World Academy of Science, Engineering and Technology
International Journal of Energy and Environmental Engineering
Vol:12, No:6, 2018

Advanced Hybrid Particle Swarm Optimization for Congestion and Power Loss Reduction in Distribution Networks with High Distributed Generation Penetration through Network Reconfiguration

C. Iraklis, G. Evmiridis, A. Iraklis

Open Science Index, Energy and Environmental Engineering Vol:12, No:6, 2018 publications.waset.org/10009183.pdf

Abstract**—**Renewable energy sources and distributed power generation units already have an important role in electrical power generation. A mixture of different technologies penetrating the electrical grid, adds complexity in the management of distribution networks. High penetration of distributed power generation units creates node over-voltages, huge power losses, unreliable power management, reverse power flow and congestion. This paper presents an optimization algorithm capable of reducing congestion and power losses, both described as a function of weighted sum. Two factors that describe congestion are being proposed. An upgraded selective particle swarm optimization algorithm (SPSO) is used as a solution tool focusing on the technique of network reconfiguration. The upgraded SPSO algorithm is achieved with the addition of a heuristic algorithm specializing in reduction of power losses, with several scenarios being tested. Results show significant improvement in minimization of losses and congestion while achieving very small calculation times.

Keywords**—**Congestion, distribution networks, loss reduction, particle swarm optimization, smart grid.

I. INTRODUCTION

GREENHOUSE gases and pollutants from using fuel-based sources, are proven to cause critically high temperature rise, changes in global water cycle, warming of the oceans, ice melting and increase of climate extremes [1]. The world is experiencing fuel crisis but mostly pollution crisis while production of electric energy is getting more unreliable, unhealthy and unsafe. Renewable energy sources are penetrating the grid faster than ever to solve the previous problems [2]. A mixture of such technologies may add complexity in the network and unreliability in power distribution. To increase reliability and grid stability towards a sustainable electric network, the concept of a Smart Grid is introduced. Smart grid technologies can be deployed to manage and distribute electric energy in an efficient and cost-effective way and at the same time get rid of congestion, power flow problems and over voltages in network nodes due to this high DG penetration [3].

In this paper, the problem of distribution network power losses is being introduced. Also, congestion is analyzed by introducing two factors in order to quantify the congestion level regarding both the branch currents and the node voltages. The first factor, i.e. Current Congestion Indicator Factor (CCIF), describes the loading level of the DN in respect to its carrying capacity (ampacity). CCIF constitutes a quantified estimation about how weighted the branch currents are. By that sense, high CCIF values correspond to high congestion levels for the network. The second factor, i.e. Voltage Congestion Indicator Factor (VCIF), describes the balance of the nodes' voltage in respect to the DN nominal value. Hence, it is expected that VCIF values higher than 1 pu should indicate overvoltage for some nodes of the network. Moreover, upgraded presentations of those factors are applied to add sensitivity to great or peak values.

The whole concept of this paper is the idea of using a heuristic algorithm which specializes in power losses reduction (while at the same time having very good results in the field of congestion mitigation) to upgrade a SPSO algorithm's solution in terms of speed, accuracy and performance. In a huge distribution network with an enormous number of nodes and branches, the calculation time is critical while at the same time, the algorithm must provide the best solution. Minimizing calculation time improves the distribution network's (DN) reliability while reduced power losses and congestion offer many economical and operational benefits to the operators.

The proposed algorithms are applied on the IEEE 69 bus system with numerous scenarios regarding the siting and sizing of Distributed Generation units. Two DG unit types have been considered based on their power factor. The first refers to DG units that produce both active and reactive power and thus influence significantly the nodes' voltage (VCIF), while the second assumes that each DG units has a power factor equal to 1. The latter is expected to affect mostly the loading level of the DN branches (CCIF).

The main contributions are:

- ✓ The significant lower simulation time of the optimization algorithm in respect to other algorithms. In big DNs with a huge number of nodes and branches, this can be an important advantage.
- ✓ Better results than other algorithms in terms of power losses reduction and voltage congestion mitigation also leading to lower costs for the DN operator.
- ✓ The ability of the algorithm to solve both power losses and congestion reduction at the same time. The decision

Christos Iraklis is with the Aristotle University of Thessaloniki, Greece (e-mail: christosiraklis@gmail.com).

to be made depends on the DN operator whether there is need for power losses reduction only, congestion mitigation only, or both using weight factors.

This paper is organized as follows: In Section II, the power losses problem is being introduced. In Section III, the congestion problem in a distribution network is being presented, and the factors which characterize voltage and current congestion in a mathematical way are being proposed. In Section IV, the simple PSO algorithm is presented, along with another modification, a SPSO algorithm. Moreover, a simple heuristic algorithm which specializes in power losses reduction is proposed. The results of heuristic algorithm simulation are being used to modify and upgrade the SPSO algorithm improving its calculation speed and performance. At the end of this section, the advanced hybrid SPSO-Heuristic algorithm is proposed which constitutes of both heuristic and SPSO factors and attributes along with the weighted summary target function. In Section V, the results are being presented and finally in Section VI, some conclusions are exported.

II. POWER LOSS PROBLEM

Active power electrical losses in power systems are proportional to the square of branch current. The problem of losses minimization in distribution networks can be written for a fixed operational point in a simple form as follows [4]:

$$minimize \ \sum_{k=1}^{m} R_k |I_k|^2 \tag{1}$$

Subject to:

$$I_k \le I_{k_{max}} \ with \ (k = 1, \dots, m) \tag{2}$$

$$V_i^{min} \le V_i \le V_i^{max} \tag{3}$$

where n is the total number of nodes, m is the total number of branches, ns is the number of sources, I is the m-vector complex branch current with, V_i is the voltage at node i, I_k is the RMS current of branch k, and $I_{k_{max}}$ is the maximum thermal RMS current of branch k and R_k is the Ohmic resistance of branch k. Equation (2) essentially indicates the thermal limits of the conductors that must not be violated. Equation (3) defines the lower and upper thresholds of voltage in each node. It is also clarified that the link between voltages and currents is expressed by the power flow equations which are utilized for power flow analysis by the proposed algorithm.

III. CONGESTION PROBLEM

A. Congestion in DNs

The efficient operation of existing networks is in need of a more sophisticated approach by controlling the voltage and power flow any time let alone when there is high penetration of new power plants mainly by private renewable energy entrepreneurs [5]. So, the problem of congestion is being regenerated in the sense that constraints and technical operating conditions reach their maximum limit. In other the words, problems may occur due to failure of the transmission or distribution system for further transport, power generators can reach the maximum charge, or there may be fluctuations in electricity demand [6].

Unlike transmission systems (high voltage) that take in account primarily economic factors in terms of network congestion, the electricity prices in distribution systems are the same, and so, measures are likely to be taken for the network to meet the technical constraints and problems (power losses, congestion, voltage levels and currents) that will certainly lead to economic benefits for the company – operator [7].

High penetration of renewable energy sources will lead some parts of the network or some lines to their operational limits resulting in the following problems:

- Non-permissible voltage levels
- Non-stabilized voltage
- Currents much greater than current capacity of lines

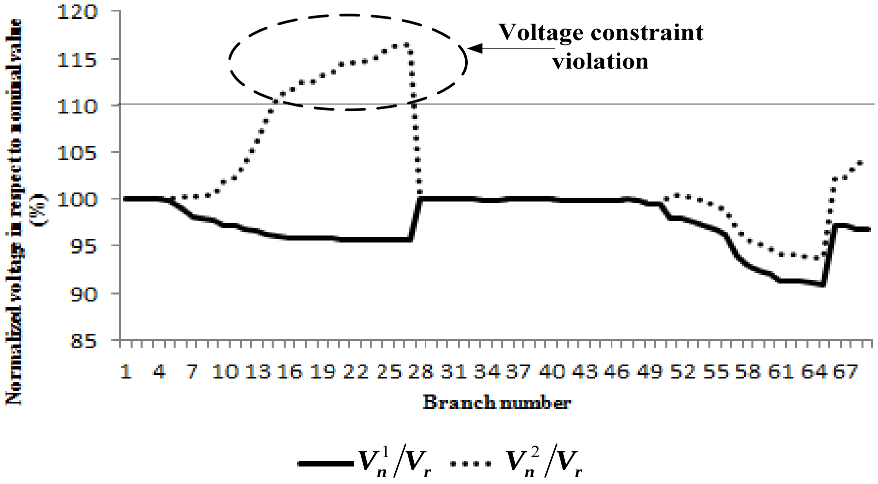

Fig. 1 Node voltages before and after DG installation

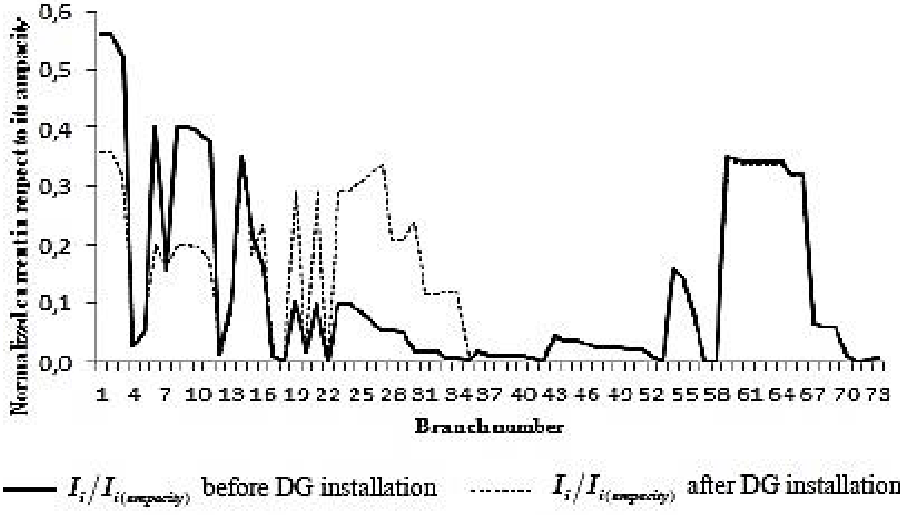

Fig. 2 Normalized branch currents before and after DG installation

Figs. 1 and 2 show the voltage of the nodes and the normalized currents of the branches to their maximum capacity at a 69-bus network before and after the penetration of DGs. As shown, after the insertion of units, congestion issues begin to occur since voltage levels no longer meet the limitations and constrains. The profile of the currents is changed, and a series of spikes are transferred in different branches. This is a fact that can lead to electric durability and distribution reliability problems.

In some branches, the current is decreased, in some other increased while in a few it has remained the same. These sectors where the current is increased compared with the previous situation face reverse power flow. Therefore, it is necessary to assess the status of the network and its overall performance with congestion factors.

B. Voltage Congestion Factor

In this section, a factor that indicates voltage congestion levels is being introduced (VCIF = Voltage Congestion Indicator Factor). This factor describes the overview of the network with respect to the voltage profile of nodes. A bigger difference of this value from 1 represents higher elevation (for negative values of the difference) or greater voltage drop (for positive values of the difference).

$$VCIF = \sqrt{\frac{\sum_{i=1}^{n}\left|1-\frac{V_n^k}{V_r}\right|^2}{n}} \tag{4}$$

where: *VCIF*: voltage indicator factor, *n*: number of nodes, *k*: the network status for a specific load and condition of switches, V_n^k: current value of voltage of node n for the state k (rms value), V_r: nominal voltage of network (rms value).

C. Current Congestion Factor

In this section, a factor that indicates current congestion levels is being introduced (CCIF = Current Congestion Indicator Factor). This factor describes the charging of network branches. This means that for two separate loads or network topologies with different CCIF, the smallest value thereof represents less congestion. This does not necessarily mean reduced currents for all branches of the network but certainly for a percentage of them.

$$CCIF = \sqrt{\frac{\sum_{i=1}^{[(n-1)+t]}\frac{{I_{b_i}^k}^2}{{I_{a_i}}^2}}{(n-1)+t}} \tag{5}$$

where: *CCIF*: current indicator factor, *n*: number of nodes, *t*: number of connection switches, *k*: the network status for a specific load and condition of switches, *i*: number of branches, i = 1, …, [(n-1) + t], $I_{b_i}^k$: the current value of the current of branch i for the state k (rms value), I_{a_i}: maximum current capacity of branch i (rms value).

The square of the terms of the numerators offers sensitivity of coefficients from great values. To make this more understandable, an example is provided. Even two values of conditions $\frac{I_{b_i}^k}{I_{a_i}}$ are equal to 0.1 and 0.9, and the two others are equal to 0.5 and 0.5, respectively. Clearly the first case is the most undesirable. This is not visible with the first definition of CCIF since both values will give a summary of 1 and therefore the same value to the function. With the numerators squared, the first gives a result of 0.82, while the other is 0.5, showing that the first situation is closer to congestion.

The operator for calculating the square root of the functions is applied to them for scaling and better presentation of results in charts due to very small values after the squaring of the currents and voltages.

IV. PROPOSED METHODOLOGY

A. Simple Particle Swarm Optimization Algorithm

A considerable amount of incredible social behavior characteristics and great intelligent exists in nature with many examples including ant colonies, bird flocking, animal herding and fish schooling. In all cases, although the ability of individual is limited, the population as a whole can achieve great results through cooperation, although there is no centralized control of the masses. The behavior of each individual depends on the interaction with others and with their surrounding environment only. Simple interactions among individuals can guide the whole population in a behavioral global manner, called swarm intelligence. The PSO algorithm is one of the many optimization algorithms utilizing the concept of such kind of global behavior [8].

In the PSO algorithm, a population of a predefined number of particles is set. During every iteration, each particle within the whole group is looking for a solution based on two values, its recorded personal best solution (known as Pbest) and the entire population's recorded global best solution (known as Gbest) [9]. When those two values are calculated, every particle updates its velocity and position in space based on (6) and (7). An example of the particles' movement in space is illustrated in the vector diagram (Fig. 3) [10].

$$v_{id}^{new} = wv_{id} + c_1 rand(pbest - x_{id}) + c_2 rand(gbest - x_{id}) \quad (6)$$

$$x_{id}^{new} = x_{id} + v_{id}^{new} \quad (7)$$

where:

$$w = \frac{w_{min} - w_{max}}{iterations - 1}(j-1) + w_{max}$$

where j: running iteration number, v_{id}^{new}: new velocity of i-th particle, v_{id}: starting velocity of i-th particle, x_{id}^{new}: new position of i-th particle, x_{id}: starting position of i-th particle, c_1, c_2: acceleration factors, random numbers, $rand$: random number generating function between [0,1].

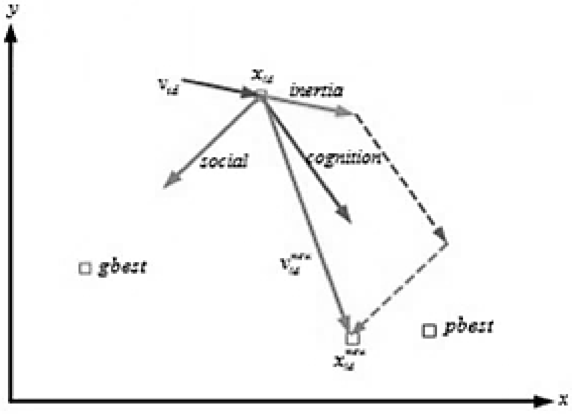

Fig. 3 Vector diagram of particle velocity

B. SPSO Algorithm

For the proposed hybrid optimization algorithm, the SPSO algorithm, a different version of the PSO algorithm, is utilized. The SPSO is using particles looking for solutions in a multidimensional space [11] after applying a transformation function on the particle velocities according to (8) and (9). In this case, the search space for each dimension is $S_d = [s_{d1}, s_{d2}, s_{d3}, \dots, s_{dn}]$ where dn is the number of selected positions in dimension d.

$$sigmoid(u_{id}^{k+1}) = dn \frac{1}{1+e^{-u_{id}^{k+1}}} \quad (8)$$

$$x_{id}^{k+1} = \begin{cases} s_{d1} & if\ sigmoid(u_{id}^{k+1})<1 \\ s_{d2} & if\ 1<sigmoid(u_{id}^{k+1})<2 \\ & \dots\dots\dots\dots \\ s_{dn} & if\ dn-1<sigmoid(u_{id}^{k+1})<dn \end{cases} \quad (9)$$

where $s_{d1}, s_{d2}, s_{d3}, \dots, s_{dn}$ are the selected values in dimension d. The velocities are then limited by V_{min} and V_{max} according to (10). Equation (11) is used so that the locking of a particle's velocity below V_{min} or above V_{max} is prevented, causing the particle to move in the search space.

$$u_{id}^{k+1} = \begin{cases} V_{\max} & if\ u_{id}^{k+1} > V_{max} \\ u_{id}^{k+1} & if\ V_{min} \leq |u_{id}^{k+1}| \leq V_{max} \\ V_{min} & if\ u_{id}^{k+1} < V_{min} \end{cases} \quad (10)$$

$$u_{id}^{k+1} = \begin{cases} rand \cdot u_{id}^{k+1} & if\ |u_{id}^{k+1}| = |u_{id}^{k}| \\ u_{id}^{k+1} & otherwise \end{cases} \quad (11)$$

D. Heuristic Algorithm

The algorithm initially provides an evaluation on the target function (power loss levels) by comparing the losses before and after the installation of distributed generation units. In this algorithm, the target function to be minimized is that of power losses. Afterwards, the algorithm performs network reconfigurations until certain criteria on power loss reduction are met. The network reconfiguration is completed in three steps:

I. Close the connection switch (ON) for the branch with the greatest bus voltage.
II. Calculate branch currents in the created loop.
III. Open the connection switch (OFF) for the branch with the lowest calculated current.

The above concept is repeated for all the connection switches. In the 69-bus system, there are five tie-switches, so five network topologies will be saved in total, among which the best solution will provide the lowest power losses. This solution will contribute in the calculation speed and the performance of the proposed hybrid SPSO-Heuristic algorithm by upgrading the particles' velocities. The flow diagram for the heuristic algorithm used is presented in Fig. 4.

The DN has got to be kept radial in every step. For this, a simple function is running inside the algorithm that provides information whether the network is radial or not. Non-radial networks are being dismissed. Also, it is checked whether every node is connected to the network or not.

D. Advanced Hybrid SPSO-Heuristic Algorithm

The best result (network with minimum power losses) from calculations of the heuristic algorithm (see section D) is used to modify and upgrade the existing SPSO algorithm. The whole concept is to provide an additional velocity in (6) giving an additional direction of the best solution from the beginning of the simulation decreasing the calculation's time, providing better results in terms of power losses minimization and congestion mitigation at the same time. Thus, (6) changes to:

$$v_{id}^{new} = wv_{id} + c_1 rand(pbest - x_{id}) + c_2 rand(gbest - x_{id}) + w_{heuristic} rand(x_{heuristic} - x_{id}) \quad (12)$$

$$x_{id}^{new} = x_{id} + v_{id}^{new} \quad (13)$$

where: $w_{heuristic} = 1.166e^{-0.04669j} - 0.113$, j : running iteration number, $x_{heuristic}$: position of particle from heuristic algorithm's solution

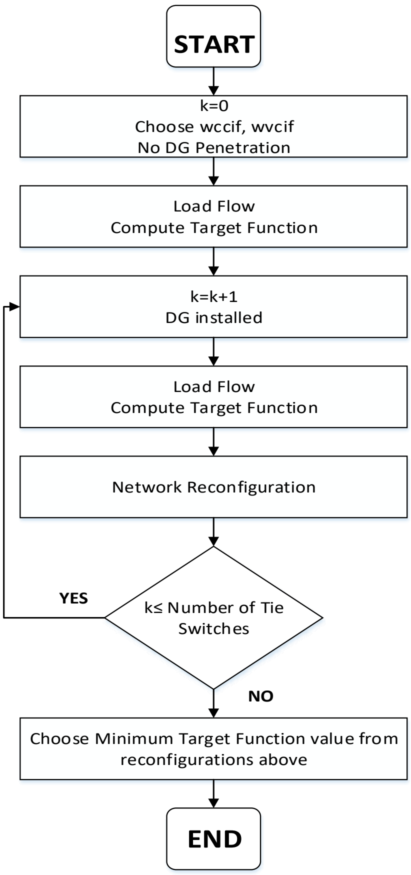

Fig. 4 Flow diagram of heuristic algorithm

For the applications of the Hybrid SPSO-heuristic algorithm in this paper, these steps are followed [12]:

1. Specification of target function.
2. Specification of problem dimensions.
3. Find the search space for each dimension.
4. Select the best solution from search sites using SPSO.

1) Specification of Target Function

The weighted summary target function to be reduced or minimized is described in (14):

$$TF = \frac{w_{loss} P_{loss}}{P_{loss_{start}}} + \frac{w_{congestion}(w_{CCIF} CCIF + w_{VCIF} VCIF)}{w_{CCIF} CCIF_{start} + w_{VCIF} VCIF_{start}} \quad (14)$$

where:

$$w_{CCIF} + w_{VCIF} = 1 \quad (15)$$

$$w_{loss} + w_{congestion} = 1 \quad (16)$$

In this way, the DN operator can choose between power losses reduction ($w_{loss} = 1$) and congestion mitigation ($w_{congestion} = 1$) only but also can run the simulation for power losses combined with congestion reduction in the same target function using weight factors. Most of the times, cooperation between power losses and congestion reduction gives good results and is well acceptable by the network operator.

2) Specification of Problem Dimensions

To specify the dimensions of this problem, all interconnecting switches are closed, and so, loops are formed in the network as many as the number of interconnecting switches. This number gives the number of dimensions of the problem.

3) Find the Search Space for Each Dimension

To explain how the search space is found, the 69-bus network is used and shown in Fig. 5 [13].

- The circuit of Fig. 5 has got 73 branches of which five are interconnecting switches.
- By closing the interconnecting switches five loops are created.
- The branches that are not participating in a loop are not represented in the search space and thus the network is simplified to that shown in Fig. 6.
- The number of dimensions is equal to the number of loops (smaller ones formed), so in this case, it is equal to five.
- The search space for each dimension will be the branches involved in the loop. In this case:
  - ➢ d1 = [46,47,48,49,72,58,57,56,55,54,53.52,8,7,6,5,4]
  - ➢ d2 = [3,35,36,37,38,39,40,41,42,69,10,9,8,7,6,5,4]
  - ➢ d3 = [43,44,45,71,14,13,12,11,69]
  - ➢ d4 = [21,22,23,24,25,26,73,64,63,62,61,60,59,58,57,56,55,54,53,52,9,10,11,12,70]
  - ➢ d5 = [15,16,17,18,19,20,70,13,14]

The radiality of the distribution network is achieved in every step because network reconfiguration occurs for every string-dimension, and so, there is no need for a function to check if the network is radial. Non-radial networks produce very big values of the target function, and so, the algorithm rejects them and moves on to the next iteration without them to influence the whole process. A disconnected node results in inability of Matpower/MATLAB to calculate the power flow in the network, and an invalid value is given to the target function. In this case, the current network topology is rejected. The flow diagram of this algorithm is presented in Fig. 7.

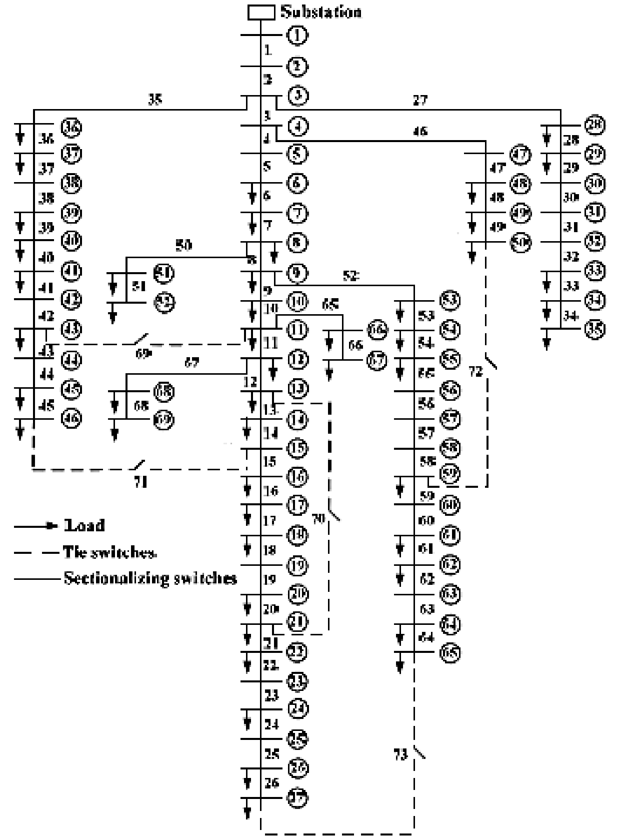

Fig. 5 Original 69 bus system with five interconnecting switches

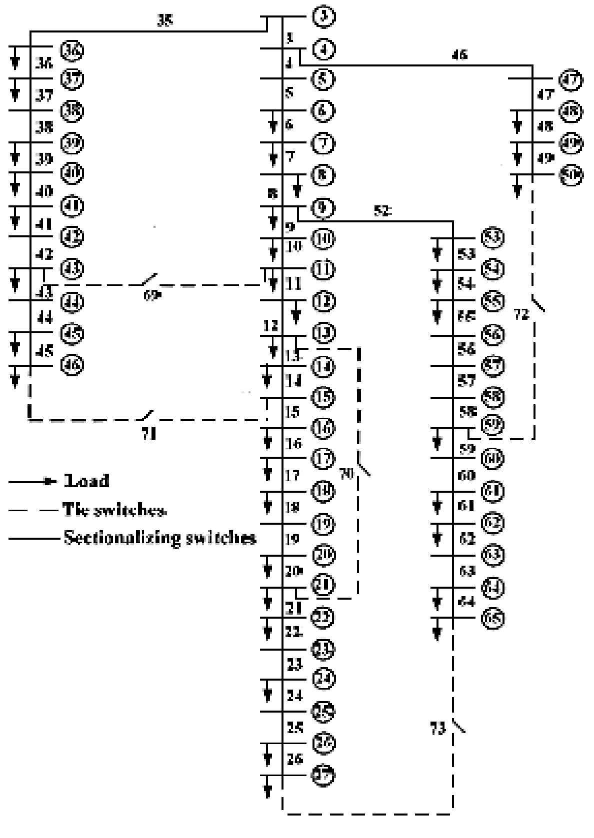

Fig. 6 Simplified 69 bus system with five interconnecting switches

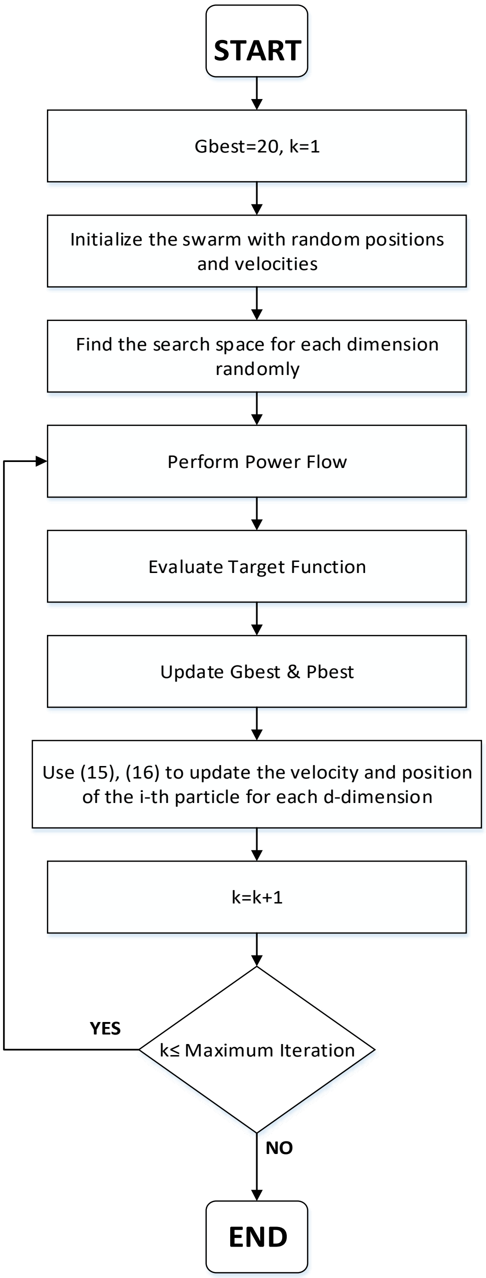

Fig. 7 Flow Diagram of hybrid SPSO-heuristic algorithm

V. SIMULATION RESULTS

The proposed algorithms were applied to the 69-bus system for 100 scenarios regarding random allocation of 35 DG units (50% penetration) and random power production between 300 and 700 kW with a fixed power factor equal to 0.9. The PSO part of the proposed hybrid algorithm has the characteristics given in Table I.

Figs. 8-10 present the simulation results, comparing the performance of the upgraded hybrid SPSO algorithm with the performance of the simple SPSO algorithm, for different target functions including power losses, voltage congestion and combined power losses with voltage and current congestion, for cases with weighting factors shown in Table II.

TABLE I
PARAMETERS OF PSO

Number of particles	20
Number of iterations	1000
Acceleration factor c_1	0.5
Acceleration factor c_2	0.5
Weight factor $w_{heuristic}$	0.5
w_{min}	0.2
w_{max}	0.9

TABLE II
CASES AND WEIGHTING FACTORS

No.	w_{VCIF}	w_{CCIF}	$w_{congestion}$	w_{loss}
Case 1	0	0	0	1
Case 2	1	0	1	0
Case 3	0.5	0	0.5	0.5
Case 4	0.5	0.5	0.5	0.5

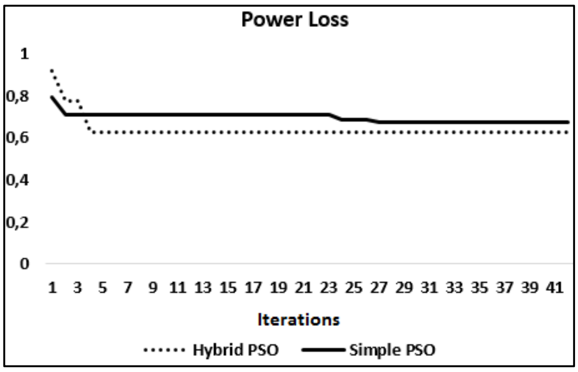

Fig. 8 Target Function for Case 1

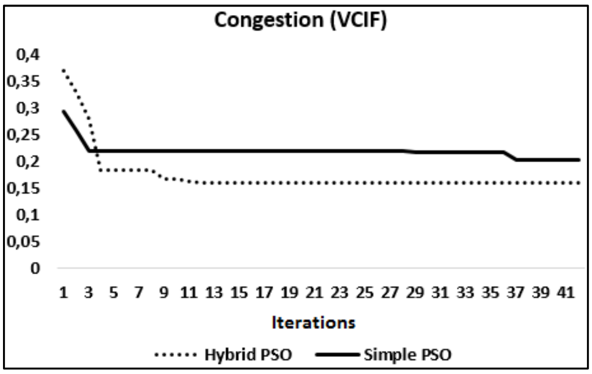

Fig. 9 Target function for Case 2

It is shown that using a heuristic algorithm as an upgrade to the simple PSO algorithm results in better algorithm efficiency and less calculation time (faster algorithm). When power loss minimization is the algorithm's target, the hybrid SPSO algorithm finds the minimum value of power losses in approximately 80% shorter period of time than the simple SPSO (that means the upgraded SPSO is 80% faster) as well, as illustrated in Fig. 8. In Figs. 9-11, the same applies for congestion (VCIF), combined power losses with voltage congestion (VCIF) and combined power losses with congestion (VCIF plus CCIF) targets with minimum values of the target functions found in 70%, 75% and 45% shorter period of time respectively. In combined cases, multiple factors are working together to achieve minimization as long as there is a relative connection. In Table III, the average calculation time needed for both algorithms to find best solution for 100 scenarios and the aforementioned pairs of weighting factors, is presented.

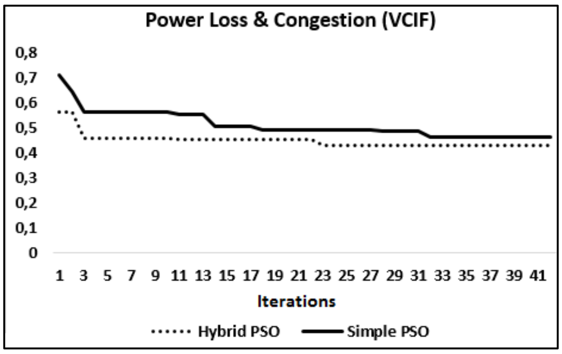

Fig. 10 Target function for Case 3

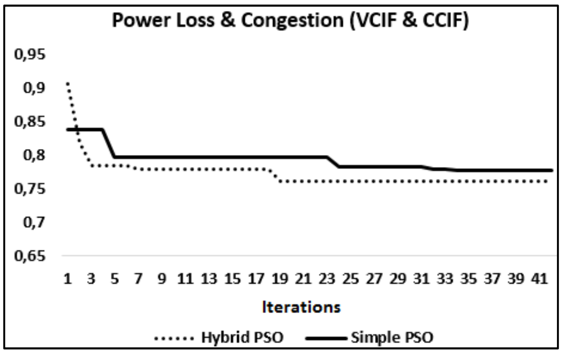

Fig. 11 Target function for Case 4

TABLE III
AVERAGE SIMULATION TIME FOR BEST SOLUTION

Average Simulation Time	
Simple PSO	20-22 seconds
Hybrid SPSO	5-7 seconds

Figs. 12-15 present the simulation results comparing the performance of the upgraded SPSO algorithm with the performance of the simple SPSO algorithm for the target functions mentioned and for 100 scenarios of randomly siting and sizing of distributed generation units.

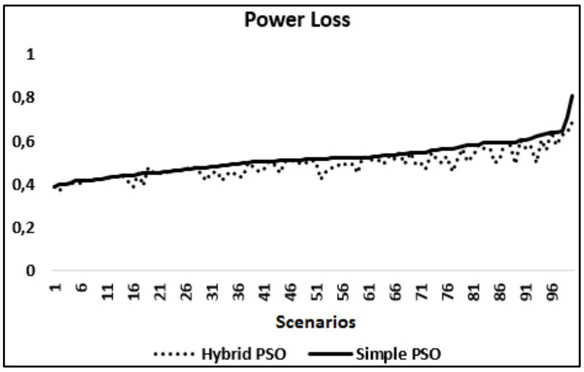

Fig. 12 Target function for 100 scenarios, Case 1

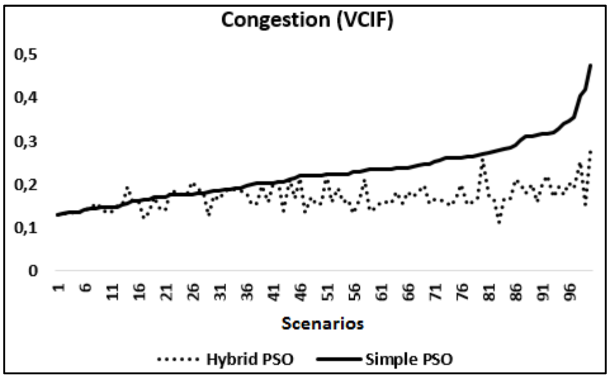

Fig. 13 Target function for 100 scenarios, Case 2

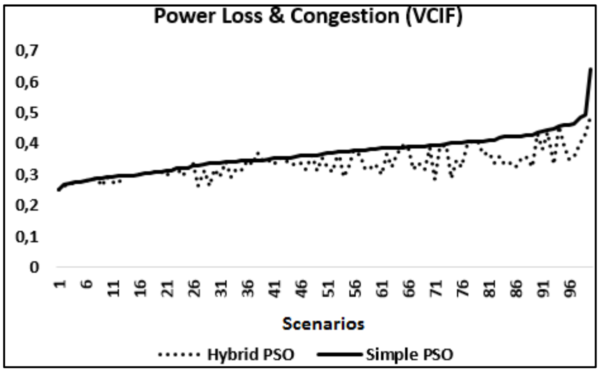

Fig. 14 Target function for 100 scenarios, Case 3

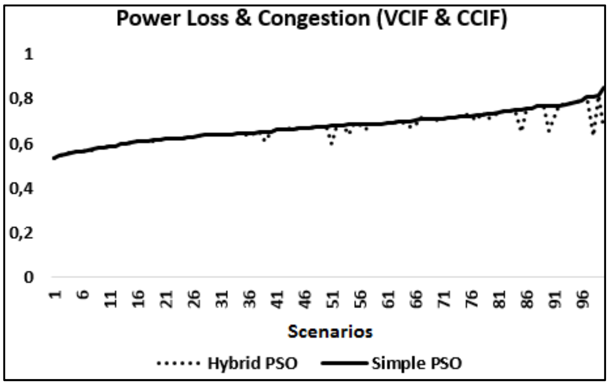

Fig. 15 Target function for 100 scenarios, Case 4

Figs. 12-15 illustrate the fact that in most cases and for different target functions and scenarios, the upgraded PSO algorithm produces better results by means of minimized target functions. When the target function describes power losses, in 75% of the scenarios being tested, the upgraded algorithm produces higher minimization than the simple PSO algorithm and when the target function describes voltage congestion (VCIF) and combined power losses with voltage congestion (VCIF), the upgraded algorithm produces higher minimization than the simple particle optimization algorithm in 75% and 73% of the scenarios being simulated respectively (Figs. 12-14). For a significant number of scenarios, that of 45%, the same conclusion is made when the target function describes power losses combined with congestion (VCIF plus CCIF) simultaneously (Fig. 15).

VI. CONCLUSION

Many heuristic, meta-heuristic, and genetic optimization algorithms have been introduced in the field of electrical networks trying to solve problems of optimal distributed power generation unit placement and network reconfiguration. In this paper, an upgraded SPSO algorithm is presented which can benefit from the utilization of a simple heuristic algorithm to provide better minimization performance at lower simulation times (high calculation speed) for a distribution network and a problem of network reconfiguration. The results in this research have illustrated that great performance can be achieved in terms of power loss minimization, congestion minimization and power loss combined with congestion minimization as a weighted sum target function in a 20-kV distribution network and for random siting and sizing of distributed generation units. In most cases, the addition of a simple heuristic algorithm in a PSO algorithm's velocity upgraded the algorithm's calculation speed too.

ACKNOWLEDGMENT

Iraklis C. thanks Aristotle University of Thessaloniki for the support during this work.

REFERENCES

[1] Kuonen, Samuel. "Estimating greenhouse gas emissions from travel-a GIS-based study." *Geographica Helvetica* 70.3 (2015): 185.

[2] Lombardi, Ing Pio, et al. "Optimal management tool for micro grids with a high penetration of renewable energy sources." *power* 1.2 (2013): 1.

[3] Hau, Lee-Cheun, et al. "Smart Grid–The Present and Future of Smart Physical Protection: A Review." *International Journal of Energy, Information and Communications* 4.4 (2013): 43-53.

[4] Dahalan, Wardiah Mohd, and Hazlie Mokhlis. "Network reconfiguration for loss reduction with distributed generations using PSO." *Power and Energy (PECon), 2012 IEEE International Conference on*. IEEE, 2012.

[5] Yao, Liangzhong, et al. "Congestion management of transmission systems using FACTS." *Transmission and Distribution Conference and Exhibition: Asia and Pacific, 2005 IEEE/PES*. IEEE, 2005.

[6] Shariatkhah, Mohammad Hosein, and Mahmoud Reza Haghifam. "Using feeder reconfiguration for congestion management of smart distribution network with high DG penetration." (2012): 316-316.

[7] Bouhouras, Aggelos S., et al. "Reducing network congestion in distribution networks with high DG penetration via network reconfiguration." *European Energy Market (EEM), 2014 11th International Conference on the*. IEEE, 2014.

[8] Bai, Qinghai. "Analysis of particle swarm optimization algorithm." *Computer and information science* 3.1 (2010): 180.

[9] Samuel, G. Giftson, and C. Christober Asir Rajan. "Hybrid: particle swarm optimization–genetic algorithm and particle swarm optimization–shuffled frog leaping algorithm for long-term generator maintenance scheduling." *International Journal of Electrical Power & Energy Systems* 65 (2015): 432-442.

[10] Tsai, Men-Shen, and Wu-Chang Wu. *A novel binary coding particle swarm optimization for feeder reconfiguration*. INTECH Open Access Publisher, 2009.

[11] Lee, Sangwook, et al. "Modified binary particle swarm optimization." *Progress in Natural Science* 18.9 (2008): 1161-1166.

[12] Chang, Chung-Fu. "Reconfiguration and capacitor placement for loss reduction of distribution systems by ant colony search algorithm." *IEEE Transactions on Power Systems* 23.4 (2008): 1747-1755.

[13] Hamouda, Abdellatif, and Khaled Zehar. "Efficient load flow method for radial distribution feeders." *Journal of Applied Sciences* 6 (2006): 2741-2748.