Critical Buckling Load of Carbon Nanotube with Non-Local Timoshenko Beam Using the Differential Transform Method

Tayeb Bensattalah, Mohamed Zidour, Mohamed Ait Amar Meziane, Tahar Hassaine Daouadji, Abdelouahed Tounsi

***Abstract*—**In this paper, the Differential Transform Method (DTM) is employed to predict and to analysis the non-local critical buckling loads of carbon nanotubes with various end conditions and the non-local Timoshenko beam described by single differential equation. The equation differential of buckling of the nanobeams is derived via a non-local theory and the solution for non-local critical buckling loads is finding by the DTM. The DTM is introduced briefly. It can easily be applied to linear or nonlinear problems and it reduces the size of computational work. Influence of boundary conditions, the chirality of carbon nanotube and aspect ratio on non-local critical buckling loads are studied and discussed. Effects of nonlocal parameter, ratios L/d, the chirality of single-walled carbon nanotube, as well as the boundary conditions on buckling of CNT are investigated.

***Keywords*—**Boundary conditions, buckling, non-local, the differential transform method.

I. Introduction

In most structures and nanostructures, the displacements increase gradually with increased applied load. If the applied load is too large (particularly for compressive structures); a small increase in applied load can lead to a sudden large increase in the displacements. Buckling refers to this transition to large, often catastrophic displacements also leading to the sudden failure of a mechanical component and structural instability, which is often called buckling. Buckling can occur due to thermal or mechanical loads. Sometimes this abrupt behavior can be exploited for useful purposes. But currently, carbon nanotubes (CNTs) have a wide application eventuality with the potential advantages on mechanical and thermal properties [1]-[4]. The first investigation by Iijima [5], [6] was single-walled carbon nanotube (SWNT) and multi-walled carbon nanotube (MWNT). Varieties of experimental, theoretical, and computer simulation approaches indicate that CNTs can be used in nanocomposite [7], nanoelectronics, and nanodevices [8].

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Many investigators have applied the continuum mechanics theory with successfully for analysis the behaviour of CNTs under different loading which are treated as beams, thin shells or solids in cylindrical shapes [9]-[14]. Based on the theory of nonlocal continuum mechanics, Xie et al. [15] investigated the effect of small size-scale on the buckling pressure of a simply supported MWNT. To study the responses of micro and nanostructures, the approach of continuum mechanics has been widely used for example the buckling and thermo-mechanical analysis of CNTs [16]-[18], the static and dynamic [19], [21], Recently, Bensattalah et al. [22] and Zidour et al. [23] have used the nonlocal elasticity constitutive equations to study vibration and buckling of CNTs.

Such study of buckling analysis of CNTs is of interest for better understanding of mechanical responses of CNTs. Sudak [24] carried out buckling analysis of multi-walled CNTs. Sears and Batra [25] investigated the buckling behavior and critical axial pressure of single walled and multi-walled CNTs by continuum mechanics models and molecular mechanics simulations. Semmah et al. [26] used the nonlocal continuum theory for the analysis of the effect of the chirality on critical buckling temperature of zigzag SWCNTs. Ranjbartoreh et al. [27] studied the buckling behaviour and critical axial pressure of the DWCNT. Kocaturk et al. [28] study the post-buckling analysis of Timoshenko beams with various boundary conditions under non-uniform thermal loading.

In the past 50 years, linear and nonlinear problems which appeared in physical, chemistry, mechanics, engineering applications, and various scientific areas are modeled and they are investigated by using so many approximating methods. Some of these numerical methods are DTM. This method was first proposed by Zhou [29] in solving linear and non-linear initial value problems in electrical circuit analysis. Several researchers have applied DTM method [30]-[34] applied DTM to obtain numerical solution of differential equations.

There are three types of SWCNTs used in this study which are armchair, zigzag and chiral tubules. The Young's moduli are calculated by Xing et al. [35] based on molecular dynamics (MD) simulation. Their results are in good agreement with the existing experimental ones [36], [37]. This present analysis is concerned with the use of the non-local Timoshenko beam model to analyse the non-local critical buckling loads of CNTs with various end conditions via the DTM. The influence of the chirality of CNT, aspect ratio of the SWCNTs and various end conditions are studied and discussed.

II. Basic Structure of CNT

Fig. 1 shows the structure of CNTs. Tokio [38] defined the diameter of the tube of CNTs by the mathematical expression; this diameter d is related to m and n as

$$d = a\sqrt{3(n^2 + m^2 + nm)} / \pi \quad (1)$$

where a is the length of the carbon–carbon bond which is $1.42\,A°$.

With the values m and n, CNT can be classified into zigzag ((n or m) =0), armchair (n =m) and chiral (n≠m).

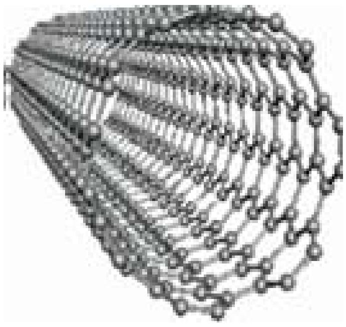

Fig. 1 SWCNT

III. Nonlocal Timoshenko Beam Theory and Boundary Conditions for Buckling of SWCNTS

The principle of virtual displacements states that if a body is in equilibrium, the total virtual work done,

$$\delta W = \delta U + \delta V \quad (2)$$

where δW, δU and δV are the total virtual work, virtual variation of the strain energy and the virtual potential energy of the axial load.

Firstly, the expression of the virtual strain energy is:

$$\delta U = \int_0^L \int_A (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xz}\delta\gamma_{xz})dAdx \quad (3)$$

where σ_{xx} is the normal stress, σ_{xz} the transverse shear stress, L the length and A the cross-sectional area of the CNT.

The strain–displacement relations are given by Wang [39]:

$$\varepsilon_{xx} = z\frac{d\phi}{dx}, \qquad \gamma_{xz} = \phi + \frac{dw}{dx} \quad (4)$$

By substituting (4) into (3), the virtual strain energy may be expressed as:

$$\delta U = \int_0^L \left(M\frac{d\delta\phi}{dx} + Q\left(\delta\phi + \frac{d\delta w}{dx}\right)\right)dx \quad (5)$$

where M and Q are the bending moment and shear force, respectively,

$$M = \int_A \sigma_{xx} z dA, \qquad Q = \beta\int_A \sigma_{xz} dA \quad (6)$$

where $\beta = 9/10$ is the shear correction factor of the Timoshenko beam theory [39].

Assuming that the nanotube is subjected to an axial compressive load P, the virtual potential energy δV of the axial external load is given by

$$\delta V = -\int_0^L P\frac{dw}{dx}\frac{d\delta w}{dx}dx \quad (7)$$

The total virtual work done, $\delta W = \delta U + \delta V$, must disappear. Thus, in view of (5) and (7), by performing integration by parts of equation $\delta W = 0$, one obtains

$$\int_0^L \left[\left(-\frac{dM}{dx} + Q\right)\delta\phi + \left(-\frac{dQ}{dx} + P\frac{d^2w}{dx^2}\right)\delta w\right]dx + \left[M\delta\phi + \left(Q - P\frac{dw}{dx}\right)\delta w\right]_0^L = 0 \quad (8)$$

In 0 < x < L, δφ and δw are arbitrary, and we obtain the following two equilibrium equations:

$$-\frac{dM}{dx} + Q = 0 \quad (9)$$

$$-\frac{dQ}{dx} + P\frac{d^2w}{dx^2} = 0 \quad (10)$$

The boundary conditions of the nonlocal Timoshenko beam theory are of the form

$$w = 0 \text{ or } Q - P\frac{dw}{dx} = 0 \quad (11)$$

$$\phi = 0 \text{ or } M = 0 \quad (12)$$

The stress at a reference point in the nonlocal continuum elasticity theory is considered to be a functional of the strain field at every point in the body. The classical theory of elasticity is obtained when the effects of strains at every point other than x are neglected. For homogeneous and isotropic elastic solids, this approach is given by Eringen [40] and has been widely used in various types of nanostructures (nano FGM structures, nanotube, etc.) such as the buckling [41] and free vibration by Zhao et al. [42].

Non-local relations for present nano-beams can be approximated to a one-dimensional form as

$$\sigma_{xx} - e0a^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E(\varepsilon_{xx}) \tag{13}$$

$$\tau_{xz} = G(\gamma_{xz}) \tag{14}$$

where $\sigma_{xx}, \varepsilon_{xx}, \tau_{xz}$ and γ_{xz} are the normal stress, the normal strain, the transverse shear stress and the transverse shear strain, respectively. E and G are the Young's and shear modulus. The coefficient e0a represents the nonlocal parameter.

The bending moment M and the shear force T for the non-local model can be expressed based on (4), (10), (11), (13) and (14):

$$M = EI \frac{d\phi}{dx} + e0a^2 \left(P \frac{d^2 w}{dx^2} \right) \tag{15}$$

$$Q = \beta AG \left(\phi + \frac{dw}{dx} \right) \tag{16}$$

where A is the cross-section area of the beam, ($I = \int_A z^2 dA$) is the moment of inertia.

It can obtain the following differential equation of a non-local Timoshenko beam theory by substituting (15) and (16) into (10) and (11).

$$EI \frac{d^2 \phi}{dx^2} + e0a^2 P \frac{d^3 w}{dx^3} - \beta AG \left(\phi + \frac{dw}{dx} \right) = 0 \tag{17}$$

$$\beta AG \left(\frac{d\phi}{dx} + \frac{d^2 w}{dx^2} \right) - P \frac{d^2 w}{dx^2} = 0 \tag{18}$$

TABLE I
THE ASSOCIATED BOUNDARY CONDITIONS

Simply supported ends	$w = \frac{\partial^2 w}{\partial x^2} = 0$ at x=0, L
Clamped ends	$w = \frac{\partial w}{\partial x} = 0$ **at x = 0, L**
Cantilever	$w = \frac{\partial w}{\partial x} = 0$ **at x=0** $\frac{\partial^2 w}{\partial x^2} = \frac{\partial^3 w}{\partial x^3} = 0$ **at x=L**

The dimensionless elastic buckling of nonlocal Timoshenko may be as shown:

$$\frac{d^4 \overline{w}}{d\xi^4} + \left[\frac{PL^2/EI}{1 - (P/\beta AG) - e0a^2 (P/EI)} \right] \frac{d^2 \overline{w}}{d\xi^2} = 0 \tag{19}$$

where

$$\overline{w} = w/L, \quad \xi = x/L \tag{20}$$

The associated boundary conditions handled in this paper are given in Table I. The non-dimensional boundary conditions are given in Table II.

TABLE II
NON-DIMENSIONAL BOUNDARY CONDITIONS

Simply supported ends	$\bar{W} = \frac{d^2 \bar{W}}{d\xi^2} = 0$ **at ξ = 0, 1**
Clamped ends	$\bar{W} = \frac{d\bar{W}}{d\xi} = 0$ at ξ = 0, 1
Cantilever	$\bar{W} = \frac{d\bar{W}}{d\xi} = 0$ at ξ = 0 $\frac{d^2 \bar{W}}{d\xi^2} = \frac{d^3 \bar{W}}{d\xi^3} = 0$ at ξ=1

IV. Differential Transformation Method

The DTM is an iterative process aims to find the solution of differential equations. Several authors [30]–[34] have applied DTM in different mechanical and physical problems. Using differential transformation technique, the ordinary and partial differential equations can be transformed into algebraic equations. In this method, certain transformation rules are used to both the governing differential equations of motion and the boundary conditions of the system in order to transform them into a set of algebraic equations as presented in Tables I and II.

Based on these works [30]–[34], the differential transform of the function $f(x)$ is given by

$$F(k) = \frac{1}{k!} \left(\frac{d^k f(x)}{dx^k} \right)_{x=x_0} \tag{21}$$

where $f(x)$ is the original function and $F(k)$ is the transformed function.

As in these works [30]–[34], the inverse transformation is defined as

$$f(x) = \sum_{k=0}^{\infty} (x - x_0)^k F(k) \tag{22}$$

By substituting (21) into (22), we have

$$f(x) = \sum_{k=0}^{\infty} \frac{(x - x_0)^k}{k!} \left(\frac{d^k f(x)}{dx^k} \right)_{x=x_0} \tag{23}$$

Based on finite series (23) can be written as follows:

$$f(x) = \sum_{k=0}^{m} \frac{(x - x_0)^k}{k!} \left(\frac{d^k f(x)}{dx^k} \right)_{x=x_0} \tag{24}$$

and (23) implies that

$$f(x)=\sum_{k=m+1}^{\infty}\frac{(x-x_0)^k}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0}$$

is neglected as it is small. Usually, the values of m are decided by a convergence of the results.

As in works [34], the theorems that are frequently used in the transformation of the differential equations and the boundary conditions are introduced in Tables III and IV, respectively.

TABLE III
TRANSFORMED FUNCTION BY DTM [34]

Original Function	Transformed Function
$f(x)=g(x)\pm h(x)$	$F(k)=G(k)\pm H(k)$
$f(x)=\lambda g(x)$	$F(k)=\lambda G(k)$
$f(x)=g(x)h(x)$	$F(k)=\sum_{l=0}^{k}G(l)H(k-l)$
$f(x)=\frac{d^n g(x)}{dx^n}$	$F(k)=\frac{(k+n)!}{k!}G(k+n)$
$f(x)=x^n$	$F(k)=\delta(k-n)=\begin{cases}0 & if \quad k\neq n\\ 1 & if \quad k=n\end{cases}$

TABLE IV
TRANSFORMED OF ORIGINALS BOUNDARY CONDITIONS BASED ON DTM [34]

$x=0$		$x=1$	
Original B.C.	Transformed B.C.	Original B.C.	Transformed B.C.
$f(0)=0$	$F(0)=0$	$f(1)=0$	$\sum_{k=0}^{\infty}F(k)=0$
$\frac{df}{dx}(0)=0$	$F(1)=0$	$\frac{df}{dx}(1)=0$	$\sum_{k=0}^{\infty}kF(k)=0$
$\frac{d^2 f}{dx^2}(0)=0$	$F(2)=0$	$\frac{d^2 f}{dx^2}(1)=0$	$\sum_{k=0}^{\infty}k(k-1)F(k)=0$
$\frac{d^3 f}{dx^3}(0)=0$	$F(3)=0$	$\frac{d^3 f}{dx^3}(1)=0$	$\sum_{k=0}^{\infty}k(k-1)(k-2)F(k)=0$

V. DTM FORMULATION AND SOLUTION PROCEDURE

According to the basic transformation operations of originals functions introduced in Table III using DTM, the transformed form of the governing (19) may be obtained as:

$$W(k+4)=\frac{-\left[PL^2/EI\right](k+1)(k+2)W(k+2)}{\left[1-(P/\beta AG)-e0a^2(P/EI)\right](k+1)(k+2)(k+3)(k+4)} \quad (25)$$

The buckling load of the local Euler beam model ($\beta\to\infty, e0a=0$), local Timoshenko beam model ($\beta\to 9/10, e0a=0$), nonlocal Euler beam model ($\beta\to\infty, e0a\neq 0$) and nonlocal Timoshenko beam model ($\beta\to 9/10, e0a\neq 0$).

The transformed form of boundary conditions is presented in Table IV. The buckling load may be derived by incorporating the transformed boundary conditions in (25):

$$A_{j1}^{(n)}(P)c_1+A_{j2}^{(n)}(P)c_2=0 \quad j=1,2,3,..n \quad (26)$$

Here, A_s are polynomials in terms of P corresponding to n^{th} term. Solving (26) in matrix form and studying the Existence condition of the non-trivial solutions yield the following characteristic determinant:

$A_{j1}^{(n)}(P), A_{j2}^{(n)}(P)$ are polynomials corresponding to n^{th} term.

When (26) is written in matrix form, we get

$$\begin{bmatrix}A_{11}^{(n)}(P) & A_{12}^{(n)}(P)\\ A_{21}^{(n)}(P) & A_{22}^{(n)}(P)\end{bmatrix}\begin{Bmatrix}c_1\\ c_2\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \quad (27)$$

The eigenvalue equation is obtained from (27) as

$$\begin{bmatrix}A_{11}^{(n)}(P) & A_{12}^{(n)}(P)\\ A_{21}^{(n)}(P) & A_{22}^{(n)}(P)\end{bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \quad (28)$$

Solving (28), we get $P=P_j^{(n)}$ where $j=1,2,3,..n$. The value of n is obtained by:

$$\left|P_j^{(n)}-P_j^{(n-1)}\right|\leq\varepsilon \quad (29)$$

where ε is the tolerance parameter.

If (29) is satisfied, then we have j^{th} eigenvalue $P_j^{(n)}$. In this study, the value of n= 50 was enough.

VI. NUMERICAL RESULTS AND DISCUSSIONS

Based on MD simulation the Young's moduli used in this study of three types of SWCNTs, armchair, zigzag and chiral tubules, are calculated by Xing et al. [30] (Table V).

The parameters used to investigate the effect of boundary conditions on the critical buckling loads of SWCNTs are given as follows: the effective thickness of CNTs taken to be 0.285 nm, and the Poisson's ratio υ=0.19.

In the present study, Fig. 2 depicts the influence of scale coefficients on the dimensionless critical buckling loads for pinned end beam of Zigzag nanotube (14, 0). The nonlocal parameter (e0a) values of SWCNT were taken in the range of 0–2 nm. From Fig. 2, it is observed that there is a significant influence of small scale parameter on the critical buckling loads of zigzag nanotube (14, 0) beam pinned end. Considering that nonlocal model is always smaller than the local (classical) model implies that the employment of the local model for SWCNT analysis would lead to an overprediction if the small length scale effects between the individual carbon atoms are neglected. Further, with increase in aspect ratio values, the critical buckling loads obtained become smaller compared to local model.

To analyse the difference between the nonlocal Euler (NEB) and nonlocal Timoshenko (NTB) beam model, with respect to length-to-diameter ratio loads ratios (PE/PT) of three types of SWCNTs, armchair, zigzag and chiral tubules are illustrated in Fig. 3. Fig. 3 shows that if L/d > 40 then the shear effect is negligible and if L/d < 40 then the shear effect is significant on the ratio (PE/PT).

The effect of the boundary conditions, on the non-local critical buckling load for different chirality of SWCNTs, armchair, zigzag and chiral is presented in figures 4-6. The ratio of the length to the diameter (L/d), is taken as 5 and 60 and small scale effects are considered (e0a=2 nm). It is clearly seen from the figures that the ranges of the critical buckling loads for these boundary conditions of SWCNTs are quite different, the range is the largest for clamped end beam, but the range is the smallest for clamped-free beam. it can be clearly seen that the boundary conditions effect reduces the buckling loads.

There are three types of SWCNTs is used in this analyses which are, armchair(20,20), zigzag(14,0) and chiral(16,8), the ranges of the non-local critical buckling loads for these chirality obtained of SWCNTs are also quite different. The reason for this difference perhaps is attributed to the increasing or decreasing of CNT diameter.

TABLE V
THE VALUES OF YOUNG'S MODULUS OF SINGLE CNT FOR DIFFERENT CHIRALITY'S [35]

(n,m)	Young's modulus (SWNT) (GPa) [35]	(n,m)	Young's modulus (SWNT) (GPa) [35]
Armchair		Zigzag	
(8,8)	934.960	(14,0)	939.032
(10,10)	935.470	(17,0)	938.553
(12,12)	935.462	(21,0)	936.936
(14,14)	935.454	(24,0)	934.201
(16,16)	939.515	(28,0)	932.626
(18,18)	934.727	(31,0)	932.598
(20,20)	935.048	(35,0)	933.061
Chiral			
	(12,6)	927.671	
	(14,6)	921.616	
	(16,8)	928.013	
	(18,9)	927.113	
	(20,12)	904.353	
	(24,11)	910.605	
	(30,8)	908.792	

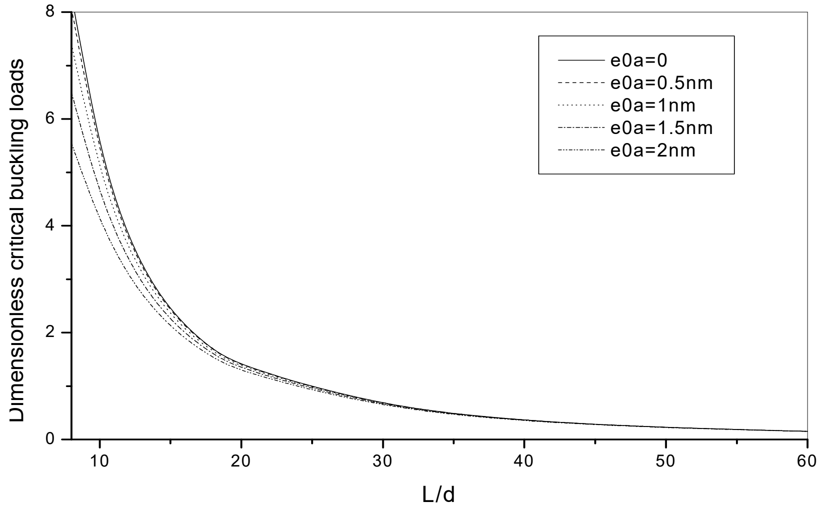

Fig. 2 Relation between the values of dimensionless critical buckling loads and the aspect ratio (L/d) for pinned end beam of Zigzag nanotube (14,0) with different scale coefficients

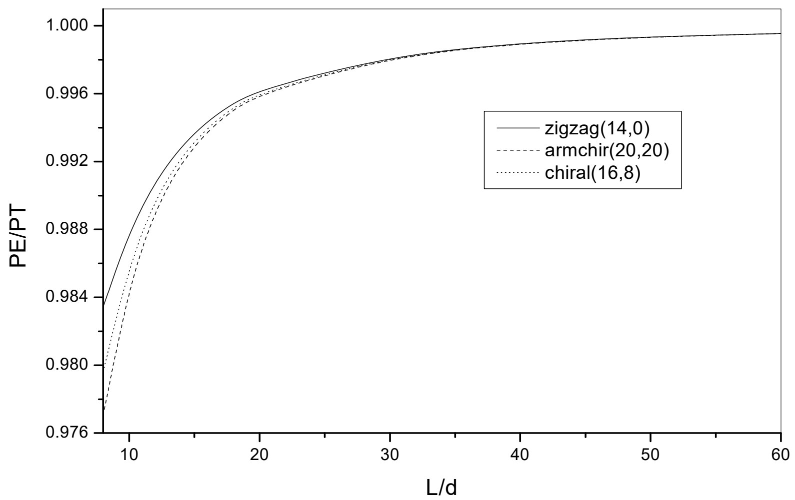

Fig. 3 Relation between the values of ratio (PE/PT) and the aspect ratio (L/d) for pinned end beam with different chirality's

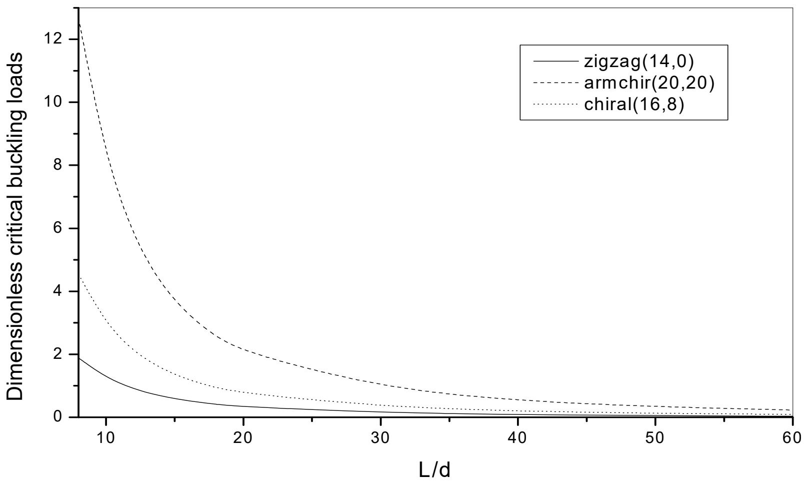

Fig. 4 Effect of chirality's and the aspect ratio (L/d) on the dimensionless critical buckling loads for clamped-free beam

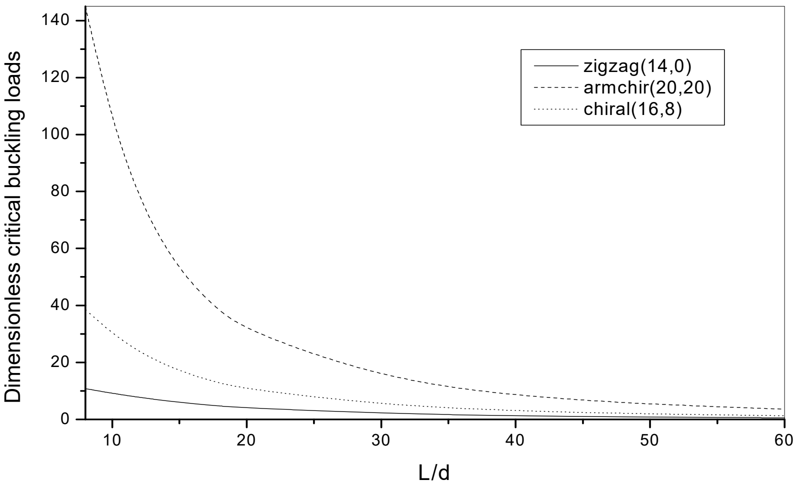

Fig. 5 Effect of chirality's and the aspect ratio (L/d) on the dimensionless critical buckling loads for clamped end beam

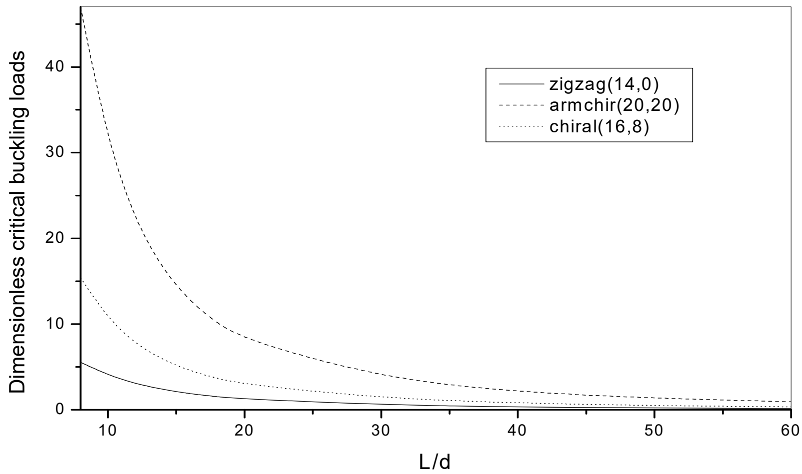

Fig. 6 Effect of chirality's and the aspect ratio (L/d) on the dimensionless critical buckling loads for pinned end beam

The variation of dimensionless critical buckling loads of SWCNTs armchair, chiral and zigzag chirality with different length-to-diameter ratios and different boundary conditions from e0a =2 nm based on the non-local Timoshenko beam model are listed in Tables VI-VIII. Their results show the dependence of the different chirality's of CNT, aspect ratio and, effect of boundary conditions on the non-local critical buckling loads.

TABLE VI
NON-LOCAL DIMENSIONLESS CRITICAL BUCKLING LOAD FOR DIFFERENT ARMCHAIR CHIRALITY'S

Armchair	pinned end beam		clamped end beam		clamped-free beam	
	L/ d =10	L/d =20	L/d =10	L/d =20	L/d =10	L/d =20
(8,8)	3.943	1.224	8.8577	3.943	1.224	0.326
(10,10)	6.769	1.969	17.324	6.769	1.969	0.513
(12,12)	10.29	2.880	28.886	10.29	2.880	0.742
(14,14)	14.50	3.958	43.455	14.50	3.958	1.012
(16,16)	19.47	5.225	61.184	19.47	5.225	1.330
(18,18)	24.92	6.608	81.119	24.92	6.608	1.677
(20,20)	31.13	8.186	104.123	31.138	8.186	2.073

TABLE VII
NON-LOCAL DIMENSIONLESS CRITICAL BUCKLING LOAD FOR DIFFERENT CHIRAL CHIRALITY'S

Chiral	Pinned end beam		clamped end beam		clamped-free beam	
	L/d=10	L/d=20	L/d =10	L/d =20	L/d=10	L/d=20
(12,6)	5.4576	1.625	13.3004	5.4576	1.624	0.426
(14,6)	7.0875	2.049	18.3939	7.0875	2.049	0.533
(16,8)	10.561	2.945	29.8769	10.561	2.945	0.758
(18,9)	13.814	3.779	41.086	13.814	3.779	0.967
(20,12	19.166	5.136	60.4167	19.166	5.136	1.307
(24,11)	23.827	6.3217	77.4401	23.8279	6.3217	1.6050
(30,8)	30.3705	7.9833	101.5937	30.3705	7.9833	2.0218

TABLE VIII
NON-LOCAL DIMENSIONLESS CRITICAL BUCKLING LOAD FOR DIFFERENT ZIGZAG CHIRALITY'S

Zigzag	**pinned end beam**		**clamped end beam**		**clamped-free beam**	
	L/d =10	L/d =20	L/d =10	L/d =20	L/d =10	L/d =20
(14,0)	4.0635	1.2578	9.1861	4.0635	1.2578	0.3344
(17,0)	6.4996	1.900	16.4645	6.4996	1.8999	0.4960
(21,0)	10.5553	2.9476	29.7529	10.5553	2.9476	0.7591
(24,0)	14.1634	3.8706	42.2553	14.1634	3.8706	0.9907
(28,0)	19.7653	5.2975	62.3055	19.7653	5.2975	1.3487
(31,0)	24.5653	6.5173	79.8365	24.5653	6.5173	1.6547
(35,0)	31.7526	8.3414	106.4312	31.7526	8.3414	2.1122

The critical buckling load increases as one transits from the armchair (20,20) to the zigzag (14,0) and then chiral (16,8), when the diameter of nanotube is decreasing. This reduction in the non-critical buckling load is affected by the diameter or long of the nanotube, which results in a more significant distortion of (C–C) bonds and low critical loads. In additional, the ranges of the critical buckling loads for various boundary conditions of SWCNTs are quite different, and this variation is pronounced in the larger long and diameter.

VII. CONCLUSIONS

This article studies the influence of various boundary conditions, the aspect ratio and the chirality of SWCNTs on the dimensionless nonlocal critical buckling loads using non-local Euler Bernoulli and Timoshenko beam theory. The different parameters are included in the formulations and the governing equations are solved by the DTM and the non-local critical buckling loads are obtained.

For this study, it is observed that the nonlocal critical buckling loads increases by increasing the diameter of SWCNTs and the variation of boundary conditions. Besides, the increasing or decreasing of long of SWCNTs affects the critical load. This phenomenon is that a CNT with higher long has a larger curvature, so it results in a more significant distortion of (C–C) bonds and low critical loads. In additional, with increase in aspect ratio values, the non-local critical buckling loads decrease and become smaller compared to local model. The present study is helpful in the use of SWCNTs, as nanoelectronics, nanocomposites and mechanical sensors.

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