_k are the degree-variances, l_{max} is the summation limit and reflects the degree to which the spherical harmonic coefficient information is considered reliable for the region under study [6]. For our case, the maximum summation has been fixed to 700, larger number did not give reliable information. The model of degree-variances generally used [7] is defined as follows:

$$\sigma_k = \frac{A}{(k-1)(k-2)(k+24)} \quad k > 2 \tag{7}$$

where A is a constant related to the variance of the disturbing potential T.

The expression of the local covariance model requires the evaluation of the three quantities, R_B, a and A. The problem consists therefore of adjusting the values of these three parameters in order to come closer as much as possible to the empirical values.

For that, one uses a nonlinear iterative adjustment for R_B, and linear one for the two other quantities a and A.

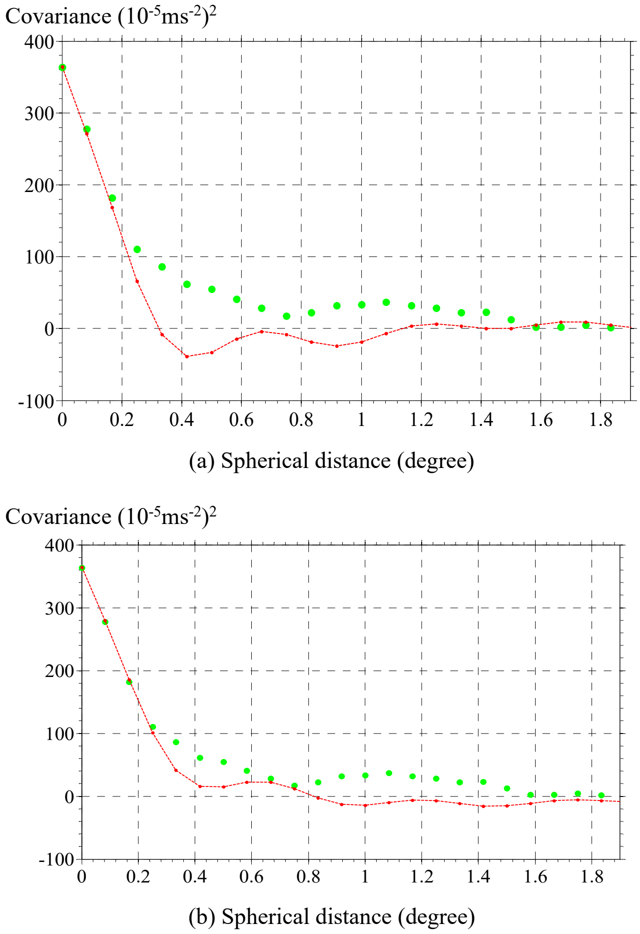


Fig. 5 (a) Covariance functions of the residual signals from GECO model contribution. (b) covariance functions of the residual signals from EGM 2008 contribution. Empirical functions (green points) and their closed forms (red dashed line)

According to Fig. 5, it seems that the empirical covariance functions of residual gravity anomalies for both models follow the same behavior; however their corresponding analytical covariances are slightly different. The GECO closed form seems to be more stable and decrease monotonically. This is due to the difference in harmonic coefficients structure regarding both geopotential models under study, particularly after including GOCE data in GECO model.

C. Constant Bias Estimation between Levelling Datum and GGM Height Anomalies

Most studies for the Earth geopotential model's evaluation are based on a comparison of the height anomalies to the GPS/Levelling. The long wavelength part of ζ_{EGM} is deduced from the fully normalized spherical harmonic coefficients of the geopotential model considered and the Bruns' formula [8]. We use generally the Least Squares adjustment that uses the following basis model:

$$h_i - H_i - \zeta_i = \boldsymbol{a}_i^T \boldsymbol{x} + v_i \quad (8)$$

where h_i is the ellipsoidal height, H_i the orthometric height, x is the n vector of unknown parameters, a_i is a n×1 vector of known coefficients, and v_i represent the residue.

The parametric part is supposed to describe all the possible datum inconsistencies and other systematic effects in the data sets [9]. The linear part of the discrepancies is modelled through the following observation equation:

$$\boldsymbol{a}_i^T \boldsymbol{x} = x_0 + x_1(\varphi - \varphi_0) + x_2(\lambda - \lambda_0)\cos\varphi \quad (9)$$

where φ and λ are the geographical co-ordinates of the control points, φ_0 and λ_0 are the co-ordinates of arbitrary reference points, x_0, x_1, x_2 are the three parameters to be estimated by least squares adjustment.

The statistics of the adjusted residuals $\{\hat{\boldsymbol{v}}_i\}$ with the three parameters, of the geoid adjustment to GPS/Levelling regarding both geopotential models are presented in Table III.

TABLE III
STATISTICS OF THE DIFFERENCES IN HEIGHT ANOMALIES

Param.\ ζ diff.	$\zeta_{\mathrm{EGM08}} - \zeta_{\mathrm{GNSS-Lev}}$	$\zeta_{\mathrm{GECO}} - \zeta_{\mathrm{GNSS-Lev}}$
x_0 (m)	0.138 (bias)	0.109 (bias)
x_1 (10^{-6})	0.403	0.244
x_2 (10^{-6})	-0.506	-0.197
mean (m)	0.000	0.000
min (m)	-0.634	-0.651
max (m)	0. 604	0.658
Std (m)	0.247	0.256

IV. CONCLUSION

Despite the fact that GECO geopotential model has been deduced from the EGM2008 but the additional GOCE data included in its conception have improved the subtraction of the low and medium frequency information from the gravity anomalies. According to the results presented in Table III, particularly biases and slopes in East and North directions, an interesting agreement has been noticed between the local leveling datum and GECO model in the area under study. We recommend to use this geopotential model for gravimetric geoid determination over Algeria.

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