World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:11, No:6, 2017

Multivariable System Reduction Using Stability Equation Method and SRAM

D. Bala Bhaskar

***Abstract*—**An algorithm is proposed for the order reduction of large scale linear dynamic multi variable systems where the reduced order model denominator is obtained by using Stability equation method and numerator coefficients are obtained by using SRAM. The proposed algorithm produces a lower order model for an original stable high order multivariable system. The reduction procedure is easy to understand, efficient and computer oriented. To highlight the advantages of the approach, the algorithm is illustrated with the help of a numerical example and the results are compared with the other existing techniques in literature.

***Keywords*—**Multi variable systems, order reduction, stability equation method, SRAM, time domain characteristics, ISE.

I. Introduction

IN general most of the physical systems are complex and their transfer function representations are of very high orders. The analysis, control and design of those high order models are tedious and difficult. So the analysis and design of such systems is often carried out by using a low order model which retains the dominant characteristics of the original high order model.

In literature, a number of methods are available for order-reduction of linear time invariant continuous systems in time domain as well as in frequency domain [1]-[14]. Further, the extension of single-input single-output (SISO) methods to reduce multi-input multi-output (MIMO) systems has also been carried out in [15]-[30]. It is established in literature that, some proposed methods like Pade approximation method [1], continued fraction expansion method [2], Markov parameter matching method by Jonckheere [3], etc. may generate unstable reduced order models for a stable original higher order model. Stability guarantee methods like Routh approximation method [4], Routh-Pade approximation method [5] etc., are proposed; these will generate stable lower order reduced models for stable high order original models. Routh approximation method has limitations like formulation of two separate Routh tables for obtaining numerator and denominator polynomials of reduced order models. Some mixed methods like [15]-[18], [23]-[25] and [28]-[30] etc. are proposed for order reduction of higher order multivariable systems. All the proposed methods have their unique pro's and con's. In this paper the author's propped a mixed method for order reduction of SISO and multi variable systems. The method is discussed as follows: Section II includes problem statement, and proposed method. A numerical example is presented in Section III; results and ISE are in Section IV and conclusion is given in Section V.

D Bala Bhaskar, Assistant Professor, is with Electrical Engineering Department, Gayatri Vidya Parishad College of Engineering (Autonomous), Affiliated to JNTU-Kakinada, Andhra Pradesh, India (e-mail: dbbhaskar@gvpce.ac.in).

II. Proposed Reduction Procedure

Let us consider the general transfer function of a continuous time invariant system of order 'n' be defined as

$$G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{n-1} B_i s^i}{\sum_{i=0}^{n} A_i s^i} \quad (1)$$

where $B_i (o \le i \le n-1)$ and $A_i (o \le i \le n)$ are scalar quantities. Let the corresponding kth (k<n) order reduced model is of the form

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{\sum_{i=0}^{k-1} b_i s^i}{\sum_{i=0}^{k} a_i s^i} \quad (2)$$

where $b_i (o \le i \le k-1)$ and $a_i (o \le i \le k)$ are scalar quantities.

Denominator of Reduced Order Model: Stability Equation Method

For stable original system, G(s), its denominator is decomposed into even and odd components in the form of stability equations as

$$D_e(s) = \sum_{i=0,2,4..}^{n} A_i s^i = A_0 \prod_{i=1}^{m_1} \left(1 + \frac{s^2}{z_i^2}\right) \quad (3)$$

$$D_o(s) = \sum_{i=1,3,5..}^{m_2} A_1 s = A_0 \prod_{i=1}^{m_2} \left(1 + \frac{s^2}{p_i^2}\right) \quad (4)$$

where m_1 and m_2 are the integer parts of $\frac{n}{2}$ and $(\frac{n-1}{2})$ respectively and $z_1^2 < p_1^2 < z_2^2 < p_2^2 ......$

As the effect of poles and zeros placed far away from origin will be less so discard the factors with large magnitudes of z_i^2 and p_i^2 in (3) and (4), then the stability equations of k^{th} order reduced models are

$$D_e(s) = A_0 \prod_{i=1}^{m_3} (1+\frac{s^2}{z_i^2}) \tag{5}$$

$$D_o(s) = A_1 s \prod_{i=1}^{m_4} (1+\frac{s^2}{p_i^2}) \tag{6}$$

where m_1 and m_2 are the integer parts of $\frac{k}{2}$ and $(\frac{k-1}{2})$ respectively Combining these reduced stability equations and therefore proper normalizing it, the k^{th} order denominator $D_k(s)$ of reduced model is

$$D_k(s) = D_e^k(s) + D_o^k(s) = \sum_{i=0}^{k-1} a_i s^i + s^k \tag{7}$$

Determination of the Numerator of the Reduced Model: SRAM

After obtaining the reduced denominator $D_k(s)$, the numerator of the biased model, which will retain the first 't' time moments and 'm' Markov parameters is found as:

$$\begin{aligned} N_k(s) &= N_{kt}(s) + N_{km}(s) \text{ with } k = t + m \\ &= T_1 + T_2 s + T_3 s^2 + ... + T_t s^{k-m+1} + \\ &\quad M_m s^{k-m} + ... + M_2 s^{k-2} + M_1 s^{k-1} \end{aligned} \tag{8}$$

in general

$$T_t = \frac{a_0}{A_0} B_{t-1} \tag{9}$$

and

$$M_m = \frac{1}{A_n} \begin{Bmatrix} \sum_{i=0}^{m} [B_{n-i} a_{k-(m-i)}] - \\ \sum_{j=0}^{m-1} [M_j A_{n-(m-j)}] \end{Bmatrix} \tag{10}$$

with M_0=0. Now finally, the k^{th} order Reduced model is given by

$$R_k(s) = \frac{N_k(s)}{D_k(s)}$$

$$= \frac{T_1 + T_2 s + T_3 s^2 + ... + T_t s^{k-m+1} + M_m s^{k-m} + ... M_2 s^{k-2} + M_1 s^{k-1}.}{D_k(s)} \tag{10}$$

Extension to Multi Variable Systems

Let the transfer function matrix of original n^{th} order system having 'p' inputs and 'q' outputs as

$$[G(s)] = \frac{1}{D_n(s)} = \begin{bmatrix} a_{11}(s) & a_{12}(s) & a_{13}(s) & ... a_{1p}(s) \\ a_{21}(s) & a_{22}(s) & a_{23}(s) & .... a_{2p}(s) \\ : & : & : & .... \; : \\ a_{q1}(s) & a_{q2}(s) & a_{q3}(s) & ... a_{qp}(s) \end{bmatrix}$$

$$[G(s)] = [g_{ij}(s)] = \frac{a_{ij}(s)}{D_n(s)} = \frac{B_0 + B_1 s + ... B_{n-1} s^{n-1}}{A_0 + A_1 s + ... A_n s^n}$$

where i=1,2,…q and j=1,2…..p

Let the transfer function matrix of reduced k^{th} order system having 'p' inputs and 'q' outputs as

$$[R(s)] = \frac{1}{D_k(s)} = \begin{bmatrix} b_{11}(s) & b_{12}(s) & b_{13}(s) & ... b_{1p}(s) \\ b_{21}(s) & b_{22}(s) & b_{23}(s) & .... b_{2p}(s) \\ : & : & : & .... \; : \\ b_{q1}(s) & b_{q2}(s) & b_{q3}(s) & ... b_{qp}(s) \end{bmatrix}$$

$$[R(s)] = [r_{ij}(s)] = \frac{b_{ij}(s)}{D_k(s)} = \frac{b_0 + b_1 s + ... b_{n-1} s^{n-1}}{a_0 + a_1 s + ... a_n s^n}$$

where i=1,2,…q and j=1,2…..p

III. Numerical Example

To ascertain the flexibility and effectiveness of the proposed method, the following example is considered.

Consider the 4^{th} order system transfer function given by [12], [13].

$$G(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}$$

It is proposed to obtain a second order model for the given original high order system using the proposed reduction method

By decomposing the denominator into even and odd parts, the stability equations are

$$D_e(s) = s^4 + 35s^2 + 24 = 24\left(1 + \frac{s^2}{0.6997}\right)\left(1 + \frac{s^2}{34.3004}\right),$$

$$D_o(s) = 10s^3 + 50s = 50s\left(1 + \frac{s^2}{5}\right)$$

Now by discarding the factors with large magnitudes of z_i^2 and p_i^2 in $D_e(s)$ and $D_o(s)$ then the stability equations of second order reduced model is

$$D_e^2(s) = 24\left(1 + \frac{s^2}{0.6997}\right) \tag{12}$$

$$D_o^2(s) = 50s \tag{13}$$

Combining and normalizing (12), (13) and from (7) the reduced denominator is

$$D_2(s) = s^2 + 1.45771s + 0.6997$$

The second order reduced model numerator using SRAM which retain 't' time moments from G(s) where t+m=2, are given by:

For t=2, m=0 $\quad N_2^1(s) = 0.6997s + 0.6997$

Then the reduced 2nd order model by using proposed method is obtained as

For t=2, m=0 $\quad R_2^1(s) = \dfrac{0.6997s + 0.6997}{s^2 + 1.45771s + 0.6997}$

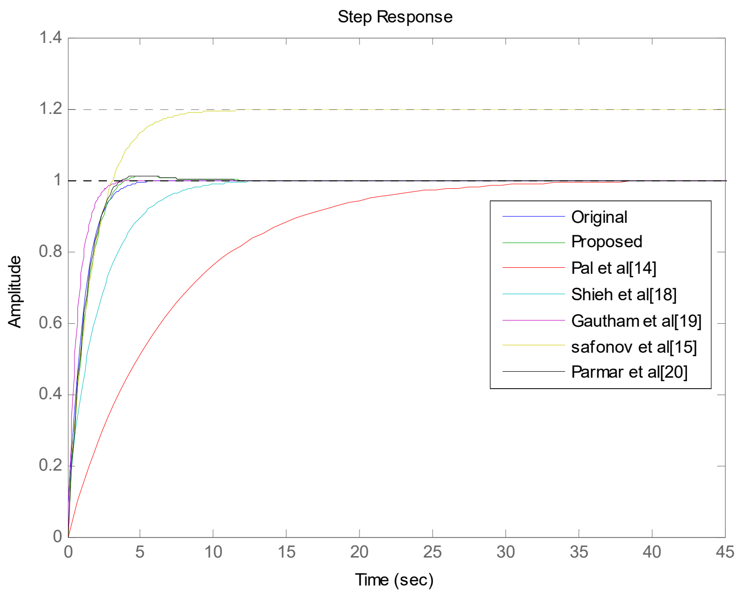

Fig. 1 Comparison of Step Responses of Original, Proposed and Other Existing Reduced Models of SISO System

TABLE I
COMPARISON OF TIME DOMAIN SPECIFICATIONS, INTEGRAL SQUARE ERROR (ISE) OF SISO SYSTEM

System/Method	t_r (sec)	t_s (sec)	ISE
Original	2.26	3.93	
Proposed	2.3	3.41	0.00458
Safonov et al. [22]	3.8	8.84	0.045161
Shieh et al. [25]	4.95	6.75	0.14256
Gautam et al. [26]	1.54	2.73	0.045593
Pal et al. [21]	15.4	27.4	1.5342
Parmar et al. [27]	2.19	3.23	0.00164

The proposed method is also applied for multi variable systems by taking some examples from the literature:

Numerical Example

Consider a two input two output system is given by transfer function matrix [10]-[17]

$$[G(s)] = \begin{bmatrix} \dfrac{2(s+5)}{(s+1)(s+10)} & \dfrac{(s+4)}{(s+2)(s+5)} \\ \dfrac{(s+10)}{(s+1)(s+20)} & \dfrac{(s+6)}{(s+2)(s+3)} \end{bmatrix} = \frac{1}{D(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix}$$

The common denominator D(s) is

$$D(s) = (s+1)(s+10)(s+2)(s+5)(s+3)(s+20)$$

$$= s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000$$

and $a_{11}(s) = 2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000$

$$a_{12}(s) = s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400$$

$$a_{21}(s) = s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000$$

$$a_{22}(s) = s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000$$

The proposed algorithm is successively applied to the given multivariable system and the reduced order models $r_{ij}(s)$ are obtained as

$$[R(s)] = \frac{1}{D_2(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) \\ b_{21}(s) & b_{22}(s) \end{bmatrix}$$

where $D_2(s) = s^2 + 1.34952s + 0.6181$ and $b_{11}(s) = 0.79323s + 0.6181$, $b_{12}(s) = 0.42855s + 0.24724$, $b_{21}(s) = 0.38116s + 0.3091$, $b_{22}(s) = 0.93745s + 0.6181$ for all t=2 and m=0.

IV. Results

A. Comparison of Step Responses, Time Domain Characteristics and Integral Square Error

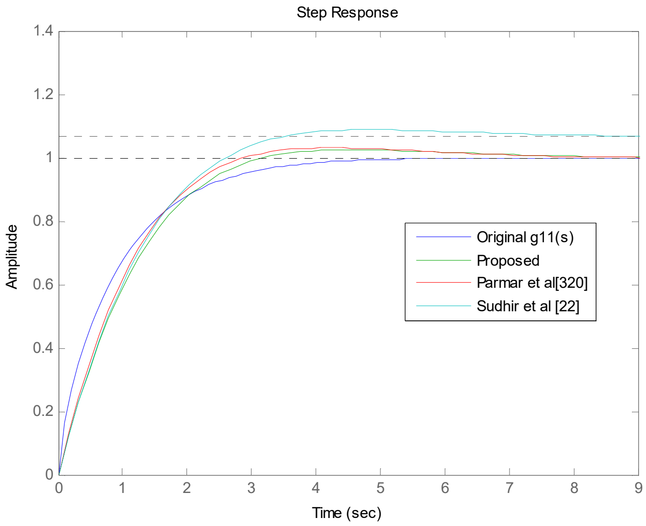

Fig. 2 Comparison of Step Responses $G_{11}(s)$, Proposed Reduced and Other Existing Methods

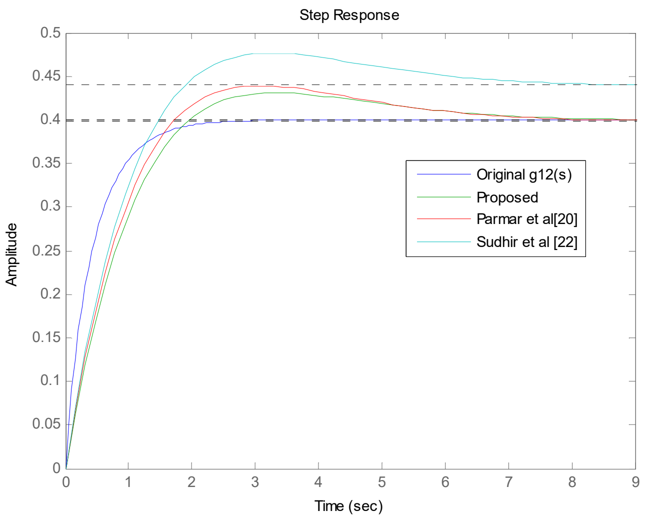

Fig. 3 Comparison of Step Responses $G_{12}(s)$, Proposed Reduced and Other Existing Reduced Models

TABLE II
COMPARISON OF TIME DOMAIN SPECIFICATIONS, ISE OF $G_{11}(S)$, PROPOSED AND OTHER EXISTING METHODS

System/Method	**t_r (sec)**	**t_s (sec)**	**ISE**
Original $G_{11}(s)$	2.12	3.8	
Proposed	2.02	5.56	0.01248
Sudhir et al. [22]	2.16	4.91	0.011517
Parmar et al. [20]	1.86	5.82	0.014498

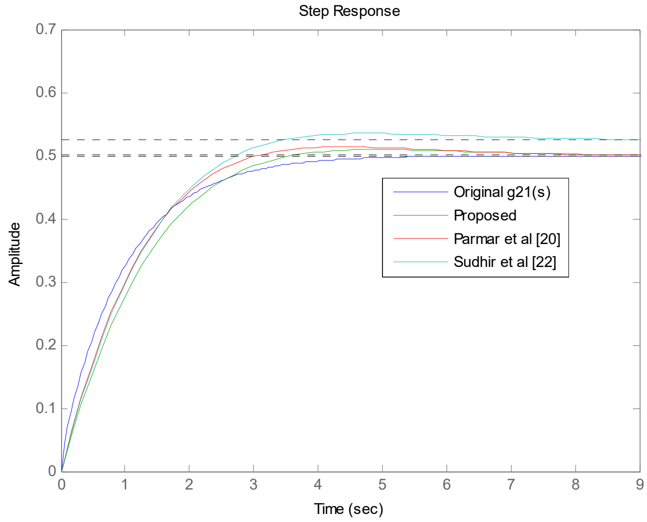

Fig. 4 Comparison of Step Responses of $G_{21}(s)$, Proposed Reduced and Other Existing Methods

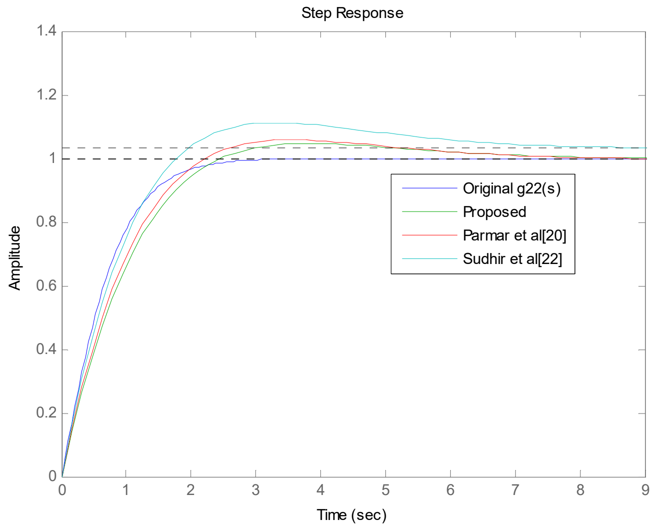

Fig. 5 Comparison of Step Responses of $G_{22}(s)$, Proposed Reduced and Other Existing Methods

TABLE III
COMPARISON OF TIME DOMAIN SPECIFICATIONS, ISE OF $G_{12}(S)$, PROPOSED AND OTHER EXISTING METHODS

System/Method	**t_r (sec)**	**t_s (sec)**	**ISE**
Original $G_{12}(s)$	1.02	1.87	
Proposed	1.39	6.28	0.008129
Sudhir et al. [22]	1.34	6.31	0.007521
Parmar et al. [20]	1.24	6.39	0.008744

TABLE IV
COMPARISON OF TIME DOMAIN SPECIFICATIONS, ISE OF $G_{21}(S)$, PROPOSED AND OTHER EXISTING METHODS

System/Method	**t_r (sec)**	**t_s (sec)**	**ISE**
Original $G_{21}(s)$	2.18	3.86	
Proposed	2.21	3.18	0.002098
Sudhir et al. [22]	2.14	5.11	0.002106
Parmar et al. [20]	1.94	5.69	0.002538

TABLE V
COMPARISON OF TIME DOMAIN SPECIFICATIONS, ISE OF $G_{22}(S)$, PROPOSED AND OTHER EXISTING METHODS

System/Method	t_r (sec)	t_s (sec)	ISE
Original $G_{22}(s)$	1.34	2.28	
Proposed	1.66	6.05	0.016569
Sudhir et al. [22]	1.39	6.28	0.017903
Parmar et al. [20]	1.53	6.17	0.015741

V. CONCLUSIONS

The proposed algorithm combines the advantages of the Stability equation method and the SRAM to generate stable reduced order models for linear time invariant dynamic systems. The poles are determined by the stability equation method and the zeros are by simplified Routh approximation method by matching first 't' time moments. The algorithm has also been extended for the order reduction of linear multivariable systems. The proposed algorithm is simple, computer oriented and approximates the time domain specifications of original system compared to other existing methods in the literature, and the proposed algorithm improves steady state performance of the system. A numerical example was illustrated and compared with other existing methods in literature.

REFERENCES

[1] Y. Shamash (1974) Stable reduced order models using Pade type approximations, IEEE trans. On Automatic Control, Vol. 19, pp.615-616
[2] Y. Shamash (1973) Continued fraction methods for the reduction of Linear time invariant systems. In Proc.Conf. Computer Aided Control system Design, Cambridge, England, pp.220-227.
[3] E. Jonckheere and C. Ma (1989) Combined sequence of Markov parameter and moments in linear systems. IEEE Trans. Aufomuf. Confr.; vol. AC-34,pp.379-382.
[4] M. F. Hutton, B. Friendland (1999) Routh approximations for reducing order of linear time-varying systems. IEEE Trans. Automat. Control, vol.44, No.9, pp. 1782-1787
[5] Y. Shamash (1975) Model reduction using the Routh stability criterion and the Pade approximation technique. Int. J. Control, 21, pp. 475-484
[6] Hwang C (1984) Mixed method of Routh and ISE criterion approaches for reduced order modeling of continuous time systems. Trans ASME J Dyn Syst Meas Control 106: pp. 353-356
[7] B. Bandyopadhyay, O. Ismail, and R. Gorez (1994) Routh Pade approximation for interval systems. IEEE Trans. Automat. Contr., pp. 2454-2456.
[8] R. Genesio and M. Milanese, "A note on the derivation and use of reduced order models", IEEE Trans. Automat. Control, Vol. AC-21, No. 1, pp. 118-122, February 1976.
[9] M. S. Mahmoud and M. G. Singh, Large Scale Systems Modelling, Pergamon Press, International Series on Systems and Control 1st ed.,Vol. 3, 1981.
[10] M. Jamshidi, Large Scale Systems Modelling and Control Series, New York, Amsterdam, Oxford, North Holland, Vol. 9, 1983.
[11] S. K. Nagar and S. K. Singh, "An algorithmic approach for system decomposition and balanced realized model reduction", Journal of Franklin Inst., Vol. 341, pp. 615-630, 2004.
[12] V. Singh, D. Chandra and H. Kar, "Improved Routh Pade approximants: A computer aided approach", IEEE Trans. Automat. Control, Vol. 49, No.2, pp 292-296, February 2004.
[13] S. Mukherjee, Satakshi and R.C.Mittal, "Model order reduction using response-matching technique", Journal of Franklin Inst., Vol. 342, pp.503-519, 2005.
[14] B. Salimbahrami, and B. Lohmann, "Order reduction of large scale second-order systems using Krylov subspace methods", Linear Algebra Appl., Vol.415, pp.385-405, 2006.
[15] S. Mukherjee and R.N. Mishra, "Reduced order modelling of linear multivariable systems using an error minimization technique", Journal of Franklin Inst., Vol.325, No. 2, pp. 235-245, 1988.
[16] S. S. Lamba, R. Gorez and B. Bandyopadhyay, "New reduction technique by step error minimization for multivariable systems", Int. J. Systems Sci., Vol. 19,No. 6, pp. 999-1009, 1988.
[17] R. Prasad and J. Pal, "Use of continued fraction expansion for stable reduction of linear multivariable systems", Journal of Institution of Engineers, India,IE(I) Journal – EL, Vol. 72, pp. 43-47, June 1991.
[18] R. Prasad, A. K. Mittal and S. P. Sharma, "A mixed method for the reduction of multi-variable systems", Journal of Institution of Engineers, India, IE(I) Journal– EL, Vol. 85, pp. 177-181, March 2005.
[19] S. Mukherjee, and R.N. Mishra, "Order reduction of linear systems using an error minimization technique", Journal of Franklin Inst., Vol. 323,No. 1,pp. 23-32, 1987.
[20] A.K. Mittal, R. Prasad, and S.P. Sharma, "Reduction of linear dynamic systems using an error minimization technique", Journal of Institution of Engineers IE(I)Journal – EL, Vol. 84, pp. 201-206, March 2004.
[21] J. Pal, "Improved Pade approximants using stability equation method", Electronic Letters, Vol. 19, No.11, pp.426-427, May 1983.
[22] M. G. Safonov and R. Y. Chiang, "Model reduction for robust control: a Schur relative error method", Int. J. Adaptive Cont. and Signal Proc., Vol. 2, pp. 259-272, 1988.
[23] Y. Bistritz and U. Shaked, "Minimal Pade model reduction for multivariable systems", ASME Journal of Dynamic System Measurement and Control, Vol.106, pp.293-299, 1984.
[24] R. Prasad, J. Pal and A. K. Pant, "Multivariable system approximation using polynomial derivatives", Journal of Institution of Engineers, India, IE(I)Journal – EL, Vol. 76, pp. 186-188, November 1995.
[25] L.S. Shieh and Y.J. Wei, "A mixed method for multivariable system reduction", IEEE Trans. Automat. Control, Vol. AC-20, pp. 429-432, 1975.
[26] P. O. Gutman, C.F. Mannerfelt and P. Molander, "Contributions to the model reduction problem", IEEE Trans. Automat. Control, Vol. AC-27, No.2, pp. 454-455, April 1982.
[27] G. Parmar, R. Prasad, and S. Mukherjee, "Order Reduction of Linear Dynamic Systems using Stability Equation Method and GA" Int Journal of Electrical, Computer, Energetic, Electronic and Communication Engineering Vol:1, No:2, pp. 236-242, 2007.
[28] R. Prasad, J. Pal and A. K. Pant, "Multivariable system approximation using polynomial derivatives", IE(I) Journal – EL, Vol. 76, pp. 186-188, November-1995, India.
[29] Sudhir Y. Kumar, Kritika, "a noble hybrid concept for mimo system modeling using bacterial foraging optimization technique", Journal of Basic and Applied Engineering Research Print ISSN: 2350-0077; Online ISSN: 2350-0255; Volume 1, Number 7;October, 2014 pp. 101-104.
[30] Jasvir singh rana, Rajendra Prasad, R.P. Agarwal, "MIMO system order reduction using basic characteristics and factor division method", International Journal Of Electrical, Electronics And Data Communication, ISSN: 2320-2084 Volume-4.

D Bala Bhaskar was born in Guntur, Andhra Pradesh, India, in 1987. He received B.Tech in Electrical & Electronics Engineering from R.V.R & J.C College of Engineering, Guntur, India in 2008 and M.E in Control Systems from Andhra University, Visakhapatnam in India in 2010. He was working as an Asst professor in Department of Electrical & Electronics Engineering in G.V.P College of Engineering (A), Visakhapatnam, Andhra Pradesh, India.
He is a life member of International Association of Engineers (IAENG), International Society for Research and Development (ISRD), Reviewer of International Journal of Measurement and Control-SAGE Publications. His areas of interest: Model Order Reduction, Controller Design, Control applications to Power systems and power electronics, Interval Systems.