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Model of Transhipment and Routing Applied to the Cargo Sector in Small and Medium Enterprises of Bogotá, Colombia

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***Abstract*—**This paper presents a design of a model for planning the distribution logistics operation. The significance of this work relies on the applicability of this fact to the analysis of small and medium enterprises (SMEs) of dry freight in Bogotá. Two stages constitute this implementation: the first one is the place where optimal planning is achieved through a hybrid model developed with mixed integer programming, which considers the transhipment operation based on a combined load allocation model as a classic transshipment model; the second one is the specific routing of that operation through the heuristics of Clark and Wright. As a result, an integral model is obtained to carry out the step by step planning of the distribution of dry freight for SMEs in Bogotá. In this manner, optimum assignments are established by utilizing transshipment centers with that purpose of determining the specific routing based on the shortest distance traveled.

***Keywords*—**Transshipment model, mixed integer programming, saving algorithm, dry freight transportation.

I. Introduction

ACCORDING to the operation conditions of the SMEs in the field of dry freight transportation located in Bogota, and working throughout Colombia, some difficulties arise due to the high costs, particularly in fuels and consumables [1]; in the same manner, [2] developed a prospective study targeting these type of enterprises, where the "need to raise the operative standard level based on the high reliability of the performance and of the delivery, where the efficient use of the transportation and optimization routes was emphasised" was established. This fact gives support to the idea of developing different techniques that lead to take decisions in an efficient way about how to handle the transportation operation, and consequently, reducing its costs; then the good use of tools such as operations research (OR), which are becoming more helpful in facing challenges in the ever-changing world of business, is of paramount importance. For this reason, this work developed a mathematical model within the context of Mixed Integer Programming (MIP) that contributes to improve the planning process of goods distribution in SMEs committed to this task in Colombia.

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II. Reference Framework

A. Supply Chain and Logistics

Following the evolution of logistics, it is the supply chain which focuses on managing the integration of relationships between customers and suppliers to satisfy the demand of the final consumers.

The management of the supply chain emerges as a model that seeks to achieve synergies through integration of key business processes along the supply chain. The main objective is to serve the final costumer and other resource owners in the most effective and efficient manner as possible, i.e. with goods and/or services of greater value, to the final customer, obtained at the lowest possible costs perceived by final client [3]. In this way, the supply chain is a process that significantly lowers the costs allowing companies to have better competitiveness indices, emphasizing in the interaction between logistics activities and marketing, sales and production areas within the company and among external agents as well [4], and in other activities such as planning, design and material flow control, information and money along the supply chain to offer an added value to the final customer in an effective and efficient way [5]. In general, it can be said that supply chain is a supplier's integration; factories and distributors have the task of obtaining and transforming materials in finished products and the subsequent distribution to consumers [6]. Thus, the supply chain is the application of logistics under the strategy of teamwork of every actor, where it seeks to deliver the costumers a much higher value of the product or service with respect to its perception [4].

According to [7], several principles and concepts that rule logistics planning are derived from the unique nature of its activities, especially transportation. Among these strategies are: total cost, differentiated distribution, mixed strategies, postponement, consolidation and standardization.

Lastly, the relationships present in the supply chain tend to be complex given the diversity of participants involved and the interests of each of them. In the face of this situation, there have been efforts to propose strategies that facilitate a better integration of the actors in such a way that the supply chain is more effective and competitive [8].

B. Distribution and Transportation

Transportation is the link that connects the raw materials supply and other components required to the production process. Distribution is the second tier that moves the finished products inside the factories or distribution hubs up to the end customer. These generally take care of the activities related, directly or indirectly, to the need of placing the products in the corresponding destination points according to some safety conditions, speed and cost. This process includes the means for that goal, the preservation and conservation of the good condition of the product and the customer's perception about the product obtained and the service given [9]. On the other hand, the complexity of business and transport operations is influenced by the different applicable technologies, service demands and constantly changing international legislation. Thus, transportation spends approximately 40% of the total expenses allocated to distribution only; the remaining 60% is distributed among storage 30%, inventory 18% and managing 12% [10].

C. Mathematical Modeling

Taking into account that a model is a mathematical description of a tangible object that exists in a universe far from the abstract knowledge [11], also it is seen as a formal expression in mathematical language of the relationships between the components of a system [12], it can be said that its construction implies the selection and quantification of the components, variables, and relations present in the system in order to present it with the proper level of detail. Such a simplified representation of the reality by a mathematical system helps to improve the level of understanding of how it works [13]. Hence, it is of primary importance to use the tools, such as OR, which is more useful when addressing challenges coming out of the dynamic business world, where its implementation to solving problems in production and in reducing costs is crucial [14], thus making the best decision (optimum) in a problem with limited resources [15]. In this way, models have been found that formulate some objective functions, incorporating new parameters such as the demand coverage, cargo factors and vehicle transfers in transportation problems [16]. Currently, another point of view is being considered, such as the use of metaheuristics in the exploration of the domain of the solution, which facilitates the integration of existing models and incorporates graphic interfaces, thereby stimulating the development of new methods, which is characterized by its adaptability, interactivity, efficiency, and flexibility.

In study [17], the exact models for the case where various customers are supplied either from platforms or warehouses are very common and easy to understand; they develop formulations based on integer programming within the general problem of vehicle routing [18]. A lineal integer program for solving these problems with multiple deposits and heterogeneous fleet and time spans is presented in [19]. In [20], a model of mixed integer lineal programming (MILP) is proposed; the objective is to minimize the cost of the route with heterogeneous fleet of available vehicles. Another MILP with a branch and bound technique is proposed for a fixed fleet problem with picking-up and delivery in [21]. In [22], a mixed integer programing model is presented in which the heterogeneous fleet of vehicles is available maximizing the total net income as objective function, while maximum and minimum demand restrictions are established. Otherwise, and if there are models with some degree of algorithmic complexity, a strategy that incorporates a step by step solution is proposed; this technique consists of dividing the problem in parts and solving independently each of these parts. In logistics, the design of models through stages has found solutions to various problems such as locating platforms, keeping in mind an initial stage for locating the central platform, then selecting the means of transportation and the numbers of travels that must be done [23]. Other applications seek to minimize inventory costs along the supply chain containing nonlinear constraints [24]. There are studies for the distribution of costs in the supply chain which propose a stochastic mathematical model for the analysis in networks and multiple stages [25].

III. Application Context

The area of application for this work encompassed the distribution logistics operation of the dry freight, which consists of goods in solid form that are protected by an appropriate packaging that facilitates its rapid mobilization [26]. That study was developed for SMEs operating from Bogota, where they must be distributed from the distribution centers (DC) to different system clients (SC) using potential transshipment centers (TC) facilitating the exchange between them. The general network that shows this behavior is portrayed in Fig. 1.

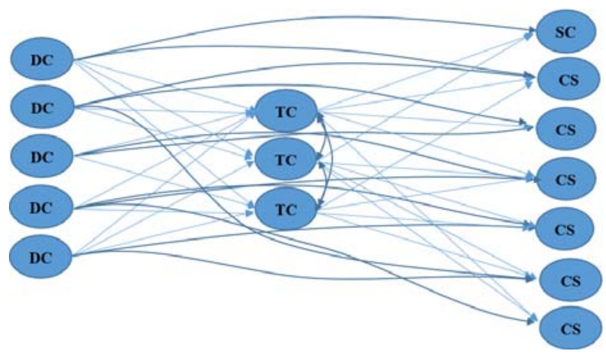

Fig. 1 General sketch of distribution operation

In the case of the enterprise chosen as reference for the research, there are five DC nodes, three TC nodes and 20 SC nodes. On the other hand, and as a characteristic case of the behavior of that system, there are units costs set for transportation, as well as fixed cost between all system components; in this manner, there is a maximum budget available to cover fixed costs for the transportation from the *Distribution Centers to different customers*, and to the transshipment centers and to the system customers, this is for restrictive conditions in the transportation along the country raising the fixed costs of the operation. Otherwise, there is also the option in some cases that the transshipment centers may have the capability to offer products for distribution or to be consumers themselves provided the great variety and dynamics of movement that merchandise has in the country. On the other hand, and as a restrictive condition, the customers can only be supplied by one distribution center DC or one transshipment center TC to have strategy of reduction of costs of freight or mobilization.

IV. Methods

The work developed is an application with a technical approach carried out through a design of a non-experimental investigation of correlational crossed type, and based on historical data obtained by direct observation and collected from secondary information sources; additionally, they were processed by theoretical foundations validated beforehand [27]. This was done by describing several variables at a given time to determine their correlation and causal relationships [28] and can propose an improvement in the analysis unit performance with the aim of increasing the efficiency. Fig. 2 depicts the methodology stages.

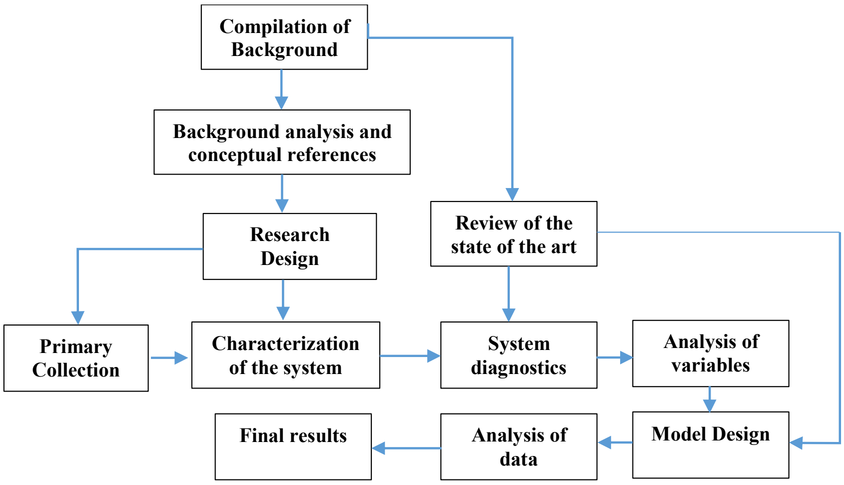

Fig. 2 Methodology stages developed in the present work

TABLE I
Source, Treatment and Use of the Information

Type of data	Source and Obtainment	Medium or tool used
- Distances between distribution centers and customers - Distances between distribution centers and transshipment centers. - Distances between transshipment centers and customers	Secondary information. Web Application	Google Maps, Global Positioning Systems (G.P.S.)
- Demand - Offer	Primary information. Enterprise in the sector	Websites and associations
- Variable and fixed costs	Primary and secondary information. Companies of the sector	Monthly price bulletins. Direct Interview
- Allocation of cargo from distribution centers to transshipment centers and customers.	Optimization Model. Computational Application	GAMS Software
- Vehicle Routes for the distribution of goods.	Heuristic model Computational Application	Programming in EXCEL®

In view of the type of information that depended on the more incidents variables in the supply operation, how they were obtained, and subsequent analysis are shown in Table I.

V. Design of the Model

A. Transshipment Hybrid Model

Based on the characterization of the system and on what showed in the in the reference frame, an optimization model was developed based on MIP that contributed to the establishment of optimum values within the logistic operations activities dedicated to the distribution and transportation of the products of good to the different customers of each enterprise. Its design began to establish the capacities of the different Distribution Centers DC of the company under study and the specific demands of the different System Customers SC taking into account the possible Transshipment Centers TC that are part of the same company in other cities of the country, in addition to the different dispatch options that the distribution centers have for each client, so that they can only be served by a single distribution center or transshipment center. As a complimentary form in the design of the model, it is clarified that, in each allocation established in the optimization model, the saving Clark and Wright algorithm was utilized to determine the specific route that each of the distribution centers should take or each transshipment center for respective distribution. On the other hand, the assumptions for the development of such models are the following:

- The reference period as planning unit is the week.
- Same vehicles of 5200 kg capacity are used.
- A unified load is established as if it were the same type of product.
- Average speed of transfer between different points is 75 km/hr.

Based on the characteristics of the cargo transport system in question, the design of the Transshipment Hybrid model and subsequent Routing was determined. It was called hybrid because it was developed based on the classical structure of the Transshipment model [29] with the *Cargo Allocation* model [30] because they were the ones that fit the operating conditions of the unit of analysis, establishing in this way a hybrid model with mathematical characteristics of these two, as can be seen below:

Sub indexes

- *i:* Refers to the *Distribution Centers* of the system
- *j:* Refers to *the Customers* of the system
- *k:* Refers to transshipment Centers

Decision Variables:

- *Xij* = Number of units to be transported of the same type, from *Distribution Center i*, to the consumption or *customer j* within the nodal network of the supply system.
- *Xik* = Number of units to be transported of the same type from *Distribution Center i,* to the *Transshipment Centers k* within the nodal network of the supply system.
- *Xkj* = Number of units to be transported of the same type, from the *Transshipment Centers k* to the consumption or *customer j* within the nodal network of the supply system.
- *Zkk* = Number of units to be transported of the same type from the *Transshipment Centers k* to another *Transshipment Centers k* within the nodal network.
- *Yij* = Binary variable that switch on if it takes a value equal to 1, or switch off if takes a value equal to 0, the use of the *Distribution Center i*, to the consumer center or *Client j* within the nodal network.
- *Ykj* = Binary variable that switch on if it takes a value equal to 1, or switch off if takes a value equal to 0, the use of *Transshipment Centers k* to the consumer center or *Client j* within the nodal network.
- *Yik* = Binary variable that switch on if it takes a value equal to 1, or switch off if takes a value equal to 0, the use of *Distribution Center i* to the *Transshipment Centers k*, and at the same time its fixed cost for this route within the nodal network.

Parameters:

- *Cij*; *CFij* = Unit Cost and Fixed Cost of transportation, from *Distribution Center i*, to the consumption or *customer j* within the nodal network.
- *Cik; CFik* = Unit Cost and Fixed Cost of transportation, from *Distribution Center i,* to the *Transshipment Centers k* within the nodal network.
- *Ckj; CFkj* = Unit Cost and Fixed Cost of transportation from the *Transshipment Centers k* to the consumption or *customer j* within the nodal network.
- $Cfmax_{i-j}$ = Maximum Fixed Cost allowed for transportation from *Distribution Center i* to the different customers, within the nodal network.
- $Cfmax_{k-j}$ = Maximum Fixed Cost allowed for transportation from the *Transshipment Centers k* to the different customers, within the nodal network.
- $Cfmax_{i-k}$ = Maximum Fixed Cost allowed for transportation from *Distribution Center I* to the different *Transshipment Centers* within the nodal network.
- *ai* = Available capacity for the supply of the *Distribution Center i.*
- *ak* = Available capacity for the supply of the *Transshipment Center k.*
- *bj* = Requirement of demand from the consumer center or *Client j.*
- *bk* = Requirement of the demand from the *Transshipment Center k.*

Mathematical Structure:

$$\text{Min: } \sum_{i=1}^{m}\sum_{j=1}^{n} Cij\,Xij + \sum_{i=1}^{m}\sum_{k=1}^{K} Cik\,Xik + \sum_{k=1}^{K}\sum_{j=1}^{n} Ckj\,Xkj + \sum_{i=1}^{m}\sum_{j=1}^{n} CFijYij + \sum_{i=1}^{m}\sum_{k=1}^{K} CFikYik + \sum_{k=1}^{K}\sum_{j=1}^{n} CFkjYkj + \sum_{k=1}^{K}\sum_{k=1}^{K} CkkZkk \quad \text{Objective Function} \tag{1}$$

Subject to

$$\sum_{j=1}^{n} Xij + \sum_{k=1}^{K} Xik \le ai \quad \forall\ i = 1, \dots, m$$

Restriction in the Distribution offering. (2)

$$\sum_{j=1}^{n} Xkj + \sum_{l=1}^{L} Zkl - \sum_{i=1}^{m} Xik - \sum_{l=1}^{L} Zlk = ak\text{-}bk \quad \forall\ k = 1,\dots,K$$

Flow Balance Restriction (3)

If *ak-bk* < 0 demand predominates at the transshipment center.

If *ak-bk* > 0 supply predominates at the transshipment center.

If *ak-bk* = 0 it is just transshipment center.

$$Xij = bjYij \wedge Xkj = bjYkj \quad \forall\ i = 1, \dots, m \wedge k = 1,\dots,K \wedge j = 1,\dots,n$$

Customer Demand Restriction. (4)

$$\sum_{i=1}^{m} Yij + \sum_{k=1}^{k} Ykj = 1 \quad \forall\ j = 1,\dots,n$$

Restriction of Connectivity from Distribution Centers and Transshipment Centers (5)

$$\sum_{j=1}^{n} CFijYij \le Cf\ \max_i \quad \forall\ i = 1, \dots, m$$

Max Cost Restriction allowed by Distribution Center. (6)

$$\sum_{j=1}^{n} CFkjYkj \le Cf\ \max_k \quad \forall\ k = 1, \dots, K$$

Max Cost Restriction allowed by Transshipment Center (7)

$\sum_{k=1}^{K} CFikYik \leq Cf\ \max_i\ \forall\ i = 1, \ldots, m$ Max Cost Restriction allowed from Distribution Center to Transshipment Center (8)

$Xik \leq YikM \quad \forall\ k = 1,\ldots,K \wedge i = 1, m$

Restriction of shipment to Transshipment Centers and the activation of their Fixed Cost (9)

Xij, Xik, Xkj, Zkk ≥ 0 Restriction Non-negativity (10)

Yij, Yik, Ykj = Binary (11)

Equation (1), which refers to the objective function to be minimized, includes variable costs and fixed costs of sending from DCs to SC and TC, as well as from TC to SC. Fixed costs are activated from DC distribution centers to SCs (6) from DCs to TCs (7) or from TCs to SCs (8), for not exceeding the budget allowed from DCs and TC respectively, providing that, by adding the variable costs to meet the demand for SCs, the lowest possible costs are reached. These restrictions on the availability of capital to cover the fixed costs of shipping from DC distribution centers to customers are due to the large distances to be traveled, which in turn is impacted in a representative manner by the high tolls and of fuel consumption mainly among other transport costs. On the other hand, (2) refers to what comes out of the DC cannot exceed the capacity of supply that they have in the respective reference period. Likewise, (3) states that what comes from transshipment centers TC including other transshipment centers l (using this new subscript to determine the direction of movement of products between TC), less what they enter to these including other centers of transshipment l, must be equal to a value equivalent to the subtraction between the supply and demand parameters of the respective TC, where if it reaches a positive value, it means that the supply predominates in the said TC, otherwise if it is negative, it means that the demand predominates in the said TC or if it is equal to zero, it means that the respective TC is only a transfer point; that is, what comes out is equal to what enters.

Talking about (4), it refers to the demand of SCs which can be supplied directly by the DC distribution centers or by the TC transshipment centers, whereby they are taken separately through the fixed expenses scheme [15] to activate the respective fixed cost in the target function if the use of the quantity variable is activated. On the other hand, (5) states that each customer can only be served by one source, either this DC or TC but not both at a time. This is done to minimize costs according to the operating strategies of the companies under study.

In (9), to activate the fixed cost if the quantity variable is activated, as in (4), only here, it is not related to another functional parameter of the model, if not with a large value M, for its activation. Moreover, in (10) and (11), the nature of the variables is established.

B. Routing Model

After establishing the allocation of loads in the optimization model to meet the demands of the SC, a heuristic based on path savings was used to establish the respective routes from each DC. This is to define in a timely manner the path to be followed from each DC for the respective delivery of the goods.

Here below and based on [31], it is established the heuristic procedure:

If there are two different routes, these can be combined to form a new route, with the purpose of generating savings (in distance), which is given by:

$$S_{ij} = C_{i0} + C_{0j} - C_{ij}$$

This algorithm is part of an initial solution, and the connections are made to give greater savings, without infringing the restrictions of the problem.

The savings algorithm has two versions: parallel and sequential. The first one is described:

1. Initialization: For each client i build the route $(0, i, 0)$
2. Savings calculation: Calculate $\boldsymbol{S_{ij}}$ for each pair of customers i and j
3. Better connection: Let $\boldsymbol{S_{i^*j^*}} = max\, S_{ij}$ where the maximum is taken between the savings that have not been considered yet. Let $\boldsymbol{r_{i^*}}$ y $\boldsymbol{r_{j^*}}$ be the routes that contain the customers $\boldsymbol{i}^*$ y $\boldsymbol{j}^*$. If $\boldsymbol{i}^*$is the last customer $\boldsymbol{r_{i^*}}$ and $\boldsymbol{j}^*$ is the first customer of $\boldsymbol{r_{j^*}}$ and the combination of $\boldsymbol{r_{i^*}}$ y $\boldsymbol{r_{j^*}}$ is feasible**,** combine.

   Remove $\boldsymbol{S_{i^*j^*}}$ from future consideration.
4. If there are savings to be examined, apply item 3, otherwise finish.

In some cases, the original saving equation can generate circular paths, which is negative. For this purpose, the following equation has been proposed:

$$S_{ij} = C_{i0} + C_{0j} - \lambda C_{ij}$$

λ is a shape parameter that restricts the connection of routes with distant customers.

VI. Results

A. Regarding the Transshipment Hybrid Model

It is clarified in the first instance that this model seeks to establish the allocation of the cargo to be covered from each DC to meet the demand of the SC passing or not by the TC at the lowest possible cost. On the other hand, in Fig. 3, the following results are obtained when running the model:

In this way, we must make 16 shipments directly from the DC to SC and 11950 units from DC 1,2 and 3 to TC.2, after which 250 units are sent to TC1, 4300 units to SC .9, 3500 to SC.14 and 4200 to SC.18. On the other hand, the TC enters 3400 units of the DC.2 and sends 3500 units to the SC.1 since the TC.3 has an offer of 100 units. Finally, TC.1 is not used, whereas DCs have a capacity utilization of: 94.3%, 99.3%, 81.7% and 100% for DC 4 and 5. With these results, a minimum cost of $ 2,922,488 per week is obtained.

B. Regarding the Routing Model

In contrast to the application of the routing model, using the Clark and Wright algorithm, it is obtained the specific routes to follow by each of the distribution centers according to the shortest distance and with limitation of travel allowed for each one of these. This can be seen in Fig. 4.

Distribution Center DC	Transshipment Center			System Clients SC								
	1	2	3	1	2	3	4	5	6	7	8	9
1		2450							4800			
2		5000	3400		4100					3650		
3												
4							3400	4000			5100	
5		4500				3800						
TC1												
TC2	250											4300
TC3				3500								

Distribution Center DC	System Clients SC										
	10	11	12	13	14	15	16	17	18	19	20
1	3800	4000					3800				
2						5800					
3				5500						4300	
4								4000			
5			3700								5000
TC1											
TC2					3500				4200		
TC3											

Fig. 3 Units to send from DC to SC using TC

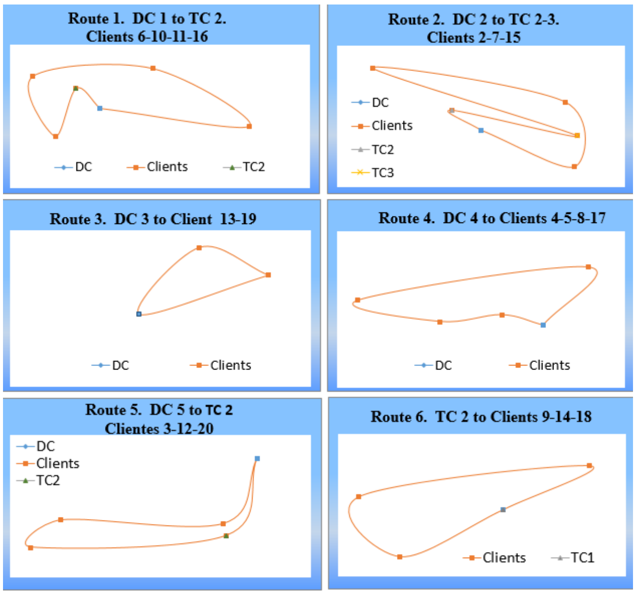

Fig. 4 Routes phase 2, applying the savings algorithm

VII. Conclusions

The design of this model consisted of the initial combination of the routing model from several sources with a classic transshipment model, to then apply the assignment of specific routes under the criterion of less distance with travel restrictions using the algorithm of Clark and Wright, what allows establish a quite satisfactory technique - according to the case where it was applied - within the use of MIP making variations in the convexity handling of the constraints, which allows efficient modeling for decision making in logistic activities where there are different allocation possibilities for the distribution and subsequent routing in a nodal network. Besides, it has the possibility of carrying out assignments of logistic operators according to the demands of the customers of that system, at the same time, it considers the capacities or availabilities from each of these operators or distribution centers of the network obtaining an optimal value with respect to the costs associated with this operation.

It is worth noting that this model includes Fixed Costs of transportation, for which it establishes basic parameters as input information that facilitate and speed up the calculation of these costs according to the distances between each of the customers and the distribution centers in the system network; this includes, for example, drivers' salaries, fuel consumption of vehicles, changes in the prices of these fuels and other items inherent to the expenditure of machinery by the mobilization of goods, which facilitates and gives greater precision in the calculation of these costs to be used as input information for the mathematical development of such a model. On the other hand, it also contemplates the Variable Costs of transportation when handling the unit value that implies the mobilization of the products from each distribution center to the different customers of the system or consumption centers; as a result, there is an integration of the cost elements involved in the development of the transport activity along with the supply chain for a company dedicated to this logistics work. Moreover, an important contribution of this model is that it had not been established previously, providing great help for those in charge of establishing the logistics of routing in companies dedicated to this activity, additionally, in those situations in which there is a great variety of options to assign the cargo to be transported.

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