# Functionally Graded MEMS Piezoelectric Energy Harvester with Magnetic Tip Mass

M. Derayatifar, M. Packirisamy, R.B. Bhat

***Abstract*—**Role of piezoelectric energy harvesters has gained interest in supplying power for micro devices such as health monitoring sensors. In this study, in order to enhance the piezoelectric energy harvesting in capturing energy from broader range of excitation and to improve the mechanical and electrical responses, bimorph piezoelectric energy harvester beam with magnetic mass attached at the end is presented. In view of overcoming the brittleness of piezo-ceramics, functionally graded piezoelectric layers comprising of both piezo-ceramic and piezo-polymer is employed. The nonlinear equations of motions are derived using energy method and then solved analytically using perturbation scheme. The frequency responses of the forced vibration case are obtained for the near resonance case. The nonlinear dynamic responses of the MEMS scaled functionally graded piezoelectric energy harvester in this paper may be utilized in different design scenarios to increase the efficiency of the harvester.

***Keywords*—**Energy harvesting, functionally graded piezoelectric material, magnetic force, MEMS piezoelectric, perturbation method.

## I. Introduction

IN line with the nature of green energy, energy harvesting in micro scale from vibrational ambient has attracted so many attentions. However, energy harvesters with linear counterpart accept limited frequency range in which their efficiency is lower in comparison with nonlinear one. Furthermore, brittleness and high rigidity of piezoelectric-ceramic materials make them unfavorable to employ for harsh environment. In this paper, to improve the performance of the piezoelectric energy harvester (PEH) in capturing energy from broader and more complex excitations, magnetoelastic structure [1] with geometric nonlinearity is presented. Moreover, to remove the brittleness of the piezoelectric-ceramics and reduce the stress concentration in layer, functionally graded piezoelectric materials (FGPMs) are introduced which are a promising solution to improve the durability of piezoelectric devices [2]-[4]. Functionally graded materials are a new class of composites that avoid the stress concentrations. As investigated in [5]-[7] ceramic-polymer composites are new class of materials which have the high electromechanical coupling coefficients while having flexibility, low density and high strength. Following [8] which studied the free vibration analysis of functionally graded PEH with magnetic interaction, the objective of the present study is to investigate FGPEH with magnetic tip mass under harmonic base excitation.

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## II. Theoretical Formulation

Fig. 1 shows a 3D view of the bimorph functionally graded piezomagnetic energy harvester (FGPMEH) with a magnetic tip mass attached at the end which is in interaction with a permanent magnet fixed above. The beam consists of two FGPM layers of thickness $h_p$ and a substructure with thickness of $h_s$. Following [9] the effective material properties in each piezoelectric layer can be found as:

$$P(z) = P_l v_l + P_u v_u \tag{1}$$

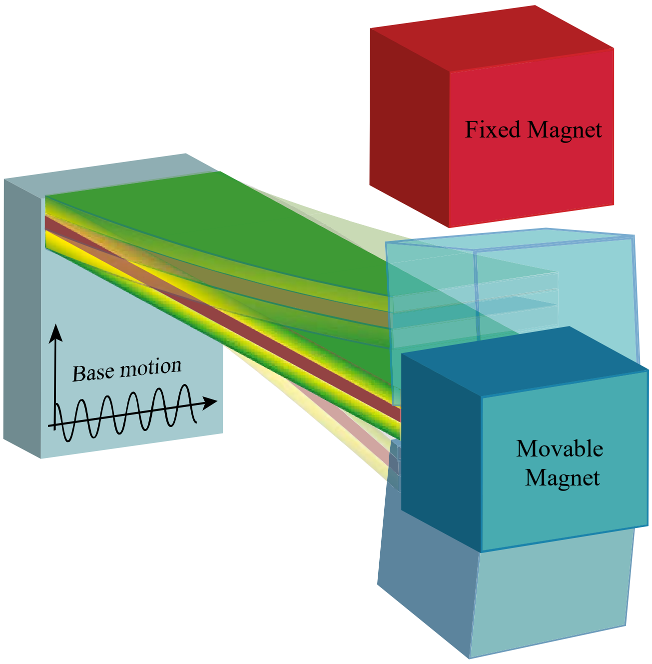


Fig. 1 Schematic view of the FGPMEH with magnetic tip mass

where $P$ represents specific properties of the piezoelectric material and $v$ is the volume fraction of the constituents material. Also, the subscripts $l$ and $u$, respectively, indicate the lower and upper surface of the piezoelectric layers. For each of the two piezoelectric layers, these are related to each other by $v_l + v_u = 1$. For the lower volume fraction we have:

$$
\begin{aligned}
v_l(z) &= \left(1+\frac{-2z+h_s}{2h_p}\right)^k \quad \text{for} \quad z\in\left[h_s/2, h_p+h_s/2\right] \\
v_l(z) &= \left(1+\frac{2z+h_s}{2h_p}\right)^k \quad \text{for} \quad z\in\left[-h_p-h_s/2, -h_s/2\right]
\end{aligned} \tag{2}
$$

where $z$ is a coordinate along the thickness of the beam as shown in Fig. 1 and $k$ is grading index. Fig. 2 shows the side view and the thickness of each layer of piezoelectric energy harvester.

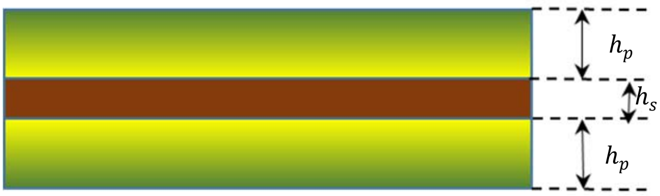


Fig. 2 Right view of the Functionally graded piezoelectric energy harvester beam

The constitutive law for piezoelectric material with electrical displacement, $\hat{D}_z$, the strain, $\hat{S}_x$, and the electric field, $\hat{E}_z$ is [1]:

$$
\begin{aligned}
\hat{\sigma}_x &= Y_{xx}\hat{S}_x - e_{31}\hat{E}_z \\
\hat{D}_z &= e_{31}\hat{S}_x + \varepsilon_{33}\hat{E}_z
\end{aligned} \tag{3}
$$

in which $Y_{xx}$ denotes the elastic modulus of elasticity, $e_{31}$ represents the coupling coefficient of piezoelectric and $\varepsilon_{33}$ is the constant of dielectric [8]. For the case in which functionally graded piezoelectric energy harvester excited from the base, the coupled electro-mechanical equations become

$$
\frac{\partial \hat{N}_{xx}}{\partial \hat{x}} = 0 \tag{4}
$$

$$
\begin{aligned}
&\frac{\partial}{\partial \hat{x}}(\hat{N}_{xx}\frac{\partial \hat{w}}{\partial \hat{x}}) + \frac{\partial^2 \hat{M}_{xx}}{\partial \hat{x}^2} - I_0(\frac{\partial^2 \hat{w}}{\partial \hat{t}^2} + \frac{\partial^2 \hat{y}_b}{\partial \hat{t}^2}) + I_2\frac{\partial^4 \hat{w}}{\partial \hat{x}^2 \partial \hat{t}^2} \\
&\quad - M_t\delta(\hat{x}-L)\frac{\partial^2 \hat{y}_b}{\partial \hat{t}^2} + F_{mag}\delta(\hat{x}-L) = 0
\end{aligned} \tag{5}
$$

$$
c_p\frac{d\hat{V}(\hat{t})}{d\hat{t}} + \chi\frac{\partial}{\partial \hat{t}}\int_0^L \frac{\partial^2 \hat{w}}{\partial \hat{x}^2}d\hat{x} + \frac{\hat{V}(\hat{t})}{R} = 0 \tag{6}
$$

where $I_0$ and $I_2$ are the mass and moment of the beam per length, $M_t$ and $I_t$ are mass and moment of the tip mass, $c_p$ is the capacitance, $\chi$ is the coupling and $R$ is the resistance of the circuit. The stress resultants are defined as:

$$
\begin{aligned}
\hat{N}_{xx} &= \int_A \hat{\sigma}_x dA = \int_A \left(Y_{xx}^E\hat{S}_x - e_{31}\hat{E}_z\right)dA \\
\hat{M}_{xx} &= \int_A \hat{\sigma}_x z dA = \int_A \left(Y_{xx}^E\hat{S}_x - e_{31}\hat{E}_z\right)z\,dA
\end{aligned} \tag{7}
$$

where $z$ is the coordinate in thickness direction and $F_{mag}$ is the magnetic force which is defined by [10]

$$
F_{mag}(t) = -\frac{3\mu_0 m_1 m_2}{2\pi\left(d_0 - w(t)\right)^4} \tag{8}
$$

where $d_0$ is the spacing distance between two magnets, $m_1$ and $m_2$ are the magnetic dipole moments and $\mu_0$ is the vacuum permeability. For attractive interaction $m_1 = -m_2$ and for repulsive interaction $m_1 = -m_2$. It has been shown that the attractive interaction between two magnets is more beneficial than the repulsive interaction [1]. Therefore, we continued our study with attractive interaction and $m_1 = 0.0192\ A/m^2$. For further calculations we can expand magnetic force in a Taylor series expansion about $w = 0$.

In order to convert partial differential equation of motion to a set of ordinary differential equation, Galerkin procedure has been applied. To do so, $\varphi(x)$ is mode shape of the energy harvester beam obtained by solving eigenvalue problem. Upon applying Galerkin method, one can transform the set of partial differential equations to sets of ordinary differential equations which are straightforward for obtaining the final expressions.

## III. Solution Procedure

A closed form expression is derived for the forced vibration analysis of the harvester under the base excitation based on the method of multiple time scales (MTS) of perturbation technique [11]. In this method, it is considered that the generalized coordinates are the function of some new time scales $T_i$ and $\varepsilon$ is the small positive nondimensional being introduced as book-keeping device. The first and the second time derivatives can then be expressed in terms of $\partial/\partial T_i$ as:

$$
\begin{aligned}
\frac{d}{dt}() &= \frac{\partial}{\partial T_0}() + \varepsilon^2\frac{\partial}{\partial T_2}() \\
\frac{d^2}{dt^2}() &= \frac{\partial^2}{\partial T_0^2}() + 2\varepsilon^2\frac{\partial^2}{\partial T_0 \partial T_2}() + \varepsilon^4\frac{\partial^2}{\partial T_2^2}()
\end{aligned} \tag{9}
$$

where $T_i$ are new time scales.

$$
\begin{aligned}
u_{tip}(T_0,T_2) &= \varepsilon^2\left(u_{tip}^{(0)}(T_0,T_2) + \varepsilon^2 u_{tip}^{(2)}(T_0,T_2)\right) \\
w_{tip}(T_0,T_2) &= \varepsilon\left(w_{tip}^{(0)}(T_0,T_2) + \varepsilon^2 w_{tip}^{(2)}(T_0,T_2)\right) \\
\mathbb{V}(T_0,T_2) &= \varepsilon\left(\mathbb{V}_0(T_0,T_2) + \varepsilon^2\mathbb{V}_2(T_0,T_2)\right)
\end{aligned} \tag{10}
$$

Employing (9), (10) and regarding small terms of damping, piezoelectric coupling coefficient and magnetic constant and the excitation frequency $\Omega = \omega_q + \varepsilon^2\sigma$ where $\sigma$ is detuning parameter[11], performing some mathematical manipulation, and looking for possible terms having high value coefficient, which are called secular terms [11], we reach to:

$$
\begin{aligned}
&-\frac{IB(T_2)\omega_q d_5 g_2}{I\omega_q + g_1} + IB(T_2)\omega_q d_1 + \frac{11B(T_2)^2 d_4 \overline{B}(T_2)\omega_q^2 F_2}{(-\omega_q^2 + F_1)(-4\omega_q^2 + F_1)} \\
&+\frac{13B(T_2)d_4\omega_q^2 d_8^2 F_2}{(-\omega_q^2 + F_1)(-4\omega_q^2 + F_1)} - \frac{3B(T_2)d_4 d_8^2 F_1 F_2}{(-\omega_q^2 + F_1)(-4\omega_q^2 + F_1)} \\
&-\frac{4B(T_2)d_4\omega_q^4 d_8^2 F_2}{(-\omega_q^2 + F_1)F_1(-4\omega_q^2 + F_1)} - \frac{8B(T_2)^2 d_4 \overline{B}(T_2)\omega_q^4 F_2}{(-\omega_q^2 + F_1)F_1(-4\omega_q^2 + F_1)} \\
&+2I\omega_q\left(\frac{\mathrm{d}}{\mathrm{d}T_2} B(T_2)\right) + 3B(T_2)^2 \overline{B}(T_2) d_3 \\
&-\frac{3B(T_2)^2 d_4 \overline{B}(T_2) F_1 F_2}{(-\omega_q^2 + F_1)(-4\omega_q^2 + F_1)} + 3B(T_2) d_8^2 d_3 + \frac{d_6 Z \mathrm{e}^{I\sigma T_2}}{2} = 0
\end{aligned}
\tag{11}
$$

where $I$ denotes the imaginary symbol and $B(T_2)$ can be written in the polar format as in (12):

$$
B(T_2) = \frac{1}{2} b(T_2) \mathrm{e}^{I\beta(T_2)} \tag{12}
$$

Next, by eliminating secular terms in **Hata! Başvuru kaynağı bulunamadı.** and performing mathematical manipulation, ordinary differential equations for $b(T_2)$ and $\beta(T_2)$ can be obtained as (13) and (14):

$$
\frac{\mathrm{d}}{\mathrm{d}T_2} b(T_2) = -\frac{1}{2} b(T_2)\left(d_1 - \frac{d_5 g_1 g_2}{g_1^2 + \omega_q^2}\right) \tag{13}
$$

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}T_2}\beta(T_2) \\
&= \frac{1}{8}\frac{1}{F_1\omega_q\left(4g_1^2\omega_q^4 + 4\omega_q^6 - 5F_1 g_1^2\omega_q^2 - 5F_1\omega_q^4 + F_1^2 g_1^2 + F_1^2\omega_q^2\right)} \\
&\times\Big(48\omega_q^6 d_8^2 F_1 d_3 - 60\omega_q^4 d_8^2 F_1^2 d_3 + 12\omega_q^2 d_8^2 F_1^3 d_3 + 3\omega_q^2 b(T_2)^2 F_1^3 d_3 \\
&-8\omega_q^6 b(T_2)^2 d_4 F_2 + 12\omega_q^6 b(T_2)^2 F_1 d_3 - 15\omega_q^4 b(T_2)^2 F_1^2 d_3 \\
&+3b(T_2)^2 F_1^3 d_3 g_1^2 - 16\omega_q^6 F_1 d_5 g_2 + 20\omega_q^4 F_1^2 d_5 g_2 - 4\omega_q^2 F_1^3 d_5 g_2 \\
&-16\omega_q^6 d_8^2 d_4 F_2 + 12 d_8^2 F_1^3 d_3 g_1^2 - 16 g_1^2 d_8^2 \omega_q^4 d_4 F_2 + 52\omega_q^4 d_8^2 d_4 F_2 F_1 \\
&-3\omega_q^2 b(T_2)^2 d_4 F_2 F_1^2 + 12 b(T_2)^2 F_1 d_3 g_1^2 \omega_q^4 - 15 b(T_2)^2 F_1^2 d_3 g_1^2 \omega_q^2 \\
&-12\omega_q^2 d_8^2 d_4 F_2 F_1^2 + 11\omega_q^4 b(T_2)^2 d_4 F_2 F_1 - 8 g_1^2 b(T_2)^2 \omega_q^4 d_4 F_2 \\
&+48 d_8^2 F_1 d_3 g_1^2 \omega_q^4 - 60 d_8^2 F_1^2 d_3 g_1^2 \omega_q^2 - 3 g_1^2 b(T_2)^2 d_4 F_2 F_1^2 \\
&-12 g_1^2 d_8^2 d_4 F_2 F_1^2 + 52 g_1^2 d_8^2 \omega_q^2 d_4 F_2 F_1 + 11 g_1^2 b(T_2)^2 \omega_q^2 d_4 F_2 F_1\Big)
\end{aligned}
\tag{14}
$$

Upon solving the differential equations, $B(T_2)$, which correspond to the transversal displacement of the functionally graded MEMS, piezoelectric energy harvester can be obtained. Further, assuming steady state solution, we can reach to the response in frequency domain. The mechanical responses include transversal displacement, while the electrical response includes output voltage. Having frequency response for the transversal displacement in hand, one can easily obtain the electrical response in frequency domain by electrical coupling which exist in the piezoelectric energy harvetser. Since the steps and resulted expressions are bulky, we avoid to present them in this paper.

## IV. Results

In order to graphically illustrate the analytical results, For the case under study, geometrical properties of the functionally graded MEMS piezoelectric energy harvester given in Table I are employed. Moreover, mechanical and electrical properties of the constituent materials PZT-5H and PVDF for piezoelectric layers and brass for the substructure in the sandwich beam of the functionally graded piezoelectric are given in Table II.

TABLE I
Geometrical Properties of Bimorph FGPMEH

| | FGPM Layer | Substructure |
|---|---|---|
| Length (μm) | 24e3 | 24e3 |
| Thickness (μm) | 265 | 140 |
| Width (μm) | 6.4e3 | 6.4e3 |

TABLE II
Mechanical and Electrical Properties of Bimorph FGPMEH

| | PZT-5H | PVDF | Brass |
|---|---|---|---|
| Density (kg/m$^3$) | 7500 | 1780 | 9000 |
| Elastic modulus (GPa) | 60.6 | 3 | 105 |
| Piezoelectric constant (C/m$^2$) | -16.1 | 0.072 | - |
| Permittivity constant (nF/m) | 25.55 | 7.97 | - |

The effect of magnetic force on frequency response function (FRF) of the outputs has been investigated by setting different values of the dimensionless parameter of initial gap between two magnets, $d_0^*$. Fig. 3 illustrates the effect of magnetic force on the electrical response of the functionally graded piezoelectric energy harvester. As it can be seen from Fig. 3, by decreasing initial gap between two magnets, or in other words increasing magnetic force, the system expresses softening effect, by which the response curve leans to the left. Also, by increasing the magnetic force, the operatory frequency range of the harvester covers more frequency range [10]. Therefore, the influence of the magnetic force on broadening the energy harvester operatory bandwidth is obvious.

Fig. 4 shows the effect of the base acceleration amplitude as an input on the output voltage response for power law index $n = 0.2$ and fixed value of magnetic force. As it is clear from this figure, by increasing the amplitude of the input acceleration, amplitude of the electrical response will increase. Moreover the dashed lines in Fig. 4 indicate the jump phenomenon, which happens in the nonlinear systems [11].

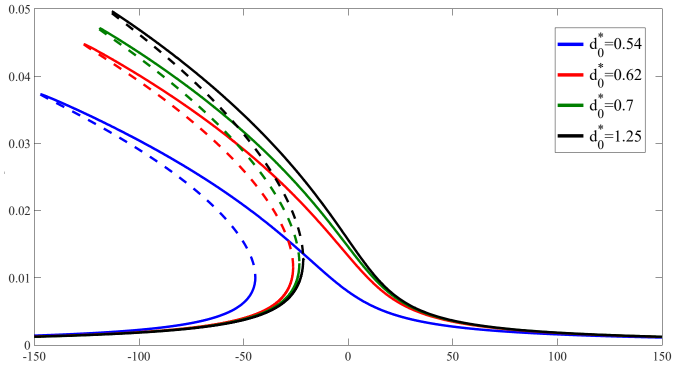


Fig. 3 Variation of voltage response of the FGPMEH beam for different values of dimensionless initial gap of the magnets for grading index n=0.2

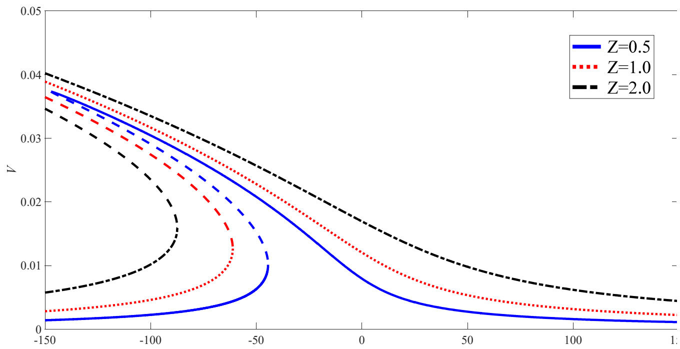


Fig. 4 Variation of amplitude of the base acceleration on frequency response of generated voltage for grading index $n = 0.2$ and $d_0^* = 0.54$

## V. Conclusion

In this paper, functionally graded piezoelectric energy harvester with tip mass attached at the free end with magnetic force is studied. It is observed from the results that the dimensionless responses of the bimorph functionally graded piezoelectric energy harvester can be influenced notably by variation of the spacing distance between two magnets. Furthermore, by increasing the input acceleration exerted to the base, electrical response of the energy harvester increases as well.

The obtained results can direct engineers for production of electromechanical structures in technical applications. This analysis showed that magnetoelastic structure with FGPM can be employed to properly choose design parameter like spacing distance of two magnets to improve the output responses.

## References

[1] A. Erturk, J. Hoffmann, and D. J. Inman, “A piezomagnetoelastic structure for broadband vibration energy harvesting,” *Appl. Phys. Lett.*, vol. 94, no. 25, 2009.

[2] A. Doroushi, M. R. Eslami, and A. Komeili, “Vibration analysis and transient response of an FGPM beam under thermo-electro-mechanical loads using higher-order shear deformation theory,” *J. Intell. Mater. Syst. Struct.*, vol. 22, no. 3, pp. 231–243, 2011.

[3] M. Komijani, J. N. Reddy, and M. R. Eslami, “Nonlinear analysis of microstructure-dependent functionally graded piezoelectric material actuators,” *J. Mech. Phys. Solids*, vol. 63, no. 1, pp. 214–227, 2014.

[4] D. J. Huang, H. J. Ding, and W. Q. Chen, “Piezoelasticity solutions for functionally graded piezoelectric beams,” *Smart Mater. Struct.*, vol. 16, no. 3, pp. 687–695, 2007.

[5] J. J. Choi, B. D. Hahn, J. Ryu, W. H. Yoon, B. K. Lee, and D. S. Park, “Preparation and characterization of piezoelectric ceramic-polymer composite thick films by aerosol deposition for sensor application,” *Sensors Actuators, A Phys.*, vol. 153, no. 1, pp. 89–95, 2009.

[6] M. Dietze and M. Es-Souni, “Structural and functional properties of screen-printed PZT-PVDF-TrFE composites,” *Sensors Actuators, A Phys.*, vol. 143, no. 2, pp. 329–334, 2008.

[7] A. Kumar, A. Sharma, R. Vaish, R. Kumar, and S. C. Jain, “A numerical study on anomalous behavior of piezoelectric response in functionally graded materials,” *J. Mater. Sci.*, vol. 53, no. 4, 2018.

[8] M. Derayatifar and M. Tahani, “Nonlinear Free Vibration of Functionally Graded Piezomagnetoelastic Energy Harvester,” in *The Twenty-Fifth Annual International Conference on Composites/Nano Engineering (ICCE-25)*, 2017.

[9] H. T. Thai, T. P. Vo, T. K. Nguyen, and J. Lee, “Size-dependent behavior of functionally graded sandwich microbeams based on the modified couple stress theory,” *Compos. Struct.*, vol. 123, pp. 337–349, 2015.

[10] A. Abdelkefi and N. Barsallo, “Comparative modeling of low-frequency piezomagnetoelastic energy harvesters,” *J. Intell. Mater. Syst. Struct.*, vol. 25, no. 14, pp. 1771–1785, 2014.

[11] A. H. Nayfeh, *Introduction to Perturbation Techniques (Ali Hasan Nayfeh)*, vol. 24, no. 3. 1982.