

# Modelling Conditional Volatility of Saving Rate by a Time-Varying Parameter Model

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**Abstract**—The present paper used time-varying parameters which are based on the score function of a probability density at time  $t$  to model volatility of saving rate. We used a scaled likelihood function to update the parameters of the model overtime. Our results revealed high diligence of time-varying since the location parameter is greater than zero. Furthermore, we discovered a leptokurtic condition on saving rate's distribution. Kapetanios, Shin-Shell Nonlinear Augmented Dickey-Fuller (KSS-NADF) test showed that the saving rate has a nonlinear unit root; therefore, it can be modeled by a generalised autoregressive score (GAS) model. Additionally, value at risk (VaR) and conditional tail expectation (CTE) indicate that 99% of the time people in Lesotho are saving more than spending. This puts the economy in high risk of not expanding. Therefore, the monetary policy committee (MPC) of Lesotho should revise their monetary policies towards this high saving rates risk.

**Keywords**—Generalized autoregressive score, time-varying, saving rate, Lesotho.

## I. INTRODUCTION

THE modern turmoil in the Asia-Pacific region has generated a universal attention in the volatility transmission between foreign exchange markets. The crisis is a good example of the financial market volatility trend which led to the development of the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. Econometric models of the first moment of a series have a complete family in finance. However, with the second moment of the series, [1] had shown that this model acquired colossal attention. In recent research, models that were developed by [2] have been largely utilized. Nonetheless, the number of reported studies concerning a multivariate GARCH (MGARCH) models stays relatively small with respect to the number of univariate studies [3].

Ever since the introduction of Autoregressive Conditional Heteroscedasticity (ARCH) models, financial time series modelling have gained much attention. On the other hand, [4] utilized GARCH and VAR models to build up a frail association between macroeconomic and stock exchange unpredictability. Their results indicated that the instability of stock exchange controls the inflation volatility. There are different methods used to model volatile series and models studied by [3] can also be used. This includes among others an autoregressive (AR) and seasonally adjusted (SA) forecasting

models. With an emphasis of [4], driving impulsiveness on such models has some drawbacks which are corrected by GARCH models.

References [5], [6] have shown the importance of modelling and forecasting stock returns and exchange rate volatility. Nonetheless, the aim of this paper is to model the volatility of saving rate using GAS model of [7]. The GAS model has an ability to predict time-varying in a time series. According to [8], time-varying is grounded on probability density of a score function at time  $t$ . For this reason, this model is attested to be robust in modelling and predicting fat-tail. Besides, dynamic correlations of this model are driven by a conditional distribution. [9]. For more readings, see for example [10]-[12]. Additionally, [7] declared that is difficult to estimate a stochastic model that is inaccurate to yield the shape of the conditional distribution of data.

The rest of the paper is as follows. Section II presents methods and procedures. Section III presents empirical analysis and lastly, Section IV presents the conclusion and recommendations.

## II. METHODS AND PROCEDURES

In this paper, the behaviour of time-varying parameters is studied by using a GAS model. Tables and graphs are used to presents results. Methods for preparing data for empirical analysis are also discussed in this section. Results obtained here are meant to give guidance about the nature of data and type of models to be estimated.

### A. Nonlinear Unit Root Test

There are tests to be executed prior model estimation in time series settings. Nevertheless, KSS-NADF of [13] is utilised to test the presence of nonlinear unit root. Scholars such as [14], [19] have shown that it is grounded on the following nonlinear model:

$$Y_t = \beta Y_{t-1} + \gamma Y_{t-1} [1 - e^{-\theta Y_{t-k}^3}] + \varepsilon_t \quad (1)$$

which yields:

$$\Delta Y_t = \varphi Y_{t-1} + \gamma Y_{t-1} [1 - e^{-\theta Y_{t-k}^3}] + \varepsilon_t \quad (2)$$

The parameter to be estimated are denoted by  $\varphi = \beta - 1, \gamma, \theta$  whereas  $\varepsilon_t$  is the error term. When  $\varphi = 0$  and  $k = 1$  are met, model (2) is augmented as:

$$\Delta Y_t = \gamma Y_{t-1} [1 - e^{-\theta Y_{t-k}^3}] + \varepsilon_t \quad (3)$$

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Hence a linear stationary hypothesis is:

$$\begin{aligned} H_0: \rho &= 0 \\ H_1: \rho &> 0 \end{aligned}$$

Moreover, [13] contended that it is unbearable to directly test a null hypothesis because there is an unknown reversion speed, therefore, model (3) ought to be estimated as a nonlinear specification. In order to achieve this, we apply the first difference to  $Y_t$  and thereafter apply a Taylor series that lead us to estimate the following model:

$$\Delta y_t = \delta y_{t-1}^3 + \sum_{k=1}^q \rho \Delta y_{t-k} + \varepsilon_t \quad (4)$$

where,  $\delta$  is coefficient for testing the presence of the unit root and  $q$  is a maximum lag length which is specified by lag length selection criterions such BIC, AIC HQ etc. Finally, KSS-NADF unit root test is given by:

$$\tau_{NL} = \hat{\delta} / se(\hat{\delta}) \quad (5)$$

The hypothesis is:

$$\begin{aligned} H_0: \delta &= 0 \\ H_1: \delta &< 0 \end{aligned}$$

A decision rule is; reject a null hypothesis if the value of  $\tau_{NL}$  exceeds the observed critical value.

TABLE I  
 CRITICAL VALUES OF KSS-NADF [13]

Significance Level	Raw	No mean	No Trended
1%	-2.82	-3.48	-3.93
5%	-2.22	-2.93	-3.4
10%	-1.92	-2.66	-3.13

### B. GAS Framework for Modelling Conditional Volatility

While modelling time-varying parameters, the GAS framework is the only outstanding structure for both univariate and multivariate time series. As a matter of fact, we use an unrestricted parameter space as in [15], to evaluate the GAS model. In order to estimate a GAS model, we let  $X_t \in \mathbb{R}^N$  be an N-dimensional random vector at time t. According [4], a GAS model with subsequent conditional distribution is given by:

$$X_t | X_{1:t-1} \sim p(X_t; \vartheta_t) \quad (6)$$

where  $X_{1:t-1} \equiv (X_1^T, \dots, X_{t-1}^T)^T$  clenches past values of  $X_t$  up to time t-1 and  $\vartheta_t \in \Theta \subseteq \mathbb{R}^J$  is a time-varying parameters' vector which fully symbolizes  $p(\cdot)$ . This solitary depends on  $X_{1:t-1}$  and  $\zeta$ . Nevertheless,  $\vartheta_t \in \Theta \subseteq \mathbb{R}^J$  symbolises a complete vector of time-varying parameters which are noted as the main feature of a GAS model which constrained by a score of a conditional distribution defined in (6). Nevertheless, [1] has incorporated the following AR component:

$$\vartheta_{t+1} \equiv \alpha + \phi \vartheta_t + \varphi \vartheta_t \quad (7)$$

where,  $\alpha, \phi$  and  $\varphi$  are the coefficient matrices of proper dimensions collected in  $\zeta$ . A proportional vector symbolized by  $q_t$  is computed b:

$$q_t \equiv v_t(\vartheta_t) \nabla_t(X_t, \vartheta_t) \quad (8)$$

Here,  $v_t = J * J$  is a positive definite scaling matrix known at time t. A score of the model which is evaluated at  $\vartheta_t$  is given by  $\nabla_t(X_t, \vartheta_t) \equiv \frac{\partial \ln p(X_t; \vartheta_t)}{\partial \vartheta_t}$ . To account for a variance of  $\nabla_t$  as seen in [9], we set a scaling matrix  $v_t$  to a power of  $\gamma > 0$ , as an inverse information matrix of  $\vartheta_t$ . According to [15], this simplified  $v_t(\vartheta_t) \equiv \varphi_t(\vartheta_t)^{-\gamma}$  with:

$$\varphi_t(\vartheta_t) \equiv E_{t-1}[\nabla_t(X_t, \vartheta_t) \nabla_t(X_t, \vartheta_t)'] \quad (9)$$

In evaluating model (9), [8] disclosed that the expected value is taken with respect to the conditional distribution of  $X_t | X_{1:t-1}$  and it is kept fixed by sub-setting  $\gamma \in \{0, 0.5, 1\}$ . As  $\gamma = 0$ , then  $v_t = I$ . Consequently, [1] declared that  $v_t$  is an identity matrix of the scaling parameter.

### III. EMPIRICAL ANALYSIS

To execute the analysis, we use a monthly saving rate for spanning period January 2010 to April 2018. For the statistical analysis, R 3.5.1 programming is utilized. Returns on savings data are displayed in Fig. 1. By visual examination, Fig. 1 uncovers that returns on saving series are non-stationary as it clearly shows some clustering volatility. This implies that saving rate can be model by a time-varying parameter model.

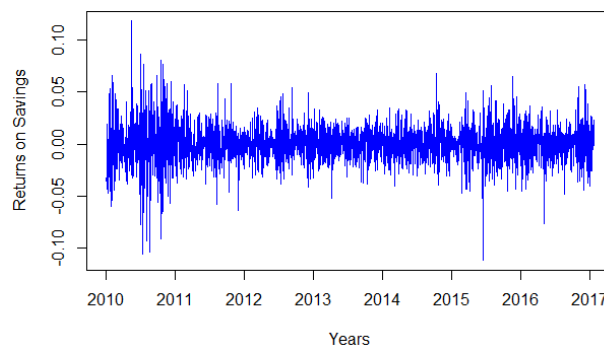


Fig. 1 Returns on savings

#### A. Exploratory Data Analysis

In this section, preliminary data analyses are conducted with a purpose of assessing data behavior. Descriptive statistics are used to provide a sound understanding and results are tabulated in Table II, also see [9], [17], [18]. Revealed in Table II are the statistical properties of the savings returns. Jarque-Bera test for normality which was developed by [12] rejects the null hypothesis of normality. This implies that the saving rate cannot be modeled by a Gaussian distribution. From our results, we further discovered that returns on savings are negatively skewed and leptokurtic as kurtosis is high above 3. The same results are obtained [13] in their study of the application of GAS model to stock returns. Likewise, the

mean of the returns on savings is -0.0016 entailing a reverse savings of 0.16% monthly. Finally, KSS-NADF test confirmed the presence of nonlinear unit root; indicating that a time-varying parameter model can be utilised to predict the nonlinear and time-variant of saving rate.

TABLE II  
EXPLORATORY DATA ANALYSIS

Statistic	Saving Rate
Mean	-0.0016
Skewness	-0.5501
kurtosis	3.7428
Jarque-Bera	0.9734(0.00)
KSS-NADF Test	
Statistic	5% Significance level
3.21	-2.22

NB: the value in () is the p-value for JB test

### B. GAS Model

Table III presents marginal coefficients of the model. Our results confirmed a strong persistence and positive reaction of volatility in conditional variance. The skewness parameter  $\eta_i$  as reported in row 7 is statistically significant and smaller than one justifying our choice of skewed innovations. In a study of switching-GAS copula models for systemic risk assessment, [2] found the same results. Because  $v_i$  is highly significant, this

confirms the presence of excess kurtosis and departure from normality. We conclude that saving rate is leptokurtic. This is also evident from the visualization of the returns properties in Fig. 1.

A quantile-quantile plot, empirical cumulative density function (CDF) and a normal probability density function (PDF) with a normal histogram significantly indicate heavy tail for saving rate. Nevertheless, [21] elucidated that returns data significantly parades this heavy tail distribution. For returns on savings,  $v$  is 0.986 inferring a highly leptokurtic distribution as the value is greater than zero.

TABLE III  
PARAMETER ESTIMATES

parameter	Estimate	Std error	t-value	prob
$\Phi_{0,i}$	0.034	0.020	1.640	0.050
$\Phi_{1i}$	0.008	0.003	2.451	0.007
$\omega_i$	0.220	0.113	1.948	0.026
$\theta_{1i}$	-2.414	0.309	-7.806	0.000
$\theta_{2i}$	0.997	0.012	8667	0.000
$\theta_{3i}$	3.949	0.007	5.869	0.002
$\eta_i$	0.359	0.057	6258	0.000
$v_i$	0.986	0.006	174.747	0.000
Unconditional Parameters				
	Location	Scale	skewness	shape
	0.052	1.780	1.055	7.776

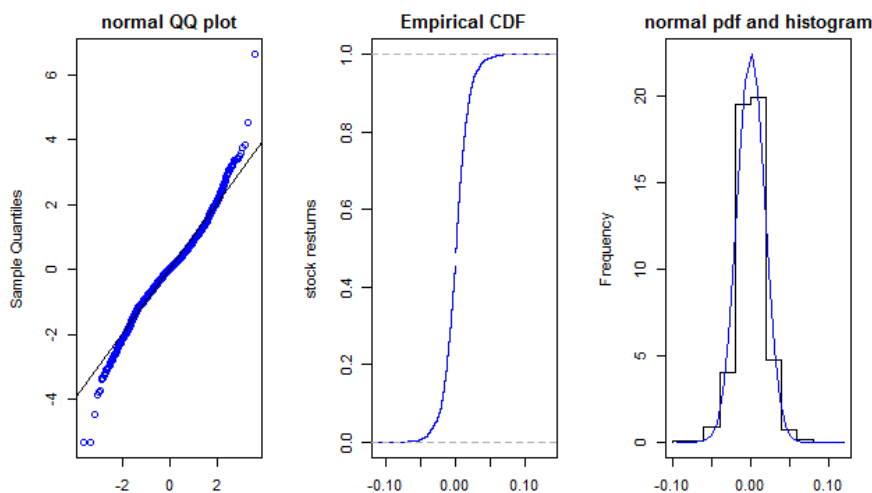


Fig. 2 The empirical properties of Stock returns

### C. The Risk for Low Saving Returns

Revealed in Table IV are risk measures for high saving returns. Both VaR and CTE are computed at 90%, 95% and 99% intervals. At 95% VaR is at 3.6; implying a gain of 3.6% in returns on savings by Basotho as the worst scenario. The same interpretation can be induced for both 90<sup>th</sup> and 99<sup>th</sup> percentiles. With CTE, the expected conditional gain is 5.37% which is higher than 3.6% of VaR This indicates that computation of economic capital using CTE is less conservative than using VaR An economic implication of this high percentage; is that, most people are not saving in Lesotho and therefore this puts the economy of the country at high risk

of not expanding rapidly.

TABLE IV  
RISK MEASURES FOR HIGH SAVING RETURNS

p	VaR	CTE
0.9	2.700	4.494
0.95	3.600	5.378
0.99	5.400	7.152

### IV. CONCLUSION

The current study aims to empirically investigate the presence of volatility in saving rates in Lesotho. To accomplish this task, the GAS model is applied to the returns

on savings. A GAS model according to [1] serves as an extension of GARCH family models which assume that a conditional distribution does not vary over time. A vibrant advantage of a GAS model is that, it exploits a full likelihood information of parameters. Taking a local density score step as a driving mechanism, time-varying parameters increase and produce a clear indication of a leptokurtic condition in returns with a shape of 7.776 in unconditional parameters. Our results generally showed that the saving rate in Lesotho is volatile and it is quite prettily modelled by a time-varying of a score function from a GAS model. Also, empirical properties of returns exposed that they are heavy-tailed in natural surroundings. The same behaviour of leptokurtic when the GAS model is applied to returns, has been realised in [2], [8]-[18]. Moreover, KSS-NADF confirmed the presence of a nonlinear unit root. This inferred that returns on saving rate can be modelled by a time-varying parameter model. Furthermore, risk measures revealed that there is a high persistence of no saving in Lesotho which puts the economy at high risk of not growing rapidly at 7.152% at 99% interval.

This study provides practical information for MPC of Lesotho to make informed policy decisions related to saving rates. Nevertheless, other scholars ought to extend this empirical analysis to the multivariate modelling by using the monetary transmission mechanism of Lesotho and utilise multivariate GAS mode to give more understanding on the nonlinear relationship of the time-varying parameter and the co-movement of saving rates in Lesotho.

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