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Generating a Functional Grammar for Architectural Design from Structural Hierarchy in Combination of Square and Equal Triangle

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***Abstract*—**Islamic culture was accountable for a plethora of development in astronomy and science in the medieval term, and in geometry likewise. Geometric patterns are reputable in a considerable number of cultures, but in the Islamic culture the patterns have specific features that connect the Islamic faith to mathematics. In Islamic art, three fundamental shapes are generated from the circle shape: triangle, square and hexagon. Originating from their quiddity, each of these geometric shapes has its own specific structure. Even though the geometric patterns were generated from such simple forms as the circle and the square, they can be combined, duplicated, interlaced, and arranged in intricate combinations. So in order to explain geometrical interaction principles between square and equal triangle, in the first definition step, all types of their linear forces individually and in the second step, between them, would be illustrated. In this analysis, some angles will be created from intersection of their directions. All angles are categorized to some groups and the mathematical expressions among them are analyzed. Since the most geometric patterns in Islamic art and architecture are based on the repetition of a single motif, the evaluation results which are obtained from a small portion, is attributable to a large-scale domain while the development of infinitely repeating patterns can represent the unchanging laws. Geometric ornamentation in Islamic art offers the possibility of infinite growth and can accommodate the incorporation of other types of architectural layout as well, so the logic and mathematical relationships which have been obtained from this analysis are applicable in designing some architecture layers and developing the plan design.

***Keywords*—**Angle, architecture, design, equal triangle, generating, grammar, square and structural hierarchy.

I. Introduction

GEOMETRIC patterns are among the most recognizable visual expressions of Islamic art and architecture [1]. The generation procedure of these patterns, their professional artificers and the applied elaborated construction techniques in creating these patterns strongly support this idea that these artificers had a comprehensive knowledge of pragmatic geometry. They knew how to subdivide a circle into twelve equal sectors, while deprived of measuring the angles with a protractor. They could construct a large geometric pattern on the dome of the architecture of mosques are sturdily attached to basic motif ornaments [2]. Their skill was not based on theory or mathematical calculation; they created the patterns by drawing circles and lines [3]. Of course, it is possible to analyze the patterns mathematically, measuring the different lines and angels, nowadays we tend to rely heavily on tools such as a protractor calculator in our perception of geometry. Mastering the basic geometric techniques will equip one with practical skills that are immensely useful in all field of design, even in the digital age [3]. Before anything else we must explain about geometry, its history and its different kinds in architecture: Geometry [4] refers to a knowledge which determines the mathematical relationship among points, lengths, surfaces and volumes and represents their proportions, derivatives and functions [5].

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II. Thesis Statement

An abundance of geometric patterns in Islamic art and architecture are based on the repetition of a single motif [6], which is designed thus and so that all the recurring components fit together in a perfect sequence. Rather than designing an elaborate pattern to cover a plenary of wall, the artificer can divide the surface into a grid of squares or hexagons, moreover for instance he can repeat the individual motif in each unit [7] (as shown in Table I). All of the patterns fit into either a square or a hexagon, from which larger geometric designs can be created by repeating the unit. Each pattern (as presented in Table III) is taken from a particular building, together with its location and date. There is a wide array of geometric styles, and preferences for definite patterns varied relying on the interval and region. SHAMSEH and CHALIPA are among the most famous Islamic geometric patterns. 4 evolution stages of the pattern (1)-(4) and its trigonometric relationship (as shown in Figs. 1 and 2) is presented below:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad (1)$$

$$R^2 = a^2 + a^2 \quad (2)$$

$$R = a\sqrt{2},\ A = 2a,\ R = \frac{A\sqrt{2}}{2}$$

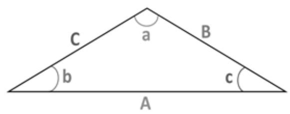


Fig. 1 Fix trigonometric relationships

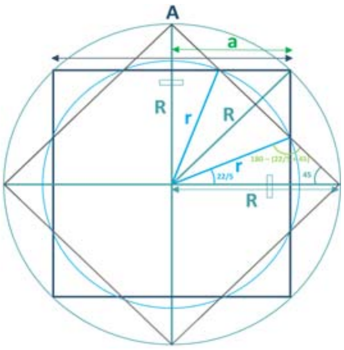


Fig. 2 Fix trigonometric relationships in circle

$$\frac{R}{\sin 112/5} = \frac{r}{\sin 45} = \frac{R*sin45}{\sin 112.5} \tag{3}$$

$$r = \frac{A\sqrt{2}}{2} * \frac{\sin 45}{\sin 112.5} \tag{4}$$

The pattern was modeled in Grasshopper (Fig. 3) and for the further analyzing, its output codes were entered into the MATLAB software. The mathematical expression in grasshopper software is:

R * sin (pi/4) / sin (5 * pi/8)

There is a wide range of instances in this regard, a few of which are in the following figures (Table III) [8]:

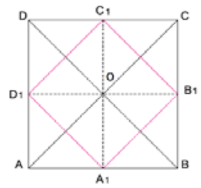
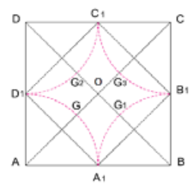
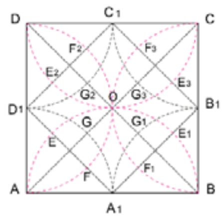
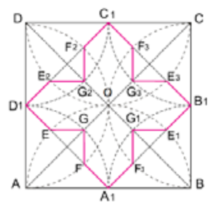
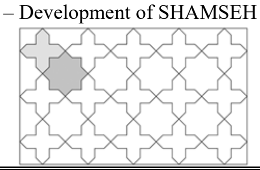
III. Research Literature

A. Islamic Pattern

Theoretical history of architecture well indicates the proportionate division of volumes in architectural forms and shapes. Symbolic concepts for these basic shapes and volumes are abundant in different cultures. The basis of these geometrical shapes is circle which is a picture of perfection leading to regular polygons if divided equally taking the shape of regular stars [9]. From Islamic philosophy point of view, this manner of conform, displacement or transference of proportions is consistent with monotheism where unity is the origin and the final point is joining all multiplicities. Circle is a universal symbol, endless and timeless without no beginning or end [10]. In Islamic culture it indicates the dome of heaven (Fig. 4, 5). In contrast, square is the earth, finite and ending. Circle states the initial form. It is the heaven and life of the world. Spirit and light are initials. Cube is symbol of stability and firmness and expresses perfection. From ancient times, geometry in east and west, from glorious historical buildings to simple and common houses, from royal gardens to small yards, from divine concepts to understanding human shape and body, geometry has been reflected mysteriously to express the beauty of mosques and shrines [11]. These patterns have three basic characteristics:

1. They are manufactured of an exiguous repeated geometric elements. The naive forms of the circle, square, and straight line are the foundations of the patterns. These elements are merged, duplicated, interlaced, and organized in intricate amalgamations. The vast majority of patterns are conventionally founded on one of two species of grid—one composed of equilateral triangles, the rest of squares. The third type of grid, composed of hexagons, is a modification on the triangular schema. "Regular Tessellation" is the mathematical term for these grids (deriving from Latin tesserae, i.e., mosaic segments), in which one regular polygon is duplicated to tile the plane [12].

TABLE I
Development of Shamseh Pattern

Step 1	Step 2
	
Step 3	Step 4
	
Step 5 – Development of SHAMSEH pattern	
	

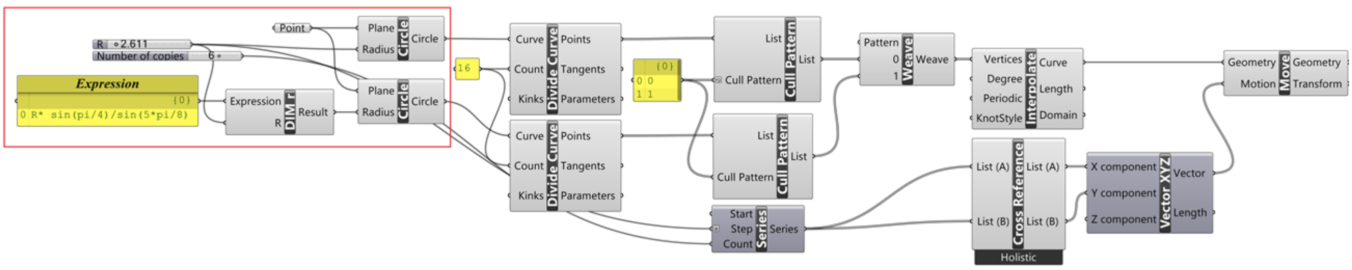
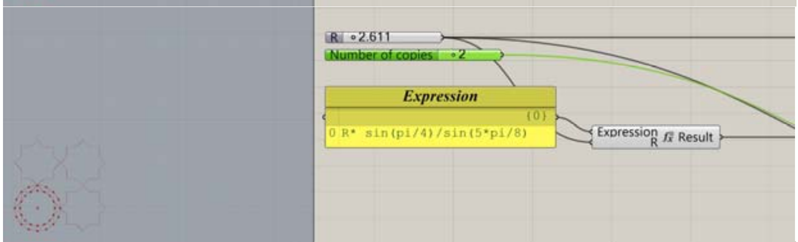
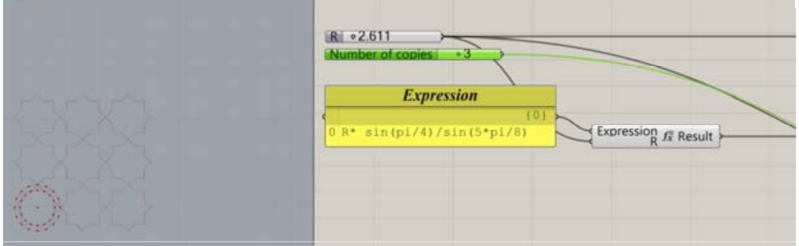
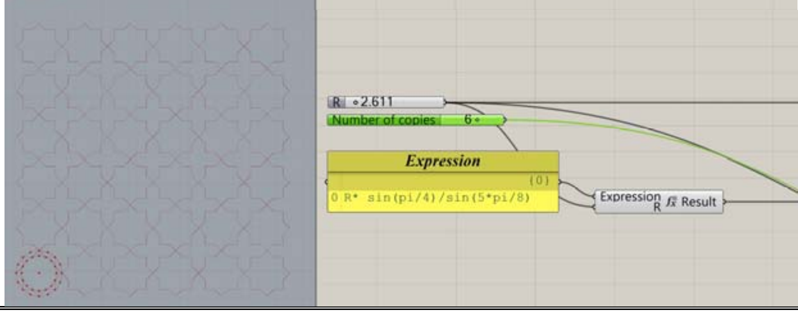


Fig. 3 Grasshopper definition of CHALIPA and SHAMSEH pattern

The growth of this model is presented in Table II.

TABLE II
GRASSHOPPER MODELING

	Number slider 1
	Number slider 2
	Number slider 3
	Number slider 4
	Number slider 5
	Number slider 6

2. They are two-dimensional. Islamic designs frequently possess a background and foreground template. The assignment of pattern upon pattern serves to flatten the space, and there is no intention to create depth. Vegetal patterns are probably an opposite set of contrasting background in which the plant like forms interlace, weaving over and under , emphasizing the foreground decoration. In other instances, the Background is substituted by the opposition between light and shade Sometimes it is inconceivable to individualize between foreground and background. Some geometric designs are created by coordinating all the polygonal shapes together resembling the pieces of a puzzle, bereft of any gaps and, thus, demanding no spatial interplay between foreground

and background. The mathematical term for this type of structure is "tessellation." The conception of space in Islamic art is entirely diverse from Western models, which typically utilize a linear perspective and split the picture space into foreground, middle ground, and background. Islamic world's artists were predominantly apathetic in linear perspective. Among the various styles of Islamic art, there was a Persian painting that a type of three-dimensional space was applied in, with the possibility of figures' interaction, but this space presented multiple perspectives and simultaneously featured views of bird's-eye and worm's-eye [13].

3. They are not designed to adapt through a frame. Geometric ornamentation in Islamic art suggests a remarkable degree of freedom [14]. The complicated adjustment and combinations of elements are infinitely expansible; the frame surrounding a pattern appears to be arbitrary and the fundamental adjustment occasionally provides a unit from which the rest of the design can be both predicted and projected [15].

TABLE III
DIFFERENT PATTERNS IN ISLAMIC GEOMETRIC PATTERNS

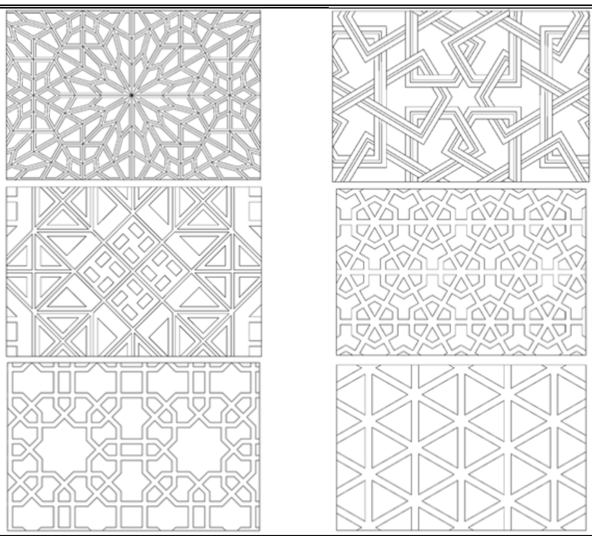


Fig. 4 Patterns in one of the dome Mosque

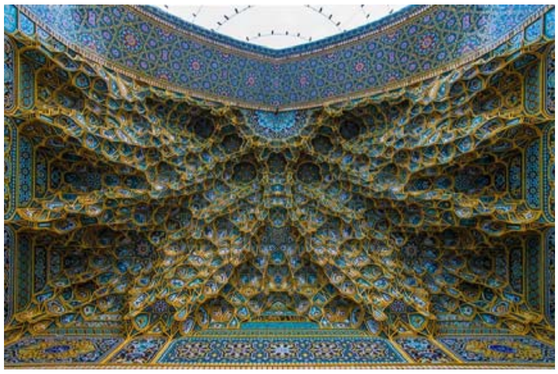


Fig. 5 Patterns in one of the dome Mosque

Furthermore, this first property guarantees the admitting an interlacing by the design. In whole crossing, every single strand can be picked out for passing over the other ones. Whenever each vertex has degree two or four, this assignment can be accomplished comprehensively in such a way that as one follows a given strand, it 50 passes alternately over and under any strands it crosses. When the design is a connected graph, the assignment can be composed of picking out the over/under relationship at a single vertex and propagating the assignment throughout the planar map. The single Boolean option at the designated vertex leads to two self-consistent, opposite assignments (when the map has multiple components, the choice must be made once per component). Furthermore, the redundancy insinuated by the equivalence of these assignments can be formalized by means of the inclusion of a new symmetry procedure, one that leaves the plane fixed but exchanges the roles of over and under everywhere. Mixing this re weaving operation in with the regular isometries produces an enrichment of the usual symmetry groups that can provide a finer classification of interlaced figures.

Aesthetic considerations prompt the latter possession rather than mathematical restraints. At a degree four vertex, the crossing is the supreme obviously rendered when the two strands, meeting there, pass right through the intersection devoid of varying course. Based on Gestalt psychology, there is an evidence in the literature supporting the aesthetic supremacy of perfect crossings. Satisfying these two conditions, the space of star patterns is however huge. A swift survey demonstrates that Bourgoin's reproduced designs are admittable of roughly 70%. Through this framework, it will be established that there is an immense space of designs to be investigated [16]. A design is an accumulation of line segments in the plane that do not intersect each other excluding perchance at their endpoints. This kind of collection may be regarded as an unbounded planar map. Owing an individual position in the plane, a set of vertices produce this map, moreover pairs of vertices are connected by a set of edges. Having a vertex as an endpoint, the number of edges define the valence of that vertex. In the case of periodic, restriction of this planar map can represent it to an individual translational unit. In what follows, we restrict out attention to the class of designs with the following two properties:

1. Every vertex has valence two or four.

2. The valence of four vertices are thorough crossings.

To create a final motif, we will need to truncate every one of these rays somewhere along its length. Because the target is the creation of uninterrupted strands, in the spot of meeting further rays, each ray will have to be truncated, either creating a curve, or sometimes a continuous straight line segment. Should all ray intersect with some other ray, a perfect motif will be composed. Based on this description, it's conceivable to designate a motif by granting a pair of rays, a set of random pairs of rays where each ray appears in exactly one time. The ultimate goal of any inference algorithm is then to choose from among all possible pairings for the one that best satisfies some sort of aesthetic goal.

B. Geometric Pattern

When we talk about geometry in architecture, undoubtedly, we consider the practical geometry [17]. Practical geometry (Figs. 6-9) is a set of techniques and principles helping designers in establishment of designs. Practical geometry can be summarized as follows: [18]

1. Drawing Geometry

It refers to a set of techniques and principles applied in the following grounds leading to the creation of these fields:

- Creating measurement system (proportions)
- Drawing aids and steps of drawing
- Establishment of guiding drawings [17].

2. Geometry of Combination

With no doubt, the greatest aspect of geometry is its application in the combination of shapes and volumes, which is the subject of this paper as well. Only in this way we may differentiate geometrical structure of architectural plans and understand the designer's creativity and dominance. Combination in imaginary arts is a background which requires free action and ignoring any quantitative discipline and mathematics. To make a plan practical requires its commitment to disciplines and quantitative rules. The contrast between action and thought makes designers and artists to find several solutions for solving this contrast and coordinating thought and action. No doubt, practical geometry has produced the majority of solutions. Any design is valuable when it can be repeated in another form and content.

In practical geometry, as we can see in Islamic art and architecture, proportions of the building result from dividing the circle into regular shapes and forms. Therefore, proportions stem from the symbol of origin, circle, bearing all creation possibilities (Figs. 6-9).

3. Geometry of Stability

Undoubtedly, gravity is one of the most significant factors for forming architecture. Discovering the course of falling the objects down was the first architectural consequence and geometrically, it led to producing the right angle [17].

In brief, the subjects of art and geometry are inclusive so that art and architecture and urban planning have been integrated with geometry [19]. All in all, the geometric principles are not restrained to appreciable temples and monuments. The geometric shapes of all buildings, regardless of how humble and simple it is. Believers state by taking notice to geometric principles and building based on them, it leads to create inspiring and comfortable dwellings. Identical to Le Corbusier's accomplishment in the United Nations Building project, it can be assumed that this is a concept in the architect's unconscious application of divine proportion.

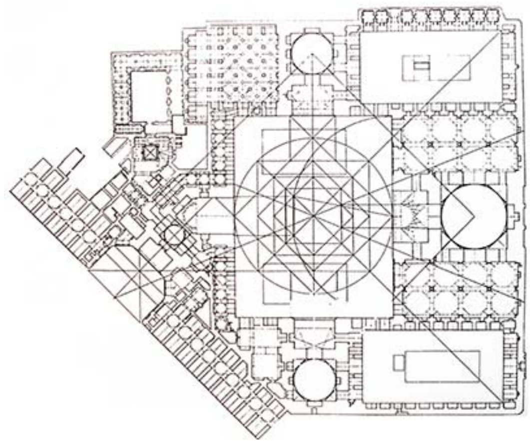


Fig. 6 Imam Masque- Isfahan -proportions of the plan [17]

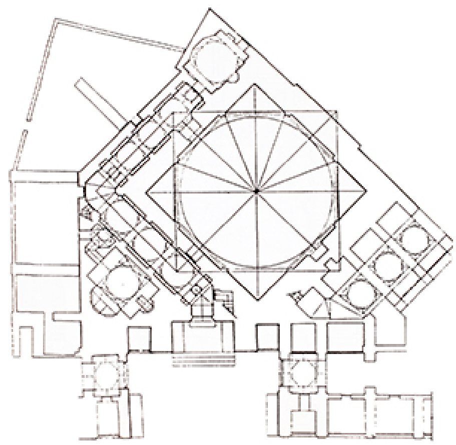


Fig. 7 Sheykh Lotf ALLAH Masque- Isfahan -proportions of the plan [17]

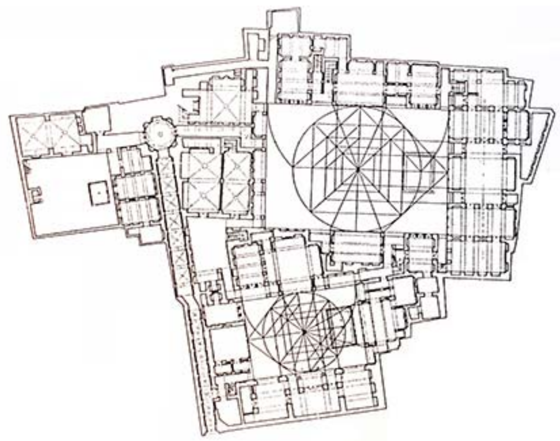


Fig. 8 Ghazviniha House, Isfahan -proportions of the plan [17]

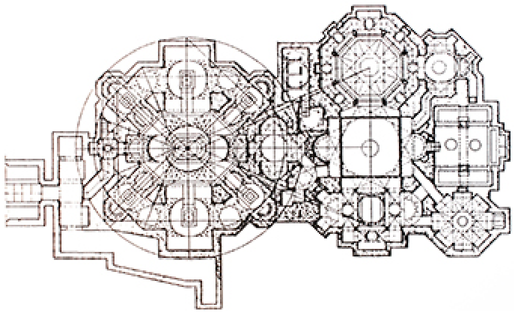


Fig. 9 Ganj Alikhan Bath, Kerman -proportions of the plan [17]

IV. Inference Mechanism (Strategy, Analysis, Solution and Design)

A. Shape Grammar

Design is evaluating with shapes and rules, and shape grammars are kinds of a mathematics science. They let us calculate them through algebras of shapes. Besides, Geometry is the basis of everything [20]. A confined set of labeled or unlabeled points, lines, planes, areas, or solids compose a shape. With this in mind, like Chomsky's phrase structure grammars, in linguistics, a language of Design is generated of conventional rules set applied to the shapes, termed Shape Grammar, which supports the visualization of the required form, the rules function and creating architectural forms [21]. Just similarly, to an alphabet of symbols, which defines the phrase structure grammars and generates one-dimensional strings of symbols, an alphabet of shapes defines shape grammars and generates n-dimensional shapes. Thus, and so the standard definition of phrase structure grammars is followed by definition of shape grammars [22]. Demonstrating the shape grammar, if A and B be shapes, a rule can be written in this form: A → B. When this rule A → B is applied, an instance of shape A is replaced with shape B [23].

Different possible designs (Fig. 10) created by the grammar [24] and Rectangular array was applied to the pattern [25] and proportions of a square generates octagon and hexadecagon (hexakaidecagon), proportions of an equilateral triangle generates hexagon and dodecagon, right triangles (isosceles right triangle and 30-60-90° triangle) are used for the rules of symmetry and proportion.

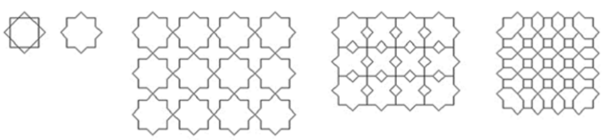


Fig. 10 Different possible designs created by the grammar [25]

B. Tensor

In mathematics, they are geometric objects describing linear relevance between scalars, geometric vectors, and other tensors. To give a preliminary illustration of such relations, the dot product, the cross product, and linear maps can be cited. In physics and engineering applications, geometric vectors are frequently applied, and scalars are tensors as well [26]. On linear algebra, Tensor calculus is a technique regarded as a follow-up. It can be assumed as generalization of classical linear algebra. Vectors and matrices are the dealing items in classical linear algebra (Fig. 11).

1. Geometrical Representation of Tensor

It makes sense to use, for the generic representation of a co-vector, two consecutive lines (or, in three dimensions, planes), such that the intersections with the axes are equal to the components of the co-vector (Figs. 12-14).

In the same vein in mathematical, tensors act upon the similar grammar, by moving a conceptual dot, as a basic vocabulary, a vector is generated and by transition of this vector a tensor is created , this argument in geometry is applied in generating the line of the point transition and furthermore a shape of the line.

There is some analysis (as shown in Table IV) based on hierarchy [27], [28] about the Islamic patterns. All of the patterns originate from square, equal triangle and circle and combination of them.

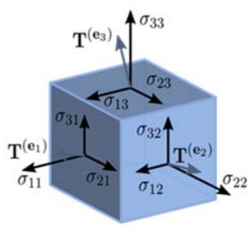


Fig. 11 Cauchy stress tensor, the tensor's components, in a three-dimensional Cartesian coordinate system

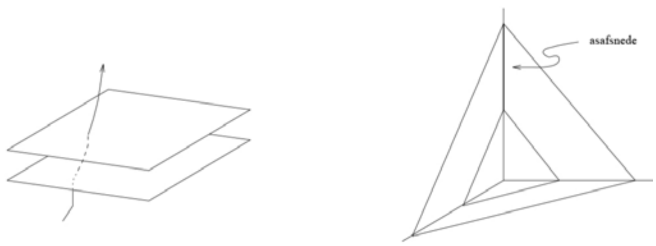


Fig. 12 Geometrical representation of a co-vector, and the determination of its components by computing the inverse of the intersection with the axes [26]

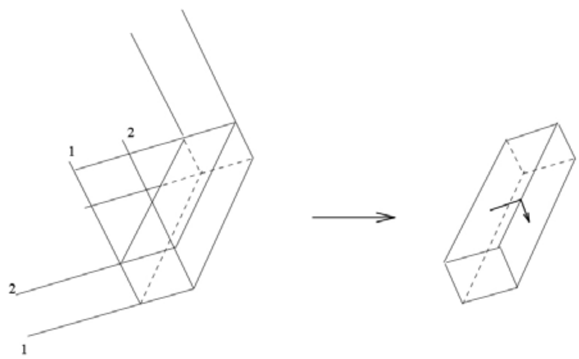


Fig. 13 Geometrical representation of an anti-symmetric covariant second order tensor [26]

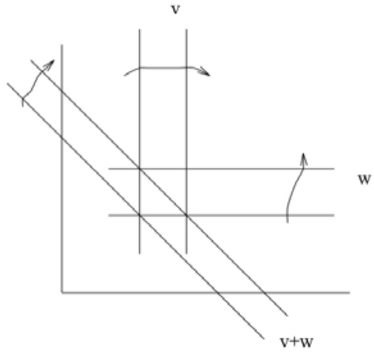


Fig. 14 Geometrical representation of the additional of co-vectors [26]

As it was mentioned, geometry is the science or doctrine of extension, or things extended; that is, of lines, surfaces, and solid. A point is the entity which has no parts, being considered in mathematics as indivisible, and may be expressed by dot. A line is that which produced by the motion of a point and has length without sensible breath of thickness. A strait line, or right line, is that which lies evenly between its extremes, without changing its direction, and is the nearest distance between the two points that terminate it. For the purpose of analyzing in the first step, it is intended squares and geometric forces (Fig. 15) and Square decomposed to the basic primitive forces.

In the second step, it is intended equal triangle and geometric forces (Fig. 16) and equal triangle decomposed to the basic primitive forces [29].

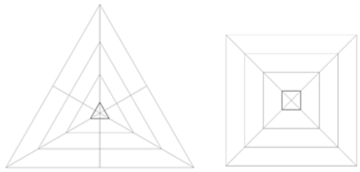
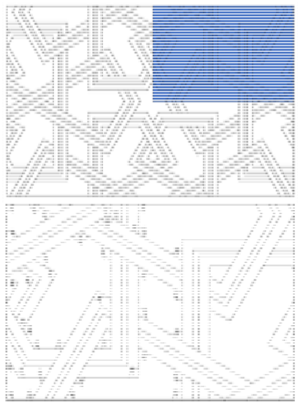
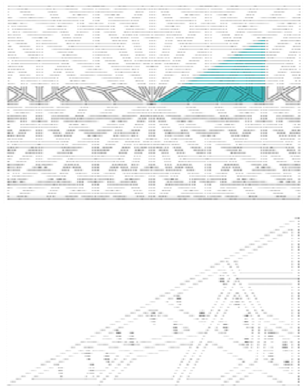
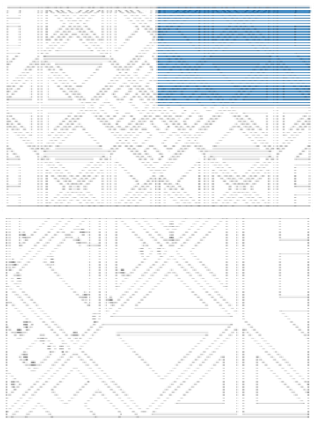
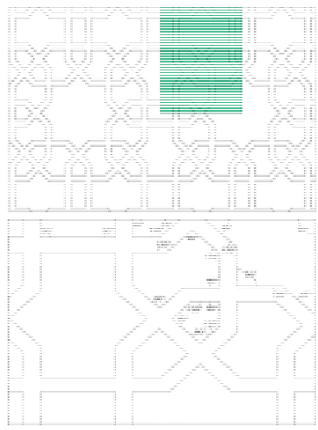
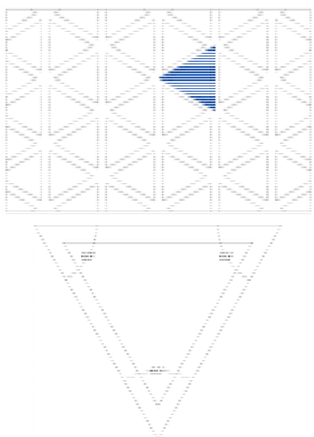
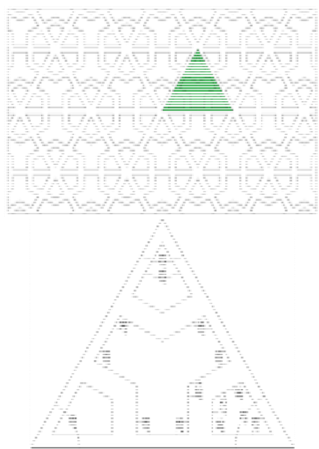
In the third step, it is intended circle and geometric forces (as shown in Fig. 17) and c decomposed to the basic primitive forces. The computational geometry chiefly affects straightly but multifaceted on the architecture, in order to architectural visualization by means of computer graphics, geographic information systems for land use planning, virtual reality for simulated computer-aided design, walkthroughs and finite element methods for structural simulation [30] for an appropriate illustration of the design informed based on latter consideration. Nevertheless, in dealing with architectural information, it leads to expect that problems emerge within any of those as well as the possibility of making precisely connections [31]. The 3D format of these analyses (as shown in Fig. 18) are as ensuing:

For creating a plan, square, equal triangle and circle combine (as shown in Fig. 19) to each other.

If any of the forms flourish (as shown in Fig. 20), the following image will be created [32].

As circle creates so many answers [33], We Omitted circle (as shown in Fig. 21) because it makes issue (question, theorem), indissoluble. The analysis has been done based on the statics and stable modes of any of geometric shapes.

TABLE IV
ANGLES ANALYSIS FOR DIFFERENT PATTERNS IN ISLAMIC GEOMETRIC PATTERNS

Base geometric shape	
	
Pattern NO.1	**Pattern NO.2**
	
Angles: 59,121, 90, 62, 118, 31, 149	Angles: 30, 60, 135, 45, 150, 90, 146, 116, 120
Pattern NO.3	**Pattern NO.4**
	
Angles: 45, 90	Angles: 45, 90, 135
Pattern NO.5	**Pattern NO.6**
	
Angles: 60	Angles: 60, 90, 120

Standing mode: Feasible standing status for a geometric shape is defined by the location of the center of mass in close relation to the intended base of support. The base of support is specified by the convex hull of all the points of the mesh projecting on the ground, which is referred as the support polygon**.** This is determined by the set of the lowest mesh vertices with respect to the gravity direction. Alternatively we flatten faces surrounding the support, within a threshold, considering expand support polygons which are too small to result in a stable configuration [34].To maintain static equilibrium [35] the center of mass of an object must project along the gravity direction into the support polygon [36].The design of self-supporting masonry is an elegant and ancient method that combines the form and function of geometry. The stability of structure, rather than intensity of reinforcement materials such as steel, precisely is dependent on its geometry. Architects and engineers have explored the link between geometry and mechanics since antiquity, and recent computer graphics approaches have developed this effort and created design tools that are informed by or optimized for structural targets [37].

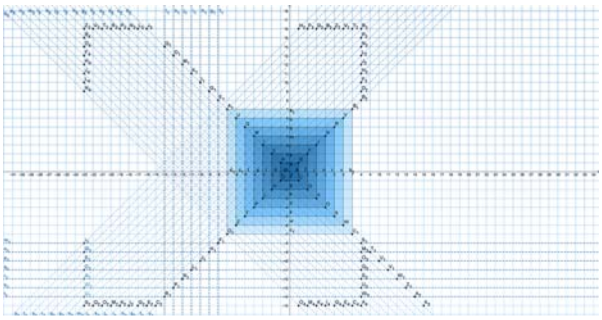


Fig. 15 Square decomposed to the basic primitive forces

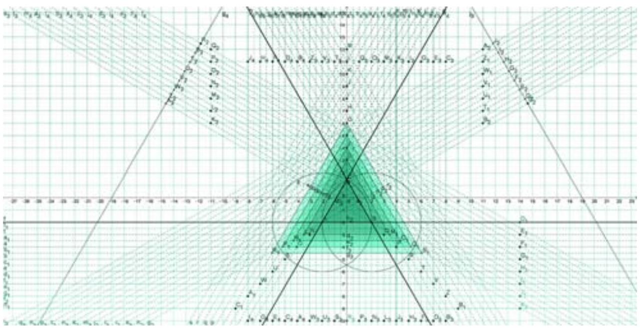


Fig. 16 Equal triangle decomposed to the basic primitive forces

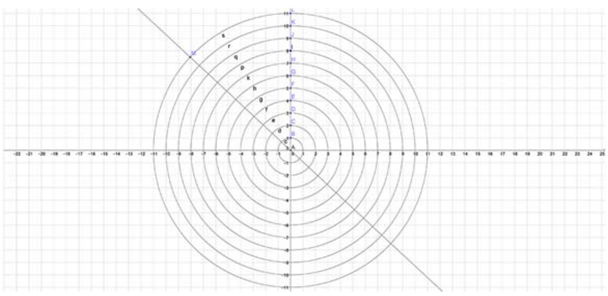


Fig. 17 Circle decomposed to the basic primitive forces

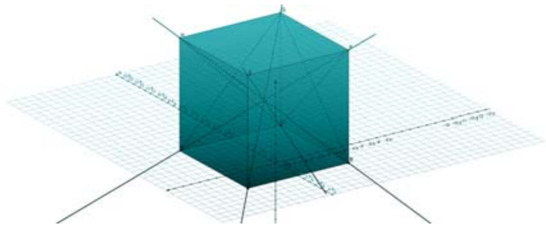


Fig. 18 Forces in Cubic

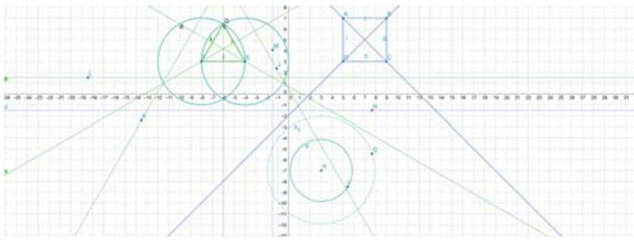


Fig. 19 Combination of square, equal triangle and circle's forces

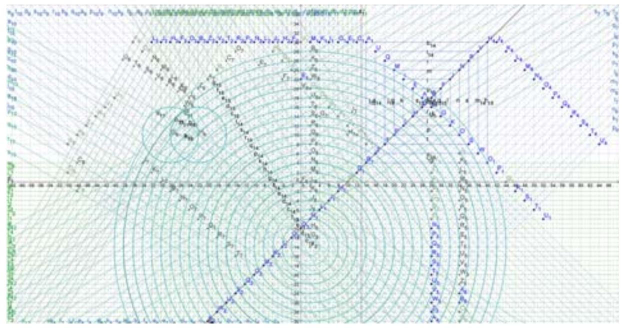


Fig. 20 Spreading the forces in combination of square, equal triangle and circle

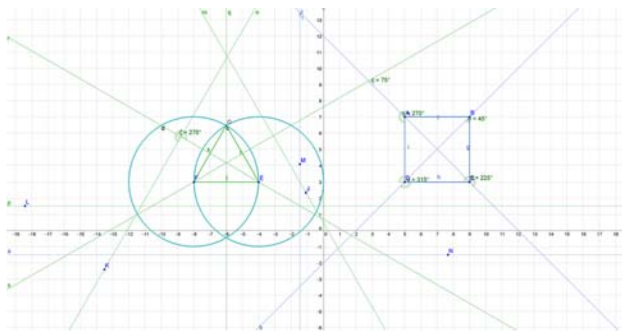


Fig. 21 Omitting the circle from analyzing process

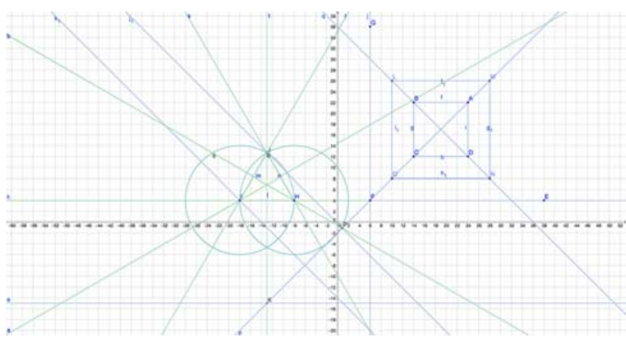


Fig. 22 All possible angles between square and equal triangle

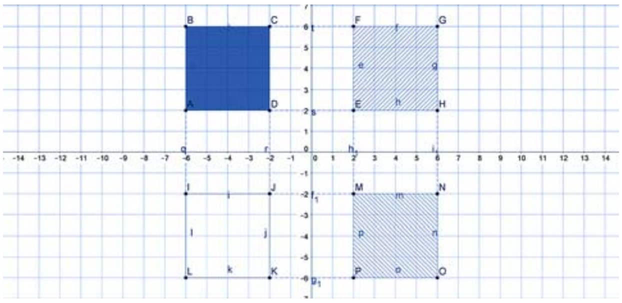


Fig. 23 Square transformation

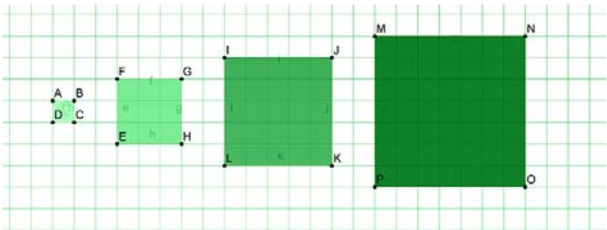


Fig. 24 Square scaling

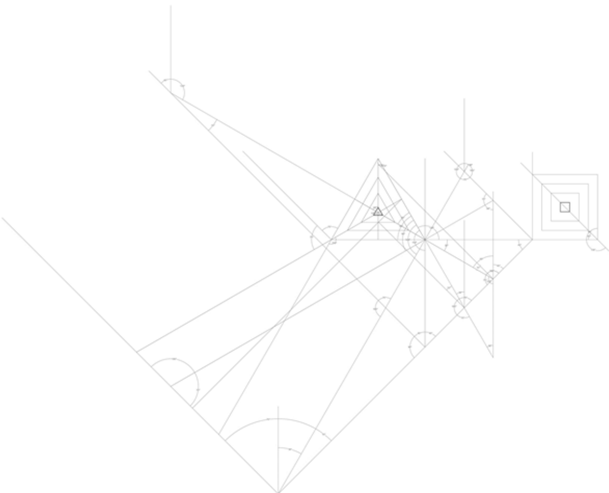


Fig. 25 An example of combination

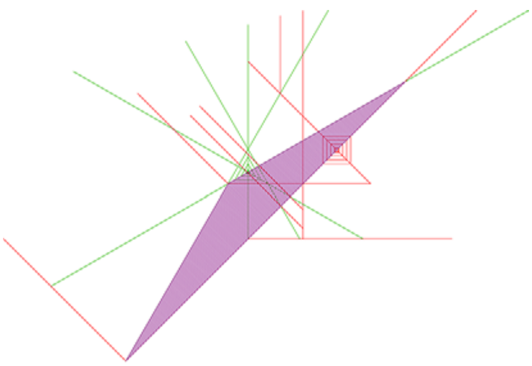


Fig. 26 An example of combination /Angles created: 15, 150

All possible scenarios are shown in Fig. 22 and Table V [38]. The angle created between the lines was measured in all cases and the results presented in Tables VI and VII were obtained. Some of these angles are not applied to the design of architectural space. Because in these angles there is no possibility of creating functional furniture. So, these angles can be removed after the analysis of the design process. If square and equal triangle transform [39]-[41] (Fig. 23), mirror, scale (Fig. 24), rotate or inscribed to each other, answers will remain unchanged.

TABLE V
EQUATION ANALYSIS

Shape	Expression	Equation	Category
Conic	C: $(x+18)^2 + (y-4)^2 = 100$ d: $(x+8)^2 + (y-4)^2 = 100$	$ax^2 + bxy + cy^2 + dx + ey = f$ $ax^2 + bxy + cy^2 + dx + ey = f$	
Line	a: $-5x + 8.66y = 124.64$ b: $-5x + 8.66y = 5.36$ e: $y = -15$ j: $x = 6$ j_1: $1366.03x + 1366.03y = -339.75$ k: $y = 4$ k_1: $x + y = -14$ p: $x - y = 2$ q: $x + y = 36$ r: $8.66x + 5y = -175.88$ s: $8.66x + 5y = -49.28$ t: $x = -13$	$y = mx + b$	Parametric form
Point	A = (24,22) B = (14,22) C = (14,12) D = (24,12) E = (38,4) F = (6,4) G = (6,36) H = (-8,4) I = (-18,4) J = (-13,12.66) K = (-13,-15) L = (10, 26) M = (28,26) N = (28,8) O = (10,8)		Polar coordinates
Segment	f = 10 $f_1 = 18$ g = 10 $g_1 = 18$ h =10 $h_1 = 18$ i = 10 $i_1 = 18$ l = 10 m = 10 n = 10		Polar coordinates

This combination can be done in any case as pointed out in Fig. 25. A few different modes of composition are shown in Figs. 26-34.

V. Discussion and Conclusion

Prior to addressing progressing mathematical statements in algebra and geometry, possessing an exhaustive understanding of geometric shapes is crucial. Square and triangle are the two original established shapes. Any of these shapes is formed by combining specific numbers of lines and/or curves.

TABLE VI
SORTING THE ANGLES BETWEEN FORCES OF SQUARE AND EQUAL TRIANGLE

	Internal	External
Square	45	45+45=90
	90	45+45+45=135
		45+45+45+45=180
		45+45+45+45+45=225
		45+45+45+45+45+45=270
		45+45+45+45+45+45+45=315
		45+45+45+45+45+45+45+45=360
Triangle (Equilateral triangle)	30	30+30=60
	60	30+30+30=90
		30+30+30+30=120
		30+30+30+30+30=150
		30+30+30+30+30+30=180
		30+30+30+30+30+30+30=210
		30+30+30+30+30+30+30+30=240
		30+30+30+30+30+30+30+30+30=270
		30+30+30+30+30+30+30+30+30+30=300
		30+30+30+30+30+30+30+30+30+30+30=330
		30+30+30+30+30+30+30+30+30+30+30+30=360
Circle	Omitted	Omitted
Combination of Square and triangle		105
		75
		15
		45
		60
		30
		120
		150
		30+75+105=210
		135
		345

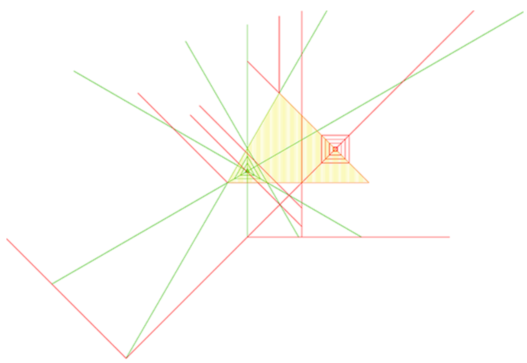


Fig. 27 an example of combination /Angles created: 45, 60, 75

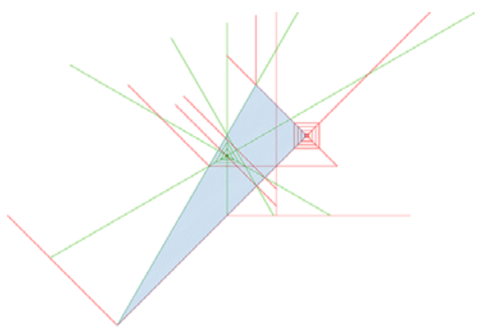


Fig. 28 An example of combination / Angles created: 15, 75, 90

TABLE VII
FINAL SORTING THE ANGLES BETWEEN FORCES OF SQUARE AND EQUAL TRIANGLE

Sort scheme	
Main number	**Add number**
15 (it can be removed, because it is not a functional angle for architectural plan)	+15
30 (it can be removed, because it is not a functional angle for architectural plan)	+15
45 (it can be removed, because it is not a functional angle for architectural plan)	+15
60	+15
75	+15
90	+15
105	+15
120	+15
135	+15
150	+30
180	+30
210	+15
225	+15
240	+30
270	+30
300	+15
315	+15
330	+15
345	+15
360	

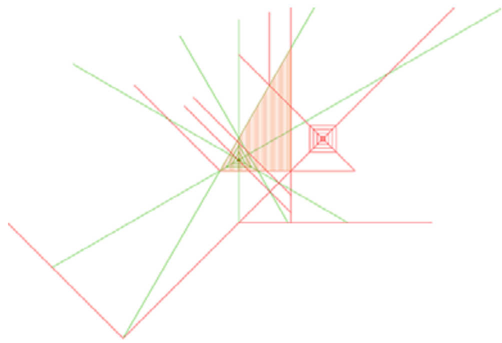


Fig. 29 An example of combination / Angles created: 30, 60, 90

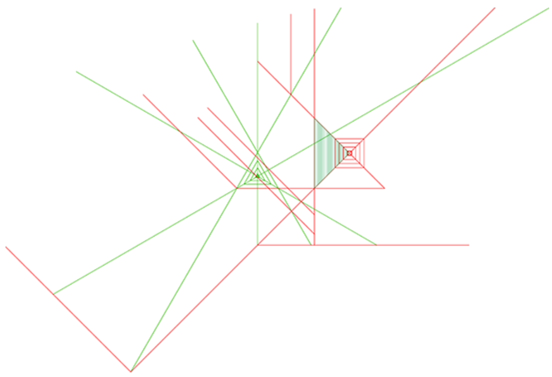


Fig. 30 An example of combination / Angles created: 45, 90

A square, for instance, is a four-edged figure formed by connecting four-line segments. All lines have the same value and are connected to each other to form the square.

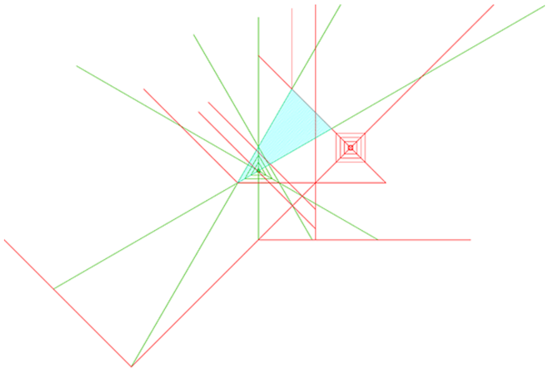


Fig. 31 An example of combination / Angles created: 30, 75

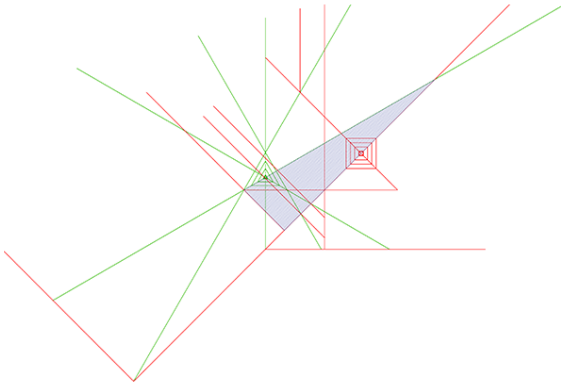


Fig. 32 An example of combination / Angles created: 15, 75, 90

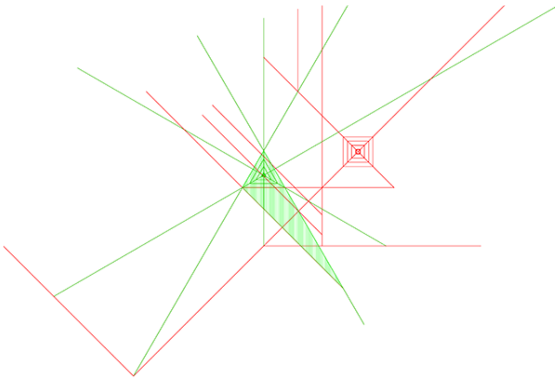


Fig. 33 An example of combination / Angles created: 15, 60, 105

A triangle consists of three line segments connected. Dissimilar to a square and a rectangle, the angles in a triangle can be of diverse measurements and not always right angles. Triangles frequently take their terms from the type of angles found within the triangle itself. A triangle with one right angle can be referred to as a right triangle.

A general category of two-dimensional primitives includes lines, points, and polygons; however, some people prefer to define triangles primitives, in view of the fact that each polygon can be created from triangles. All other graphic elements are constructed from these primitives. In three dimensions, in order to model complex 3D forms, primitives, polygons or triangles, located in three-dimensional space can be applied. Curves such as circles, Bezier curves, etc. In some cases, might be determined primitives; in other cases, ample straight, primitive shapes create composite forms of curves.

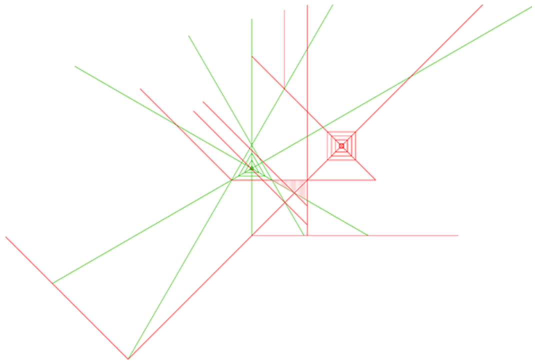


Fig. 34 An example of combination / Angles created: 30, 60, 90

Commonly used geometric primitives include:

- Points
- Lines and line segments
- Planes
- Triangle and other polygons

Note that Primitive forms and geometric shapes may appear in the form of a three-dimensional image in the following ways:

- Sphere
- Cube or boxes
- Pyramids

The aforementioned statements lead us to the conclusion that possessing own rules, every shape applies them in its composition likewise the rules. Thus, with the purpose of analyzing the cited cases, the geometry of each primitive shape individually has been perused.

- Squares representing all 4 polygons
- Triangle representing all 3 polygons

Owing to the fact that these rules in 2 dimensions can be applied to 3 dimensions, therefore, it may be regarded as certain that:

- Squares representing all 4 polygons and cubic
- Triangle representing all 3 polygons and pyramid

In addition to being able to recognize basic geometric shapes, it is significant to be able to categorize the shapes. in this process, it is necessary to look for what common characteristics they share. Shapes can be sorted and categorized in a wide array of methods.

The classification of geometric shapes, based on their formulation rules, forms the common principles that allow the architects to create scientific and mathematical rules in order to design the architectural plans layouts. This provides a better conception contingency of the human being's surrounding world. For instance, via the conception of various shapes, patterns or other tiled images can be generated. A variety of functions will be too complicated to be performed if a perfect knowledge of similarities among shapes does not exist.

A profound understanding can be based on the conception of these oversimplified properties or classifications; to give an illustration, within a bunch of shapes, recognizing the concept of parallel lines will let the wider conclusion be drawn. In this respect, if shapes with parallel lines are combined longer, wider, and larger parallel lines then exist. Consequently, composite shapes with parallel lines autonomously generate likewise further parallel lines. As a principle, it can be said that the combination of parallel lines remains parallel, which is the base fact in the field of geometry. A summary of the studies (except circle) is given below (Fig. 35).

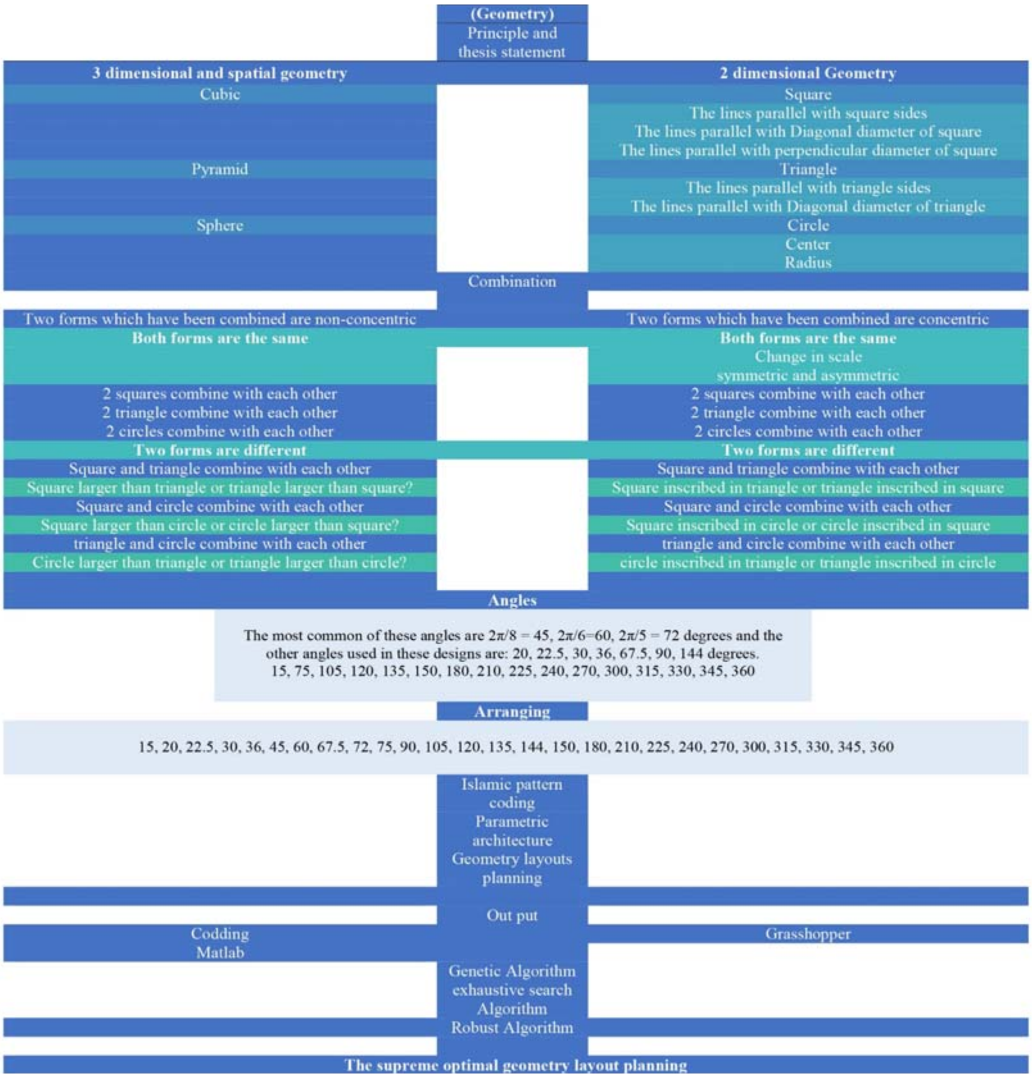


Fig. 35 The rules of combining geometric shapes

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