

Fuzzy Multi-Criteria Decision-Making Based on Ignatian Discernment Process

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Abstract—Ignatian Discernment Process (IDP) is an intense decision-making tool to decide on life-issues. Decisions are influenced by various factors outside of the decision maker and inclination within. This paper develops IDP in the context of Fuzzy Multi-criteria Decision Making (FMCDM) process. Extended VIKOR method is a decision-making method which encompasses even conflict situations and accommodates weightage to various issues. Various aspects of IDP, namely three ways of decision making and tactics of inner desires, are observed, analyzed and articulated within the frame work of fuzzy rules. The decision-making situations are broadly categorized into two types. The issues outside of the decision maker influence the person. The inner feeling also plays vital role in coming to a conclusion. IDP integrates both the categories using Extended VIKOR method. Case studies are carried out and analyzed with FMCDM process. Finally, IDP is verified with an illustrative case study and results are interpreted. A confused person who could not come to a conclusion is able to take decision on a concrete way of life through IDP. The proposed IDP model recommends an integrated and committed approach to value-based decision making.

Keywords—Analytical hierarchy process, fuzzy multi-criteria decision making, Ignatian discernment process, Ignatian discernment, multi-criteria decision making, VIKOR.

I. INTRODUCTION

DECISION making is one of the most frequently carried out human activities. Ignatian discernment is the decision-making process proposed by Ignatius of Loyola – the founder of the ‘Society of Jesus’ (Jesuits) [2]. It is one of the rich Jesuit heritages that have caught the attention of corporate world particularly in the area of decision making. Discernment is made in an ‘indifference’ attitude which means inner freedom. Unlike most of the decision techniques; IDP includes sensual, emotional and spiritual aspects along with the intellect in the decision-making process. We consider IDP as a decision-making tool that aims at an uncompromised solution unlike other MCDMs. The ultimate purpose of discernment is to arrive at ‘the better choice’. ‘The better choice’ for Ignatius originally meant ‘for the greater glory of God’ [2]. The meaning of ‘for the greater glory of God’ has evolved and is expressed in different words and phrases applicable to the context. In this paper we attempt to understand IDP and use it in the FMCDM environment.

In the second section, general understanding of FMCDMs is presented. One of the MCDMs, Analytical Hierarchy Process (AHP) and its different variants are discussed. One case study has been carried out in AHP which is used for later comparison.

In Section III, we present the IDP as given by St. Ignatius and discuss three ways of decision making; discernment and tactics of ‘Spirits’. In Section IV, we discuss two types of situations in decision making. In Section V, we propose the adaptation of IDP in various decision-making contexts. In sixth section, we illustrate IDP in the framework of VIKOR method with an example.

II. FUZZY MULTI-CRITERIA DECISION MAKING

Multi-criteria Decision Making is one of the most widely used decision making techniques in various fields such as; business, medicine, energy and environment, economy, production [6]. FMCDM is introduced particularly for the application-oriented problems where decision makers are faced with conflicting objectives and non-commensurable criteria. Therefore, one of the most common characteristics of FMCDMs is to highlight these conflicts and find a compromise solution.

The foundations of modern Multi-criteria Decision Making (MCDM) were developed in 1950s and 1960s and the development of MCDM research accelerated during the 1980s and early 1990s [6]. In 1965, Roy [14] introduced Elimination and Choice Expressing Reality (ELECTRE) and it was extended to ELCETRE I, II, III, IV, IS, TRI for choosing, ranking and sorting. Satty in 1971 introduced AHP [15]. In 1979, Opricovic S. introduced Vise Kriteriumska Optimizacija I Komproisno Resenje (VIKOR) [9] to arrive at a compromise solution in the context of conflict. In 1980, Bransintroduced [6] Preference Ranking Organization METHod for Enrichment Evaluation (PROMETHEE). In 1981, Technique for Order Preference to Similarity to Ideal Solution (TOPSIS) was introduced by Hwang and Yoon [17]. AHP was extended to Analytic Network Process (ANP) by Satty in 1996 [15]. In 2015 Best Worst Method (BWM) was introduced by Rezaei [16].

It is accepted that a certain degree of subjectivity is involved in the MCDM decision making process [12]. Subjectivity in MCDM is due to the preference relations introduced, weight values defined for the criteria based on the experience of the individual and so on. Classical models based on rationality are founded on two valued logic and do not capture the importance of subjectivity. Hence most of the MCDMs have been extended to FMCDM. Fuzzy decision-making tool allows solving problems dealing with imprecise and uncertain data. Kumar and Pathinathan extended AHP to Stratified Fuzzy Analytic Hierarchy Process (SFAHP) [11] which is developed using

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Pentagonal Fuzzy Number [10] for quantifying the qualitative data. The tool is used to assess level of poverty in the four districts of Bihar. Pathinathan and Johnson studied the problem of farming using Weight Based Intuitionistic Fuzzy Set (WBIFS) with AHP [8]. Pathinathan et al. proposed Extended VIKOR method using Pentagonal Fuzzy Numbers to study crops cultivation [9].

A. Simple Illustration of AHP Method

It is observed that every year during festivals like *Pongal* (feast celebrated in Tamilnadu, India) and *Diwali* (festival celebrated all over India), new movies are released. In general, ordinary people go to watch movies for entertainment and relaxations but for some section of the youth in Tamil Nadu, India, cinema and film star affiliations are very important matters of interest. We studied the selection procedure of Pongal 2017 Tamil movies by college students. The movies are

Bairava, *Koditta Idangalai Nirappuga (KIN)*, *Sivappu Enakku Pidikum (SEP)*. We found out the main goal of students to go for movies by Delphi method and culled out four main criteria namely actor, message (story line), review from friends, and music. We used a questionnaire to find out students' preferences. The sample is taken randomly from a group of students. We used the above mentioned four criteria to rank the three movies (alternatives) using traditional AHP.

In this case study, "entertainment" is the goal of students. The set of criteria: C1 = actor, C2 = message (story line), C3 = review from friends, C4 = music.

The set of alternatives: A₁ = *Bairava*, A₂ = *Koditta Idangalai Nirappuga (KIN)*, A₃ = *Sivappu Enakku Pidikum (SEP)*.

We use the following relative scale to quantify the qualitative response and then use it for pair-wise comparison of various criteria and alternatives with respect to each criterion. We use this scale to assign weight to each criterion.

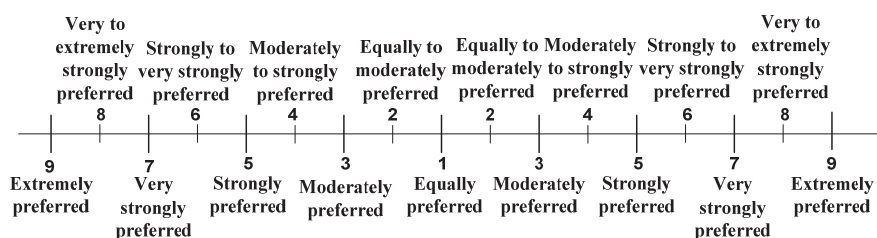


Fig. 1 Relative scale to quantify qualitative response

Step1. We collect various opinions from the students for the four main criteria and by pair-wise comparison; we obtain the matrix as in Table I.

TABLE I
 PAIR-WISE COMPARISON MATRIX OF CRITERIA

	C ₁	C ₂	C ₃	C ₄
C ₁	1	3	7	9
C ₂	$\frac{1}{3}$	1	3	7
C ₃	$\frac{1}{7}$	$\frac{1}{3}$	1	3
C ₄	$\frac{1}{9}$	$\frac{1}{7}$	$\frac{1}{3}$	1
sum	$\frac{100}{63}$	$\frac{94}{21}$	$\frac{34}{3}$	20

Step2. We sum each column in matrix in Table I and then divide each element of the matrix with the sum of its column and thus obtain normalized comparison matrix as in Table II.

Step3. We take average of the entries along each row of the normalized matrix in Table II and obtain normalized principal eigenvector which is also called priority vector (W) as in (1):

Step4. To check the consistency of students' opinion, we obtain Principal Eigen value by summing up the products of each element of Priority vectors and the sum of columns of the pair-wise comparison matrix from Table I and denote it by λ_{max}

TABLE II
 NORMALIZED MATRIX

	C ₁	C ₂	C ₃	C ₄
C ₁	$\frac{63}{100}$	$\frac{63}{94}$	$\frac{21}{34}$	$\frac{9}{20}$
C ₂	$\frac{21}{100}$	$\frac{21}{94}$	$\frac{9}{34}$	$\frac{7}{20}$
C ₃	$\frac{9}{100}$	$\frac{7}{94}$	$\frac{3}{34}$	$\frac{3}{20}$
C ₄	$\frac{7}{100}$	$\frac{3}{94}$	$\frac{1}{34}$	$\frac{1}{20}$

$$W = \frac{1}{4} \begin{bmatrix} \frac{63}{100} + \frac{63}{94} + \frac{21}{34} + \frac{9}{20} \\ \frac{21}{100} + \frac{21}{94} + \frac{9}{34} + \frac{7}{20} \\ \frac{9}{100} + \frac{7}{94} + \frac{3}{34} + \frac{3}{20} \\ \frac{7}{100} + \frac{3}{94} + \frac{1}{34} + \frac{1}{20} \end{bmatrix} \quad (1)$$

$$W = \begin{bmatrix} 0.5866 \\ 0.2620 \\ 0.1007 \\ 0.0453 \end{bmatrix} \quad (2)$$

$$\lambda_{max} = \frac{100}{63}(0.5866) + \frac{94}{21}(0.2620) + \frac{34}{3}(0.1007) + 20(0.0453) \\ \lambda_{max} = 4.1511 \quad (3)$$

Step5. We obtain eigenvalue and eigenvector using MATLAB and compare the largest eigenvector with the value of λ_{\max} . We observe that the largest eigenvalue is 4.0990 which is close to $\lambda_{\max} = 4.1511$ as in (3). We obtain the principal eigenvector \bar{w} which is the eigenvector that corresponds to the highest eigenvalue.

$$\bar{w} = \begin{bmatrix} 0.9060 \\ 0.3909 \\ 0.1481 \\ 0.0664 \end{bmatrix} \quad (4)$$

By dividing each eigen element of \bar{w} by the total 1.5114 we obtain the normalized principal eigenvector w^* .

$$w^* = \begin{bmatrix} 0.5994 \\ 0.2586 \\ 0.0980 \\ 0.0439 \end{bmatrix} \quad (5)$$

From (1) and (5) we observe that w^* is very close to W .

Step6. To check the consistency of students' opinion, we calculate Consistency Index (CI) and Consistency Ratio (CR) using the following formula:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \quad (6)$$

$$CR = \frac{CI}{RI} \quad (7)$$

where λ_{\max} is Principal Eigen value, n is the number of criteria and RI is random CI given by Table III.

N	RI
1	0
2	0
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45
10	1.49

Using (5) and (6) we obtain:

$$CI = 0.0532 \quad (8)$$

$$CR = 0.0591 \quad (9)$$

CR is less than 10%. Therefore, result is considered consistent.

Step7. The value of λ_{\max} , CI and CR are calculated using Comparison Matrix (CM).

	A ₁	A ₂	A ₃
A ₁	1	9	7
A ₂	$\frac{1}{9}$	1	3
A ₃	$\frac{1}{7}$	$\frac{1}{3}$	1
sum	$\frac{79}{63}$	$\frac{31}{3}$	11

$$\lambda_{\max} = 3.4103, CI = 0.2052, CR = 0.3537$$

	A ₁	A ₂	A ₃
A ₁	1	$\frac{1}{3}$	5
A ₂	3	1	7
A ₃	$\frac{1}{5}$	$\frac{1}{3}$	5
sum	$\frac{21}{5}$	$\frac{31}{21}$	13

$$\lambda_{\max} = 3.0969, CI = 0.0485, CR = 0.0836$$

	A ₁	A ₂	A ₃
A ₁	1	3	5
A ₂	$\frac{1}{3}$	1	3
A ₃	$\frac{1}{5}$	$\frac{1}{3}$	1
sum	$\frac{23}{15}$	$\frac{13}{3}$	9

$$\lambda_{\max} = 3.0557, CI = 0.0279, CR = 0.0481$$

	A ₁	A ₂	A ₃
A ₁	1	3	3
A ₂	$\frac{1}{3}$	1	$\frac{1}{3}$
A ₃	$\frac{1}{3}$	3	1
sum	$\frac{5}{3}$	7	$\frac{13}{3}$

$$\lambda_{\max} = 3.1764, CI = 0.0882, CR = 0.1521$$

Step8. Using steps 2 and 3 we obtain priority vectors for the matrices in Tables IV-VII then obtain the composite matrix of alternatives combining all as in Table VIII. Taking the row wise average, we get the composite value of the matrix as in (10).

Composite Weight (CW):

$$CW = \frac{1}{4} \begin{bmatrix} 0.7682 + 0.2828 + 0.6333 + 0.5736 \\ 0.1527 + 0.6434 + 0.2605 + 0.1399 \\ 0.0790 + 0.0737 + 0.1062 + 0.2864 \end{bmatrix} \quad (10)$$

$$CW = \begin{bmatrix} 0.5644 \\ 0.2991 \\ 0.1362 \end{bmatrix} \quad (11)$$

TABLE VIII
 COMPOSITE MATRIX OF ALTERNATIVES

A ₁	0.7682	0.2828	0.6333	0.5736
A ₂	0.1527	0.6434	0.2605	0.1399
A ₃	0.0790	0.0737	0.1062	0.2864

According to the composite weight (CW) in equation (11), the students' order of preference is:

A₁= *Bairava*, A₂= *Koditta Idangalai Nirappuga*, A₃= *Sivappu Enakku Pidikum*. The AHP method helps us not just to rank but it helps us to understand to what degree or to what quality the alternatives are preferred from each other.

B. Observations

The procedure followed in the above example brings out the following different aspects involved in decision making.

1. Every decision has an overall purpose/goal. This goal is reached by the attainment of various objectives. In general, the objectives of MCDM are to optimize the benefit criteria and minimize the cost criteria. In our case the goal is entertainment.
2. There are various methods that define and decide the attainment of the goal. In most of the MCDMs, the criteria stem out of alternatives. In our case, all the four criteria namely actors, story line, review about the movie and music are within that movies.
3. There are finite numbers of alternatives. The decision maker chooses one that optimizes the goal.
4. It also brings to the light that there could be various types of decision makers namely:
 - i. Non-committed-type: There are those who take decision to please others. In our case, some students go for the movies because of the request made by their friends. They do not have their own focused goal. Their decision depends on the decision of others.
 - ii. (ii)Compromise-type: There are those who take decision to consider the available situations. In our case, if students are not able to watch the first day first show they try next time but do not get upset. They are ready to compromise for convenient day and time. Some students go for the movies just to oblige friends. They do not have their own focused goal.
 - iii. Uncompromised-type: Once decision is made, they pursue it by hook or crook. In our case, there are students who use any means to watch the first day, first show. If they fail, they are very upset.

Depending on contexts and issues, people take their stand while making decisions. Some decisions are made to optimize the solution, others are made for satisfaction and yet others are made out of commitment.

Most of the decisions directly related to life demand genuine commitment and uncompromised attitude towards our decisions. St. Ignatius proposed rules for discernment [2]

particularly to choose for and commit oneself to life preserving and life promoting choices. In the next section we briefly discuss the IDP.

III. IGNATIAN DISCERNMENT PROCESS (IDP)

One of the salient features of IDP unlike other MCDMs is its stand on uncompromised solution. While all the existing decision-making models search to settle for a compromise solution, IDP aims at arriving at the 'better' of two or more solutions. One of the yardsticks St. Ignatius used to measure the 'better' of two or more solutions is *Magis* (more or better universal good). *Magis* is one of the characteristics of the Jesuit way of decision making [3]. St. Ignatius believed that serving the more universal good gives greater glory to God than working for a more particular good [2]. The more universal good reminds us that all decisions one makes, no matter how private or personal they have implications for wider human community. This yardstick ultimately has the origin in '*Ad Majorem Dei Gloriam*' (for the greater glory of God). In order to understand the spirit of IDP readers need to reflect on the life of St. Ignatius and examine the beginning and the motivation behind IDP.

A. The Man of Discernment [4]

St. Ignatius was ready to take any challenge in order to please the king and the royal lady. The aim initially was to gain worldly fame and fortune. In 1521, the fortress of Pamplona was attacked by a very large French army. A tiny band of Spanish soldiers trying to defend the town were ready to surrender. But Ignatius single-handedly tried to fight back till a French cannonball shattered the right leg. The Spanish soldiers were defeated but Ignatius won much appreciation from the French army.

After Pamplona battle, during convalescence in the family castle, the book of Life of Christ by Ludolf of Saxony and Life of Saints were given to St. Ignatius [13] as there were no chivalric fictions available for to read. The life of Christ and the life of saints had a strong impact on St. Ignatius. Now St. Dominic and St. Francis of Assisi were two new heroes. The selfless attitude, other-centered approach and self-denying life style of Jesus and Saints were great challenges to the self-centered, self-seeking, and worldly way of life. There was a gradual progress in understanding with regard to the movements of desires, thoughts and imagination within.

With this understanding there was deeper awareness of the two kinds the inner movements within – one towards God and another one away from God. Ignatius observed during these days that reading and thinking about Saints and Christ gave lasting peace and satisfaction. On the contrary, while daydreaming of the noble lady love, worldly glory and earthly pleasures resulted with restlessness and dissatisfaction. The biblical sentence from the gospel according to St. Mathew "For what will it profit them if they gain the whole world but forfeit their life?"(Matthew 16:26) [7] was a frequent challenge to the self-centered approach to life. Self-denying experience was the beginning of inner conversion and basis for the discernment process which later on was proposed by St.

Ignatius in the classical book *Spiritual Exercises*.

The second significant lasting impact of the book by Ludolf of Saxony on St. Ignatius was the use of imagery – vivid physical description of person of Jesus, scenes, events. This provided an opportunity for involvement in the scene and assimilating the image and theme presented [12]. This attitude of being involved and immersed is one of the basic characteristics in IDP.

Another event that had life-long impact was the foundational or mystical experience at the bank of river Cardoner [2]. After recuperation, deeper philosophical questions regarding human life and life in general like – Who am I? What is the purpose of life? What is the purpose of creation? continued to bother the St. Ignatius. At the bank of the river Cardoner, the realization and conviction that everything (every human) comes from God

and goes back to God gave real deeper satisfaction. Therefore, everything else that does not lead us to God is vanity namely name and fame, vain glory, riches, honor, earthly pleasures. According to the new understanding, gaining the world meant gaining riches, name and fame, honor, earthly pleasures etc. and losing oneself meant separating oneself from God. Metaphorically we could say that a golden bed is of no use if one cannot sleep peacefully on it.

B. Three Ways of Decision Making

St. Ignatius proposed the rules or directions for discernment of ‘Spirits’ after carefully reflecting over the inner experience in life. There are three ways of arriving at any decision [2] according to St. Ignatius. These three ways of arriving at decision are presented in the following chart.

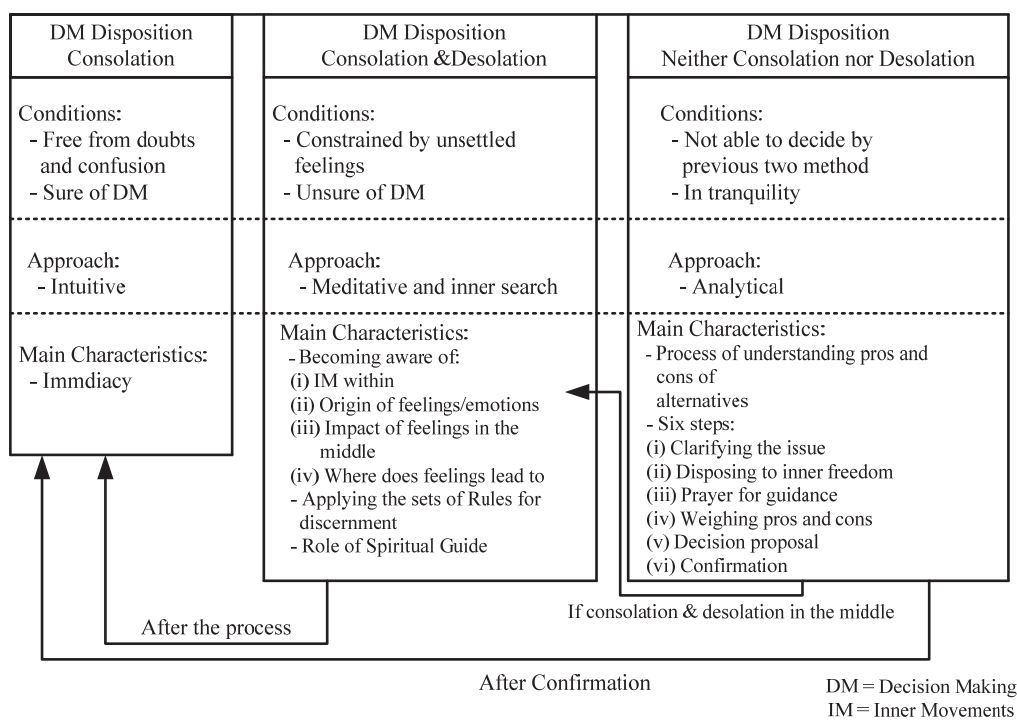


Fig. 2 Flow chart of three ways of decision making proposed by St. Ignatius

C. Basic Conditions for IDP

It is very clear that IDP is a decision-making process that involves the whole of a person – head, heart and hand. According to St. Ignatius, the use of all the human faculties while dealing with any issue is very crucial. Therefore, emotions/inner movements are taken into due consideration while making decision in IDP unlike other classical MCDMs. IDP contains certain prerequisites. The first and foremost is faith in God. Originally, Ignatius proposed discernment to find the will of God and thus, it was meant for one who believed in God. The second condition is ‘Indifference’, which means interior as well as exterior freedom i.e. freedom from passions, affections, attitudes, emotions, persons, situations and compulsions etc [1]. The third condition is openness to approach different issues with genuine broader perspective. Here openness also means to be open to oneself, to others and

to God. The fourth condition is desire to grow in commitment.

The special aspect in IDP is the role of the guide or the director with whom decision maker shares the inner movements. The guide clarifies the inner movements within and suggests the exercise to the decision maker. At every stage, the Guide is a reference point for verification.

D. Discernment of Spirit and Discernment of Will of God

St. Ignatius talks of discernment of spirits (DoS) and discernment of the will of God (DwG). DoS basically concerns inner movements within a person. DoS leads to a proper DwG. They are not in parallel or opposed but in continuity. DoS is heart and soul of the process of discernment. It is always geared towards finding the will of God. DoS is basically to become aware of inner movements within oneself and understand the tactics of spirits.

1) Inner Movements

Inner movements are interior experiences within a human being. This could be an act of intellect (thoughts, reasoning, imagining etc.) or an act of will (love, hate, desire, fear, etc.) or an act of affective feelings, impulses, inclinations, etc. [2]. According to the Greek philosopher Plato, humans are partly body and partly soul [5]. St. Ignatius believes that humans have two tendencies and finally leaning onto one of two directions – good or evil/less good. St. Ignatius calls this as good and evil spirits respectively working within. In more general terms; this could be called as spiritual and worldly nature.

2) Tactics of Spirits [2]

St. Ignatius suggests that worldly nature cannot work directly

on human will but only indirectly by tempting persons through the imagination or senses. They attack at the weakest point and proceed to other areas from there. If the heavenly nature is strong to resist the worldly nature, persons overcome worldly nature but if it happens otherwise worldly nature overcomes humans.

If the person is progressing in divine nature (selflessness, love, altruism etc.) worldly nature creates doubts, distractions and disturbances by questioning the person’s way of life [2]. On the other hand, divine nature affirms person’s life-promoting actions. Every decision maker realizes such tug of war between two natures.

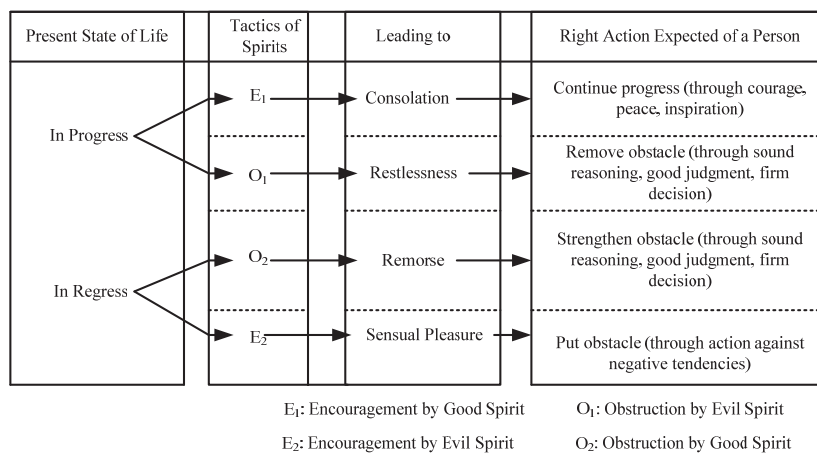


Fig. 3 Flow chart of tactics of spirits and rules to respond to it

IV. TWO TYPES OF SITUATIONS IN DECISION-MAKING

In general, we can see there are two types of situations in decision making. In the existing FMCDM/MCDM situations, the issues involved in decision making mostly lie outside of the decision maker. The decisions are made under constraints of the career prospects and the goal of the company/office. We term this situation as “Deciding for others”. Here decisions define person’s profession.

In IDP, the issues involved lie “within”. A decision is made for the decision maker the issues decided upon are intimately associated with life. We term it as “Deciding for oneself”. We can correlate traditional FMCDM/MCDM and IDP in the following manner:

One of the biggest questions that arises following the above comparison is – can we incorporate core values (personal, corporate, institutional or social) in FMCDM/FMCDA? We believe that IDP can play a vital role in the process of incorporation of core values in MCDM/MCDA.

The first step towards this could be to adapt at least the important aspects of IDP in FMCDM. Secondly, moving from narrow understanding of rationality (which takes into account only mind/reason) to deeper and broader understanding of being reasonable which also includes emotions, experiences, beliefs and values.

TABLE IX
 MCDM/FMCDM AND IDP

Traditional MCDM/FMCDM	IDP
There is a big distance between decision and decision maker.	There is a little distance between decision and decision maker.
Objectivity is the main concern.	Subjectivity is given due importance as experiences, emotions, feelings, values and actions are taken into considerations.
Outcome/decision conditions the process.	Outcome/decision emerges from the process.
State of doubt and confusion is overlooked.	State of doubt and confusion is taken for serious consideration.
Focus is on the somatic (material) level with emphasis on utilities.	Focus is on the higher level of being i.e. principles and values of life.
Focus is on attributes	Focus is on criteria.

V. ADAPTATION OF IDP

IDP in its original form is presented in the language of St. Ignatius. The words, terms and imageries used are taken from the historical period that time. Many have presented it in simplified language but mostly using Christian terminologies, which others might not understand. It is therefore important that it is expressed in a much simpler language so that Christians and non-Christians, believers and non-believers all might understand and make use of this procedure in making important decisions. For example, in the Spiritual Exercises of St.

Ignatius, it is mentioned that the goal of human life is to help the souls. The phrase ‘helping souls’ might sound very Christian but it is not particularly Christian. In the vocabulary of Ignatius’ time it was a way of speaking of the ‘human person’. So, we can interpret the phrase as ‘helping the whole human person’ or ‘working for preservation and promotion of life’.

Here are few examples of adaptation of Ignatian terminologies to the ordinary terms:

TABLE X
 ADAPTATION OF IGNATIAN TERMS TO ORDINARY TERMS

Ignatian Terms	Ordinary Terms
Discernment	Decision Making
Indifference	Neutral; free from within and outside forces
God	Ultimate destination/goal/value
Salvation	Continual progress towards the ultimate goal
Saving/helping souls	Saving/helping the whole human person
Losing one’s soul	Loosing peace of mind; Losing sight of ultimate destination/goal
Consolation	Positive orientation; increase in hope and charity
Desolation	Negative orientation; decrease in hope and charity
Magis	More universal good
Finding God in all things	Looking at the reality positively and aspiring for universal good
Interior movements	Consists of thoughts, imagination, emotion, inclinations, desires, feelings, repulsion, attraction

VI. ILLUSTRATION OF IDP IN THE EXTENDED VIKOR FRAMEWORK

We take an example to discuss IDP. We look at it in the framework of “Extended VIKOR method using Pentagonal Fuzzy Numbers” proposed by Pathinathan T., Johnson Savarimuthu, S. and Mike Dison, E.; and make a comparative study of IDP with FMCDM.

A. Different Steps in Extended VIKOR [9]

Step 1: Classify and characterize linguistic terms and relevant membership functions.

Step 2: Considering the expert’s opinion, we construct a decision matrix.

$$DM = [f_{ij}]_{n \times m} \quad (12)$$

Step 3: If many decision makers are present, then aggregation is done.

Step 4: Obtain fuzzy weights (w_i) for each criterion and if required aggregated.

Step 5: If required, the decision matrix is normalized.

Step 6: We obtain the best value f_j^+ and the worst value f_j^- using the formula:

$$f_j^+ = \max_i f_{ij} \text{ and } f_j^- = \min_i f_{ij} \quad (13)$$

if i^{th} function is benefit.

$$f_j^+ = \min_i f_{ij} \text{ and } f_j^- = \max_i f_{ij} \quad (14)$$

if i^{th} function is cost.

Step 7: We calculate the values of group utility factors S_i and group regret factors R_i by the following equations:

$$S_i = \sum_{j=1}^m w_j \left(\frac{f_j^+ - f_{ij}}{f_j^+ - f_j^-} \right) \quad (15)$$

$$R_i = \max \left[w_j \left(\frac{f_j^+ - f_{ij}}{f_j^+ - f_j^-} \right) \right] \text{ where } i = 1, 2, \dots, n; j = 1, 2, \dots, m$$

where w_j ’s are weight of criteria expressing the relative importance.

Step 8: We calculate the value of Q_i using the following relation:

$$Q_j = v \left(\frac{S_i - S^*}{S^- - S^*} \right) + (1-v) \left(\frac{R_i - R^*}{R^- - R^*} \right) \quad (16)$$

where Q_j is ranking measure; v is the weight introduced for the strategy of maximum group utility, and $1-v$ is the weight of the individual regret and where

$$S^* = \min S_i$$

$$S^- = \max S_i$$

$$R^* = \min R_i$$

$$R^- = \max R_i \quad (17)$$

Step 9: The alternatives are ranked with the help of the values of Q_j ’s. The alternative which is the best ranked by the measure of Q should satisfy the two following conditions:

- Acceptable advantage and
- Acceptable stability in decision making.

B. Case Study

We have discussed with a person who has done thirty days Ignatian Spiritual Exercises and used IDP to make decision regarding personal choice about the profession (way of life). We can summarize IDP in the following 7 steps:

Step 1: Disposing oneself for undertaking the discernment procedure.

Step 2: Looking for a suitable guide for accompaniment.

Step 3: Becoming aware of the purpose, principles and values of life.

Step 4: Writing down the possible alternatives (finite set of alternatives).

Step 5: Becoming aware of the movements within.

Step 6: Enumerating positive and negative aspects of the consequences of various alternatives available. Important criteria that define our decisions are also enumerated (finite set of criteria).

Step 7: Assigning weights to the criteria expressing relative importance.

Step 8: Using alternatives, criteria and weights decision matrix is formed which is further calculated in the framework of Extended VIKOR method and then conclusions are drawn.

The whole IDP particularly from step 1 to step 7 takes place in four stages. We can see the dynamics of these stages in the following flow chart:

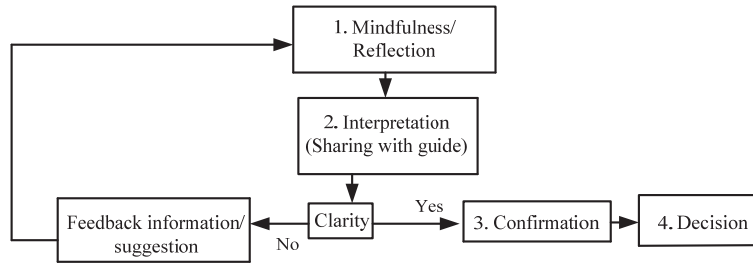


Fig. 4 Flow chart demonstrating the four stages in IDP

In the case considered, the person has the following alternatives:

- A₁ = Priesthood, A₂ = Brotherhood (a person remains religious without ordination to priesthood), A₃ = Married Life
- A₄ = Government Job

The person uses the following criteria in decision-making process:

- C₁ = Personal inner satisfaction, C₂ = better use of personal life, C₃ = In line with personal values and principles and C₄ = Security/money

The criteria are classified with the help of the linguistic variables and its fuzzy linguistic scale values as in the Table XI. The weights to the criteria are assigned as in Table XIII based on the importance given by the decision maker.

TABLE XI
 LINGUISTIC VARIABLES AND RELATED PENTAGONAL FUZZY NUMBERS

Linguistic variables	Pentagonal Fuzzy Number
Extremely High (EH)	(0.80,0.85,0.90,0.95,1.0)
Very High (VH)	(0.60,0.675,0.75,0.825,0.90)
Fairly High (FH)	(0.40,0.475,0.55,0.625,0.70)
Somewhat High (SH)	(0.30,0.35,0.40,0.45,0.50)
Just High (JH)	(0.10,0.175,0.25,0.325,0.40)

The decision maker's opinions collected in the form of linguistic variables are entered in the decision matrix as in the Table XII and which is converted into pentagonal decision matrix as in Table XIV.

TABLE XII
 DECISION MATRIX WITH LINGUISTIC VARIABLES

	C ₁	C ₂	C ₃	C ₄
A ₁	EH	VH	EH	JH
A ₂	SH	SH	EH	JH
A ₃	FH	SH	VH	FH
A ₄	FH	SH	FH	VH

TABLE XIII

WEIGHT OF CRITERIA (W_i)

C ₁	0.3
C ₂	0.3
C ₃	0.3
C ₄	0.1

TABLE XIV
 PENTAGONAL DECISION MATRIX

	C ₁	C ₂	C ₃	C ₄
A ₁	(0.8,0.85,0.9,0.95,1)	(0.6,0.675,0.75,0.825,0.9)	(0.8,0.85,0.9,0.95,1)	(0.1,0.175,0.25,0.325,0.4)
A ₂	(0.3,0.35,0.4,0.45,0.5)	(0.3,0.35,0.4,0.45,0.5)	(0.8,0.85,0.9,0.95,1)	(0.1,0.175,0.25,0.325,0.4)
A ₃	(0.4,0.475,0.55,0.625,0.7)	(0.3,0.35,0.4,0.45,0.5)	(0.6,0.675,0.75,0.825,0.9)	(0.4,0.475,0.55,0.625,0.7)
A ₄	(0.4,0.475,0.55,0.625,0.7)	(0.3,0.35,0.4,0.45,0.5)	(0.4,0.475,0.55,0.625,0.7)	(0.6,0.675,0.75,0.825,0.9)

TABLE XV
 DECISION MATRIX WITH AVERAGE VALUE OF 5-TUPLES

	C ₁	C ₂	C ₃	C ₄
A ₁	0.90	0.75	0.90	0.25
A ₂	0.40	0.40	0.90	0.25
A ₃	0.55	0.40	0.75	0.55
A ₄	0.55	0.40	0.55	0.75

TABLE XVI

BEST (f_j^+) AND WORST (f_j^-) VALUE	C ₁	C ₂	C ₃	C ₄
f_j^+	0.90	0.75	0.90	0.75
f_j^-	0.40	0.40	0.55	0.25

Using (15) we calculate S_i and R_i, where i=1,2,3, and 4 as in Table XVII.

TABLE XVII
 VALUE OF S_i AND R_i

$S_{11} = 0.00$	$S_{12} = 0.00$	$S_{13} = 0.00$	$S_{14} = 0.10$	$S_1 = 0.10$	$R_1 = 0.10$
$S_{21} = 0.30$	$S_{22} = 0.30$	$S_{23} = 0.00$	$S_{24} = 0.10$	$S_2 = 0.70$	$R_2 = 0.30$
$S_{31} = 0.21$	$S_{32} = 0.30$	$S_{33} = 0.13$	$S_{34} = 0.04$	$S_3 = 0.68$	$R_3 = 0.30$
$S_{41} = 0.21$	$S_{42} = 0.30$	$S_{43} = 0.30$	$S_{44} = 0.00$	$S_4 = 0.81$	$R_4 = 0.30$

Using (17) we get values of S^* , S^- , R^* , R^- as in the table XVIII.

TABLE XVIII
 VALUE OF S^* , S^- , R^* , R^-

$S^* = 0.1$	$S^- = 0.81$	$R^* = 0.1$	$R^- = 0.1$
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Using (16) we calculate Q_i 's as in the Table XIV.

TABLE XIV VALUE OF Q_i			
$Q_1=0$	$Q_2=0.922$	$Q_3=0.908$	$Q_4=1$

The best option is arrived on the basis of lowest rank. Thus, alternative A_1 (Priesthood) is the best option to be chosen by the decision maker.

VII. OBSERVATION

Using the illustration, we can enumerate the following defining characteristics of IDP:

- i. Integrated Approach: Unlike most of the FMCDMs/MCDMs, IDP integrates personal emotions and individual subjective reasons in the process of decision making. IDP includes sensual, emotional and spiritual aspects along with intelligence in decision making process.
- ii. Means leads to the end: All the means for arriving at the main goal are directly proportional to the attainment of goal. There is a great danger of confusing means with the end. Therefore, Ignatius of Loyola literally warns not to confuse means with the end [2].
- iii. Responsible Decision: IDP engages decision makers in dialogues with oneself, with others and with the Divine (ultimate or universal goal). Thus, it allows others opinion and ideas in the process of clarification and thus shares accountability. Moreover, by valuing the role of feelings in decision making, the decision maker is willing to be responsible for ourselves and for others at the psychological level.
- iv. Criteria beyond the Objects/Alternatives: In most of the decision-making techniques, criteria taken are centered on the alternatives. In IDP, we also look for the criteria which lie beyond the object. They are rather centered on the subject that is decision maker.
- v. Decision with Uncompromised Solution: All the criteria in IDP are progressively supportive of each other. The attainment of one objective helps us to attain another objective. Finally, the objective is to commit oneself to the decision made. Thus, one is led to commitment without compromise.
- vi. Counter check for confirmation: The important characteristic of IDP is that the whole process is guided by

an experienced person (Guide). At every stage, the guide clarifies and helps the decision maker to confirm to the final decision.

VIII. CONCLUSION

In this paper, we have tried to understand IDP in the environment of FMCDM. One case study is carried out in AHP and various aspects involved in decision making are elaborately discussed. We have made study on the origin and various features involved in IDP as proposed by Ignatius of Loyola. We have correlated and compared IDP with FMCDM. We have proposed some simpler terms and phrases for complicated and highly Ignatian terms and have suggested how we could adapt IDP in our daily decision making. We have illustrated IDP in the framework of Extended VIKOR method with an example. With this study we have affirmed that IDP is an integrated approach to decision making. With the help of IDP we can incorporate core values (personal, corporate, institutional or social) in FMCDM. This gives us space to work further on IDP and refine the model for common man's use in this modern world.

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