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Perturbation Based Modelling of Differential Amplifier Circuit

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Abstract**—**This paper presents the closed form nonlinear expressions of bipolar junction transistor (BJT) differential amplifier (DA) using perturbation method. Circuit equations have been derived using Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL). The perturbation method has been applied to state variables for obtaining the linear and nonlinear terms. The implementation of the proposed method is simple. The closed form nonlinear expressions provide better insights of physical systems. The derived equations can be used for signal processing applications.

Keywords**—**Differential amplifier, perturbation method, Taylor series.

I. Introduction

DIFFERENTIAL circuit and topology is useful in many wireless applications as they are more immune to electromagnetic coupling than the single ended architecture. They also present the improved even order linearity and double voltage swing, which is important feature for integrated circuits with low supply voltage. BJT DA is a basic component in many integrated circuits (ICs). It is used for power amplifier circuit in phased array transceiver IC for frequency modulated continuous wave radar [1], CMOS temperature to digital converter [2], phased array IC for 5G communication [3], vector gain amplifier used in vector modulator in 5G communication [4] etc.

Generally, nonlinear circuit components are approximated by linear model for different applications. But, use of linear model causes signal distortion, which affect the performance of the nonlinear circuit component. This requires nonlinear closed form expression of circuit components. Nonlinear circuit analysis can be classified into two different classes: (1) Numerical method and (2) Analytical method. Numerical methods include shooting Newton (SN), time domain method, harmonic balance (frequency domain method), mixed frequency-time method, linear time varying analysis and envelope method etc. [5]-[7]. Analytical methods include perturbation method and Volterra series method etc. Perturbation method and Volterra series have been widely used for analysis of nonlinear systems. Lee et al. [8] used Volterra series to analyze digital to analog converter. Braithwaite [9] used Volterra series method for digital predistortion of power amplifier used in radio frequency. Song et al. [10] implemented memory polynomial model and Volterra model based on dynamic deviation to compensate nonlinear distortion of the power amplifier. Though the Volterra series is nonlinear extension to a linear impulse model, but the large number of parameters associated with the Volterra model limits the practical implementation to the problems having only modest values of memory. In general, Volterra series are applicable to systems having fading memory. Fading memory state that past inputs have a weakening effect on the present output. This implies that all the kernels of a system should decay to zero in a finite period of time. The modelling of signals using Volterra series needs excessive computational resources as the number of coefficients to be determined increases exponentially with the model degree of nonlinearity and the Volterra filter length. The perturbation method has the advantage of easy implementation as the problem is solved through a mathematical model of approximation. It is used for different applications. Wu et al. [11] used iterative perturbation to predict direct mode to common mode conversion that occur in differential interconnects due to asymmetry and nonuniformity of manufacturer. Afifi and Dussaux [12] used perturbation and small slope approximation to derive the expression of scattering amplitude and scattered intensities of electromagnetic wave from a two dimensional surface that separates the vacuum from a perfect electromagnetic conduction. Mishra and Yadava [13] presented the effects of internal and external noisy perturbations on chaotic colpitt oscillator. Liu et al. [14] presented design of centralized fusion and weighted measurement fusion Kalman predictors for multisensor systems. For this, they used stochastic parametric uncertainties by decomposing each stochastic parameters in the state space model into a deterministic parameter (mean) plus a random perturbation with zero mean. In [15], analysis of exponential stability and passivity of switched system with varying time delays have been studied using nonlinear perturbation. Buonomo and Schiavo [16] presented the closed form expressions of the harmonics of intermodulation products of multistage amplifier and continuous time transconductor filter by approximating the circuit nonlinearities by cubic polynomials. Wang et al. [17] proposed a memresistor based oscillator using perturbation projection vector. The time delay and use of approximate model cause uncertainties in neutral systems. Intrinsic and extrinsic noise also cause nonlinear perturbation in neutral systems. References [18] and [19] studied the stability of neutral system due to these nonlinear perturbations and time delay. Rathee et al. [20] presented the stochastic modelling of nonlinear circuits, which is based on perturbation theory and compared this with perturbation based on deterministic model of the nonlinear circuit and obtained the noise process component. Rathee and Parthasarathy [21] presented nonlinear distortion in weakly

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nonlinear circuits using perturbation based Fourier series model. We used perturbation method to obtain the first order nonlinear expression of BJT DA.

The paper is organized as follows: A brief introduction to perturbation theory is given in Section II. Section III presents nonlinear analysis of BJT DA using perturbation method. Results and conclusions are given in Sections IV and V, respectively.

II. Perturbation Theory

Perturbation method is widely used for analysis of nonlinear circuits. It is implemented by small deformation of a model that can be solved exactly. The problem is solved through a mathematical model of approximation. This method provides the solution by continuously improving the previously obtained approximate solution. In this way, the method allows to implement the computational efficiency of idealized systems to more realistic problems. Also, it presents the analytic insight into complex problems.

The method is implemented by adding a small term $'\epsilon'$ to exactly solvable problem as

$$A = A_0 + \epsilon A_1 + \epsilon^2 A_2 + ... \tag{1}$$

where A_0 is the known solution to the exactly solvable problem and A_1, A_2 are higher order nonlinear terms.

III. Implementation of Perturbation Method to BJT Differential Amplifier

BJT DA circuit is shown in Fig. 1. It has transistors Q_1 and Q_2. Applying KVL and KCL to DA circuit, we have

$$\frac{V_{EE} + v_E}{R_E} + (I_{E_1} + I_{E_2}) = 0 \tag{2}$$

$$i_L R_L + \frac{1}{C_L}\int i_L dt = v_{C_1} - v_{C_2} \tag{3}$$

$$\frac{V_{CC} - v_{C_1}}{R_{C_1}} - I_L - I_{C_1} = 0 \tag{4}$$

$$\frac{V_{CC} - v_{C_2}}{R_{C_2}} + I_L - I_{C_2} = 0 \tag{5}$$

From the circuit, we have, $v_{BE_1} = v_1 - v_E$, $v_{BE_2} = v_2 - v_E$, $v_{BC_1} = v_1 - v_{C_1}$, $v_{BC_2} = v_2 - v_{C_2}$, where v_{C_1}, v_{C_2}, v_E and q_L are the state vectors. Also, $\int i_L dt = q_L$, where q_L is the charge at load. The collector current and emitter current for the transistors Q_1 i.e. I_{C_1}, I_{E_1} are

$$I_{C_1} = \beta_1 I_0 \left[e^{\left(\frac{v_{BE_1}}{V_T}\right)} - 1 \right] \tag{6}$$

$$I_{E_1} = \frac{I_0}{1-\alpha_1} \left[e^{\left(\frac{v_{BE_1}}{V_T}\right)} - 1 \right] \tag{7}$$

Similarly, the collector currents and emitter current for the transistors Q_2 i.e. I_{C_2}, I_{E_2} are

$$I_{C_2} = \beta_2 I_0 \left[e^{\left(\frac{v_{BE_2}}{V_T}\right)} - 1 \right] \tag{8}$$

$$I_{E_2} = \frac{I_0}{1-\alpha_2} \left[e^{\left(\frac{v_{BE_2}}{V_T}\right)} - 1 \right] \tag{9}$$

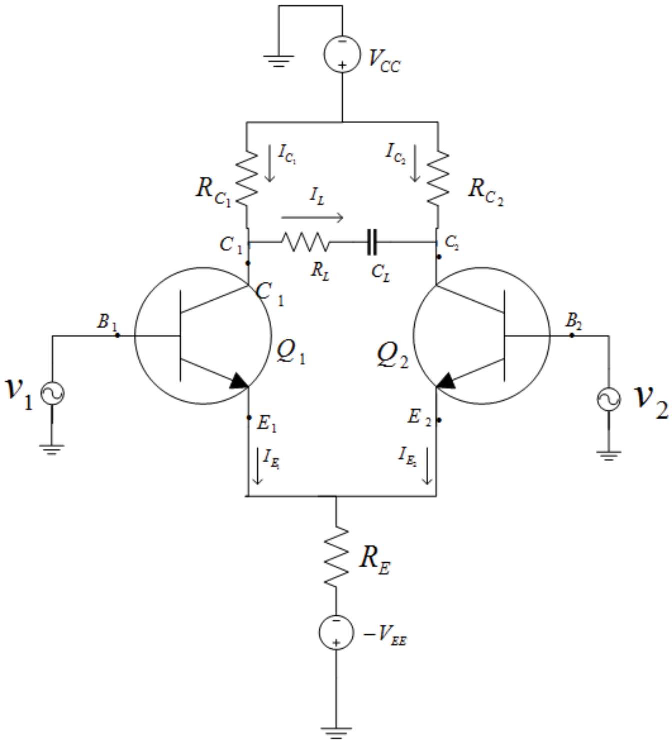


Fig. 1 Differential amplifier circuit

Expanding equations (6)-(9) in Taylor series expansion, we have

$$I_{C_1} = \beta_1 I_0 \left(\frac{v_1 - v_E}{V_T}\right) + \frac{\beta_1 I_0}{2}\left(\frac{v_1 - v_E}{V_T}\right)^2 + \text{Higher order terms} \tag{10}$$

$$I_{E_1} = \frac{I_0}{1-\alpha_1}\left(\frac{v_1 - v_E}{V_T}\right) + \frac{I_0}{2(1-\alpha_1)}\left(\frac{v_1 - v_E}{V_T}\right)^2 + \text{Higher order terms} \tag{11}$$

$$I_{C_2} = \beta_2 I_0 \left(\frac{v_2 - v_E}{V_T}\right) + \frac{\beta_2 I_0}{2}\left(\frac{v_2 - v_E}{V_T}\right)^2 + \text{Higher order terms} \tag{12}$$

$$I_{E_2} = \frac{I_0}{1-\alpha_2}\left(\frac{v_2 - v_E}{V_T}\right) + \frac{I_0}{2(1-\alpha_2)}\left(\frac{v_2 - v_E}{V_T}\right)^2 + \text{Higher order terms} \tag{13}$$

Then from (2)-(5) and neglecting the higher order terms in (10)-(13), we have

$$\frac{V_{EE} + v_E}{R_E} + \frac{I_0}{1-\alpha_1}\left(\frac{v_1 - v_E}{V_T}\right) + \frac{I_0}{2(1-\alpha_1)}\left(\frac{v_1 - v_E}{V_T}\right)^2 + \frac{I_0}{1-\alpha_2}\left(\frac{v_2 - v_E}{V_T}\right) + \frac{I_0}{2(1-\alpha_2)}\left(\frac{v_2 - v_E}{V_T}\right)^2 = 0 \tag{14}$$

$$R_L \frac{dq_L}{dt} + \frac{q_L}{C_L} - v_{C_1} + v_{C_2} = 0 \tag{15}$$

$$\frac{V_{CC}-v_{C_1}}{R_{C_1}}-\frac{dq_L}{dt}-\beta_1 I_0\left(\frac{v_1-v_E}{V_T}\right)+\frac{\beta_1 I_0}{2}\left(\frac{v_1-v_E}{V_T}\right)^2=0 \quad (16)$$

$$\frac{V_{CC}-v_{C_2}}{R_{C_2}}+\frac{dq_L}{dt}-\beta_2 I_0\left(\frac{v_2-v_E}{V_T}\right)+\frac{\beta_2 I_0}{2}\left(\frac{v_2-v_E}{V_T}\right)^2=0 \quad (17)$$

Now applying perturbation term to state variables as: $v_{C_1}=v_{C_1}^{(0)}+\epsilon v_{C_1}^{(1)}$, $v_{C_2}=v_{C_2}^{(0)}+\epsilon v_{C_2}^{(1)}$, $v_E=v_E^{(0)}+\epsilon v_E^{(1)}$ and $q_L=q_L^{(0)}+\epsilon q_L^{(1)}$

Applying perturbation method to (14)-(17) and rearranging the equations, we have

$$(v_E^{(0)}+\epsilon v_E^{(1)})\left(\frac{1}{R_E}-\frac{I_0}{V_T(1-\alpha_1)}-\frac{I_0}{V_T(1-\alpha_2)}\right)=-\frac{V_{EE}}{R_E}-\frac{v_1 I_0}{V_T(1-\alpha_1)}-\frac{v_2 I_0}{V_T(1-\alpha_2)}-(v_1-v_E)^2\frac{I_0}{2V_T^2(1-\alpha_1)}-(v_2-v_E)^2\frac{I_0}{2V_T^2(1-\alpha_2)} \quad (18)$$

$$(v_{C_1}^{(0)}+\epsilon v_{C_1}^{(1)})+(q_L^{(0)}+\epsilon q_L^{(1)})\left(R_L\frac{d}{dt}+\frac{1}{C_L}\right)+(v_{C_2}^{(0)}+\epsilon v_{C_2}^{(1)})=0 \quad (19)$$

$$(v_{C_1}^{(0)}+\epsilon v_{C_1}^{(1)}))\left(\frac{1}{R_{C_1}}\right)+(q_L^{(0)}+\epsilon q_L^{(1)})\left(\frac{d}{dt}\right)+(v_E^{(0)}+\epsilon v_E^{(1)})\times\left(-\frac{\beta_1 I_0}{V_T}\right)=\frac{V_{CC}}{R_{C_1}}+v_1\left(-\frac{\beta_1 I_0}{V_T}\right)+(v_1-v_E)^2\left(-\frac{\beta_1 I_0}{2V_T^2}\right) \quad (20)$$

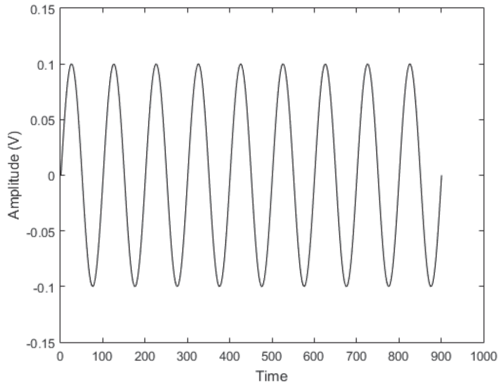
$$(v_{C_2}^{(0)}+\epsilon v_{C_2}^{(1)}))\left(\frac{1}{R_{C_2}}\right)+(q_L^{(0)}+\epsilon q_L^{(1)})\left(-\frac{d}{dt}\right)+(v_E^{(0)}+\epsilon v_E^{(1)})\times\left(-\frac{\beta_2 I_0}{V_T}\right)=\frac{V_{CC}}{R_{C_2}}+v_2\left(-\frac{\beta_2 I_0}{V_T}\right)+(v_2-v_E)^2\left(-\frac{\beta_2 I_0}{2V_T^2}\right) \quad (21)$$

A. Zeroth-Order Approximation

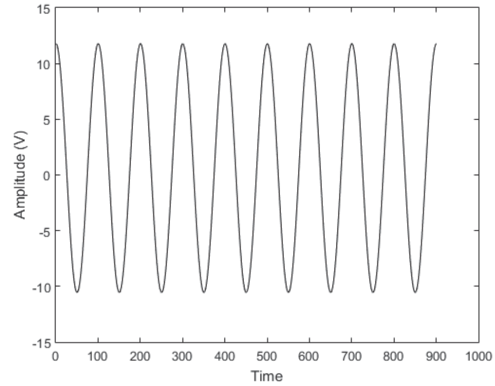
Linear terms are obtained by comparing the coefficients of $\epsilon^{(0)}$ in (18)-(21). They are:

$$v_E^{(0)}\left(\frac{1}{R_E}-\frac{I_0}{V_T(1-\alpha_1)}-\frac{I_0}{V_T(1-\alpha_2)}\right)=-\frac{V_{EE}}{R_E}-\frac{v_1 I_0}{V_T(1-\alpha_1)}-\frac{v_2 I_0}{V_T(1-\alpha_2)} \quad (22)$$

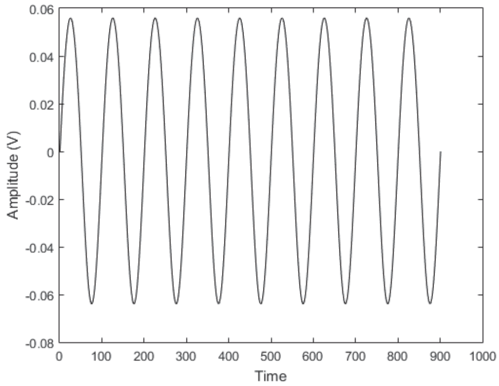
$$v_{C_1}^{(0)}+q_L^{(0)}\left(R_L\frac{d}{dt}+\frac{1}{C_L}\right)+v_{C_2}^{(0)}=0 \quad (23)$$



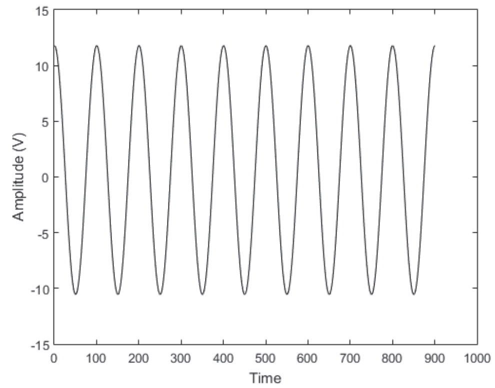
(a) Differential input to amplifier circuit



(b) Linear differential output voltage



(c) First order nonlinear differential output voltage



(d) Linear and nonlinear differential output voltage

Fig. 2 Differential amplifier output when peak to peak differential input is 0.2 V with input frequency 100Hz

TABLE I
DIFFERENTIAL GAIN AND PERCENTAGE DISTORTION

Input at Q_1 (V)	Input at Q_2 (V)	V_{PP}, Peak to peak differential input (V)	Frequency (Hz)	Gain (dB)	Percentage distortion
1	1.1	0.2	100	41.22	0.54%
1	1.15	0.3	100	41.75	0.50%
1	1.1	0.2	1000	41.14	0.53%
1	1.15	0.3	1000	42.04	0.49%
1	1.1	0.2	10000	41.43	0.53%
1	1.15	0.3	10000	41.96	0.50%

$$v_{C_1}^{(0)}\left(\frac{1}{R_{C_1}}\right)+q_L^{(0)}\left(\frac{d}{dt}\right)+v_E^{(0)}\left(-\frac{\beta_1 I_0}{V_T}\right)=\frac{V_{CC}}{R_{C_1}}+v_1\left(-\frac{\beta_1 I_0}{V_T}\right)\tag{24}$$

$$v_{C_2}^{(0)}\left(\frac{1}{R_{C_2}}\right)+q_L^{(0)}\left(-\frac{d}{dt}\right)+v_E^{(0)}\left(-\frac{\beta_2 I_0}{V_T}\right)=\frac{V_{CC}}{R_{C_2}}+v_2\left(-\frac{\beta_2 I_0}{V_T}\right)\tag{25}$$

$$A(s)X_(s)=B_1(s)v_1(s)+B_2(s)v_2(s)+C_1(s)V_{EE}+C_2(s)V_{CC}\tag{26}$$

$$X_0(s)=\begin{bmatrix} v_{C_1}^{(0)} & v_{C_2}^{(0)} & v_E^{(0)} & q_L^{(0)}\end{bmatrix}\tag{27}$$

$$A=\begin{bmatrix} 0 & 0 & A_{13} & 0\\ \frac{1}{R_{C_1}} & 0 & -\frac{\beta_1 I_0}{V_T} & \frac{d}{dt}\\ 0 & \frac{1}{R_{C_2}} & -\frac{\beta_2 I_0}{V_T} & -\frac{d}{dt}\\ -1 & 1 & 0 & R_L\frac{d}{dt}+\frac{1}{C_L}\end{bmatrix}\tag{28}$$

where $A_{13}=\frac{1}{R_E}-\frac{I_0}{V_T(1-\alpha_1)}-\frac{I_0}{V_T(1-\alpha_2)}$

$$B_1(s)=\begin{bmatrix} -\frac{I_0}{V_T(1-\alpha_1)} & -\frac{\beta_1 I_0}{V_T} & 0 & 0\end{bmatrix}\tag{29}$$

$$B_2(s)=\begin{bmatrix} -\frac{I_0}{V_T(1-\alpha_2)} & 0-\frac{\beta_2 I_0}{V_T} & 0\end{bmatrix}\tag{30}$$

$$C_1(s)=\begin{bmatrix} -\frac{1}{R_E} & 0 & 0 & 0\end{bmatrix}\tag{31}$$

$$C_2(s)=\begin{bmatrix} 0 & \frac{1}{R_{C_1}} & \frac{1}{R_{C_2}} & 0\end{bmatrix}\tag{32}$$

$$\begin{aligned}v_{C_1}^{(0)}=&\left\{-\frac{I_0}{V_T(1-\alpha_1)}\times\frac{A_{11}}{|A|}-\frac{\beta_1 I_0}{V_T}\times\frac{A_{21}}{|A|}\right\}*v_1(t)\\&+\left\{-\frac{I_0}{V_T(1-\alpha_2)}\times\frac{A_{11}}{|A|}-\frac{\beta_2 I_0}{V_T}\times\frac{A_{31}}{|A|}\right\}*v_2(t)\\&-\frac{1}{R_E}\times\frac{A_{11}}{|A|}V_{EE}+\left(\frac{1}{R_{C_1}}\times\frac{A_{21}}{|A|}\right.\\&\left.+\frac{1}{R_{C_2}}\times\frac{A_{31}}{|A|}\right)V_{CC}\end{aligned}\tag{33}$$

$$\begin{aligned}v_{C_2}^{(0)}=&\left\{-\frac{I_0}{V_T(1-\alpha_1)}\times\frac{A_{12}}{|A|}-\frac{\beta_1 I_0}{V_T}\times\frac{A_{22}}{|A|}\right\}*v_1(t)\\&+\left\{-\frac{I_0}{V_T(1-\alpha_2)}\times\frac{A_{12}}{|A|}-\frac{\beta_2 I_0}{V_T}\times\frac{A_{32}}{|A|}\right\}*v_2(t)\\&-\frac{1}{R_E}\times\frac{A_{12}}{|A|}V_{EE}+\left(\frac{1}{R_{C_1}}\times\frac{A_{22}}{|A|}\right.\\&\left.+\frac{1}{R_{C_2}}\times\frac{A_{32}}{|A|}\right)V_{CC}\end{aligned}\tag{34}$$

$$\begin{aligned}v_E^{(0)}=&\left\{-\frac{I_0}{V_T(1-\alpha_1)}\times\frac{A_{13}}{|A|}-\frac{\beta_1 I_0}{V_T}\times\frac{A_{23}}{|A|}\right\}*v_1(t)\\&+\left\{-\frac{I_0}{V_T(1-\alpha_2)}\times\frac{A_{13}}{|A|}-\frac{\beta_2 I_0}{V_T}\times\frac{A_{33}}{|A|}\right\}*v_2(t)\\&-\frac{1}{R_E}\times\frac{A_{13}}{|A|}V_{EE}+\left(\frac{1}{R_{C_1}}\times\frac{A_{23}}{|A|}\right.\\&\left.+\frac{1}{R_{C_2}}\times\frac{A_{33}}{|A|}\right)V_{CC}\end{aligned}\tag{35}$$

$$\begin{aligned}q_L^{(0)}=&\left\{-\frac{I_0}{V_T(1-\alpha_1)}\times\frac{A_{14}}{|A|}-\frac{\beta_1 I_0}{V_T}\times\frac{A_{24}}{|A|}\right\}*v_1(t)\\&+\left\{-\frac{I_0}{V_T(1-\alpha_2)}\times\frac{A_{14}}{|A|}-\frac{\beta_2 I_0}{V_T}\times\frac{A_{34}}{|A|}\right\}*v_2(t)\\&-\frac{1}{R_E}\times\frac{A_{14}}{|A|}V_{EE}+\left(\frac{1}{R_{C_1}}\times\frac{A_{24}}{|A|}\right.\\&\left.+\frac{1}{R_{C_2}}\times\frac{A_{34}}{|A|}\right)V_{CC}\end{aligned}\tag{36}$$

B. First-Order Approximation

To obtain first order nonlinear terms, we compare coefficients of $\epsilon^{(1)}$ in (18)-(21). We have

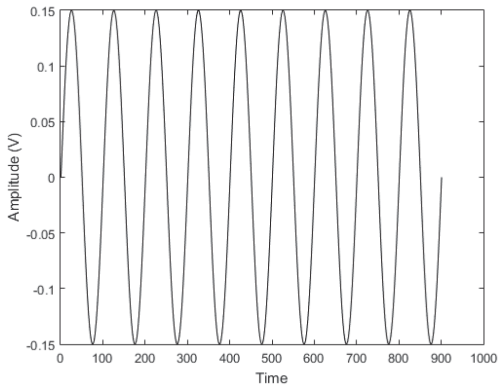
$$v_E^{(1)}\left(\frac{1}{R_E}-\frac{I_0}{V_T(1-\alpha_1)}-\frac{I_0}{V_T(1-\alpha_2)}\right)=-\frac{(v_1-v_E^{(0)})^2 I_0}{2V_T^2(1-\alpha_1)}-\frac{(v_2-v_E^{(0)})^2 I_0}{2V_T^2(1-\alpha_2)}\tag{37}$$

$$v_{C_1}^{(1)}+q_L^{(1)}\left(R_L\frac{d}{dt}+\frac{1}{C_L}\right)+v_{C_2}^{(1)}=0\tag{38}$$

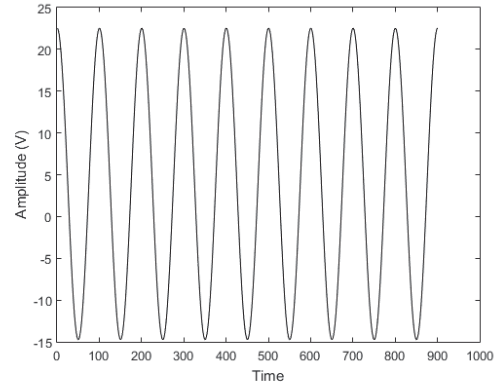
$$v_{C_1}^{(1)}\left(\frac{1}{R_{C_1}}\right)+q_L^{(1)}\left(\frac{d}{dt}\right)+v_E^{(1)}\left(-\frac{\beta_1 I_0}{V_T}\right)=-(v_1-v_E^{(0)})^2\left(\frac{\beta_1 I_0}{2V_T^2}\right)\tag{39}$$

$$v_{C_2}^{(1)}\left(\frac{1}{R_{C_2}}\right)+q_L^{(1)}\left(-\frac{d}{dt}\right)+v_E^{(1)}\left(-\frac{\beta_2 I_0}{V_T}\right)=-(v_2-v_E^{(0)})^2\left(\frac{\beta_2 I_0}{2V_T^2}\right)\tag{40}$$

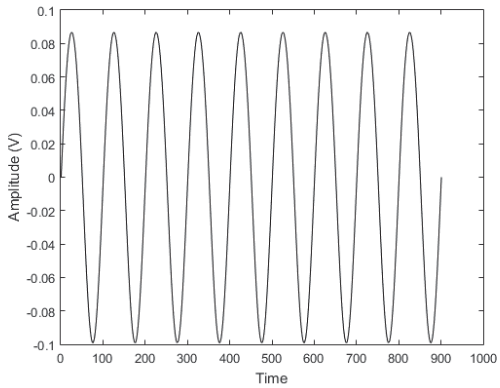
$$X_1(s)=\begin{bmatrix} v_{C_1}^{(1)} & v_{C_2}^{(1)} & v_E^{(1)} & q_L^{(1)}\end{bmatrix}\tag{41}$$



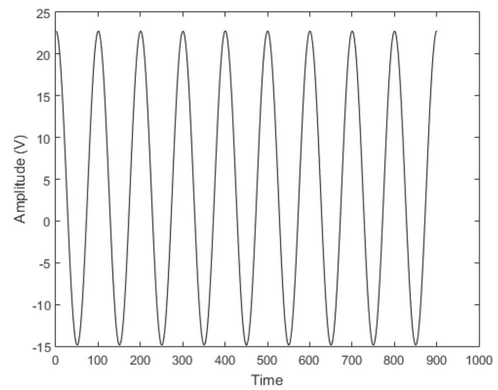
(a) Differential input to amplifier circuit



(b) Linear differential output voltage



(c) First order nonlinear differential output voltage



(d) Linear and nonlinear differential output voltage

Fig. 3 Differential amplifier output when peak to peak differential input is 0.3 V with input frequency 100Hz

$$v_{C_1}^{(1)} = \left\{ -\frac{I_0}{2V_T^2(1-\alpha_1)} \times \frac{A_{11}}{|A|} - \frac{\beta_1 I_0}{2V_T^2} \times \frac{A_{21}}{|A|} \right\} * (v_1 - v_E^{(0)})^2 + \left\{ -\frac{I_0}{2V_T^2(1-\alpha_2)} \times \frac{A_{11}}{|A|} - \frac{\beta_2 I_0}{2V_T^2} \times \frac{A_{31}}{|A|} \right\} * (v_2 - v_E^{(0)})^2 \quad (42)$$

$$v_{C_2}^{(1)} = \left\{ -\frac{I_0}{2V_T^2(1-\alpha_1)} \times \frac{A_{12}}{|A|} - \frac{\beta_1 I_0}{2V_T^2} \times \frac{A_{22}}{|A|} \right\} * (v_1 - v_E^{(0)})^2 + \left\{ -\frac{I_0}{2V_T^2(1-\alpha_2)} \times \frac{A_{12}}{|A|} - \frac{\beta_2 I_0}{2V_T^2} \times \frac{A_{32}}{|A|} \right\} * (v_2 - v_E^{(0)})^2 \quad (43)$$

IV. Simulation Results

Nonlinear equations derived for BJT DA have been implemented in MATLAB. Circuit element values used in simulations are: $V_{CC} = 20V$,$V_{EE} = -20V$, $R_{C_1} = 8k\Omega$, $R_{C_2} = 7.5k\Omega$, $R_E = 0.08k\Omega$, $R_L = 10k\Omega$, $C_L = 10\mu C$ and sampling time is 10μsec. Linear and first order nonlinear terms have been plotted for different amplitudes and input frequencies. Table I shows the percentage error due to linear term only for different input amplitudes and frequencies. Percentage distortion has been calculated using following expression

$$Distortion = \frac{v_0 - v_0^{(0)}}{v_0} \times 100\%$$

V. Conclusions

We derived the closed form expressions of first order nonlinear term using the perturbation method. Simulations show the importance of considering the nonlinear term of BJT DA. We derived first order nonlinear term only, but the method can be used to obtain higher order nonlinear closed form expressions. Though the higher order perturbation can be used to obtain the higher order nonlinear terms, but essential insight is given by this nonlinear term.

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