

# Optimal Mitigation of Slopes by Probabilistic Methods

D. De-León-Escobedo, D. J. Delgado-Hernández, S. Pérez

**Abstract**—A probabilistic formulation to assess the slopes safety under the hazard of strong storms is presented and illustrated through a slope in Mexico. The formulation is based on the classical safety factor (SF) used in practice to appraise the slope stability, but it is introduced the treatment of uncertainties, and the slope failure probability is calculated as the probability that  $SF < 1$ . As the main hazard is the rainfall on the area, statistics of rainfall intensity and duration are considered and modeled with an exponential distribution. The expected life-cycle cost is assessed by considering a monetary value on the slope failure consequences. Alternative mitigation measures are simulated, and the formulation is used to get the measures driving to the optimal one (minimum life-cycle costs). For the example, the optimal mitigation measure is the reduction on the slope inclination angle.

**Keywords**—Expected life-cycle cost, failure probability, slopes failure, storms.

## I. INTRODUCTION

MANY collapses have been produced as a consequence of slopes failure as the one in Puerto Rico in 1989 after the hurricane Hugo [1], and the one in Taiwan where the Typhoon Herb produced 1300 landslides in 1996 [2].

Monte Carlo simulation techniques and probability-based approaches have been widely used to analyze the slope stability and the parameters sensitivity [3], [4]. The rainfall-induced changes on the soil properties have been studied by using a variety of models [5]-[7].

Hazard management systems have been developed for slope stability with non-saturated soils [8], and it has been pointed out that most of the recent slope risk analyses have the deficiency that they are based on a single rainfall record for the hydro mechanical slope analysis. Instead of that, a Markov chain model has been proposed to generate a time series of rainfall records [9].

In Mexico, the States of Puebla, Veracruz, Hidalgo, Baja California and Chiapas have suffered strong consequences (in casualties and economic loss) due to rainfall induced slope failures [10], [11].

In this paper, a simplified procedure to assess, in a probabilistic manner, the cost-effectiveness of mitigation measures is proposed and applied to a slope in Mexico. One of the contributions is the calculation of the allowable failure probability in terms of the minimization of the expected life-

cycle costs.

## II. SLOPE STABILITY ANALYSIS

### A. General Formulation

The procedure is based on the general limit equilibrium (GLE) method by Fredlund [12] and the following expression [13]:

$$s = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \left[ \frac{\theta - \theta_r}{\theta_s - \theta_r} \right] \tan \phi' \quad (1)$$

where  $s$  = unsaturated soil shear strength,  $c'$  = effective cohesion of saturated soil,  $\sigma_n - u_a$  = net normal stress on the failure path,  $u_a - u_w$  = matric suction on the failure path,  $\phi'$  = effective angle of shear strength for saturated soil,  $\theta$  = volumetric water content,  $\theta_s$  = saturated water content,  $\theta_r$  = residual water content.

Monte Carlo simulation techniques are applied to account for the soil and rainfall variabilities and the SF is assessed throughout 1000 trials (see flowchart in Fig. 1). The steps are as follows:

1. Initial condition (trial with geometry and soil properties)
2. Rainfall simulation (trial from exponential distribution)
3. Slope stability analysis
4.  $i = n$ ? (if the number of trials is not yet the proposed number, the process continues to step 5, otherwise it goes to step 6)
5. Perform a new trial and goes to step 1
6. Calculate the slope failure probability (number of trials with failure divided by the total number of trials "n").

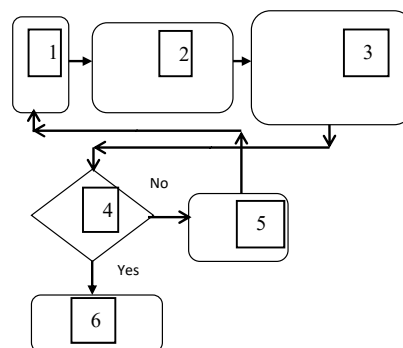


Fig. 1 Flowchart for the proposed procedure

The math is applied through commercial software by the SoilVision [14] which uses the relationships between water content and conductivity and the matric suction for each type

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of soil in the slope.

**B. Model of the Considered Slope and Data**

The shape and profile of the considered slope (in Zinacantepec, Mexico) is shown in Fig. 2.

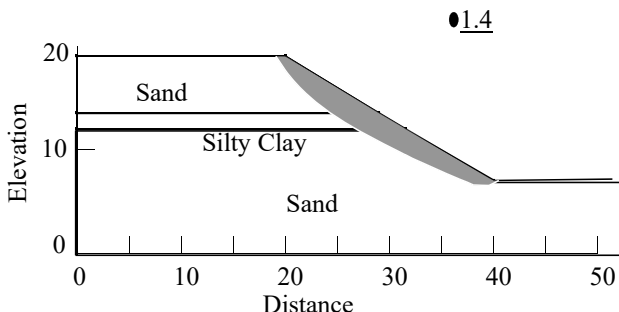


Fig. 2 Slope model (initial condition). Dimensions in m.

Table I shows initial data of soil conditions, without rain, for sand and clay.

TABLE I  
SOIL PROPERTIES (INITIAL CONDITION)

Soil	Cohesion	Friction angle (°)	Volumetric Weight
Sand	2 kPa	35	18 kN/m <sup>3</sup>
Silty Clay	10 kPa	25	19 kN/m <sup>3</sup>

**C. Rainfall Modeling**

According to the World Meteorology Organization, a period of 30 years is representative for the simulation of rainfall series. Also, in this work, it is considered that the rain falls only on top of the slope (Fig. 2), the water slides on the seepage face and the pressure zero level corresponds to the highway level.

Data taken (1982 to 2014) from the meteorological station located in Zinacantepec, Mexico State, Mexico serve to fit an exponential distribution to these records.

$$f_R(r) = 1/\mu \exp(-r/\mu) \tag{2}$$

where  $\mu = 1.665$ .

**D. Conductivity and Water Content Functions**

Lab tests serve as a basis to get the curves of conductivity and water content for each material. In this paper, the ones for conductivity and water content for sand and clay are shown in Figs. 3-6.

**III. MC SIMULATION AND ACCEPTABLE FAILURE PROBABILITY**

Monte Carlo simulation is performed to calculate the slope failure probability, modifying the soil properties and rainfall intensity each trial. Slope final condition may be seen in Fig. 4.

The resulting slope failure probability is  $9 \times 10^{-2}$ . Now, the acceptable failure probability is obtained from the minimization of the expected life-cycle cost  $E(C_L)$  [15]:

$$E(C_L) = C_i + E(C_f)P_f \tag{3}$$

where the initial cost  $C_i$  is expressed [16]:

$$C_i = C_1 - C_2 \ln(P_f) \tag{4}$$

where  $C_1$  and  $C_2$  are constants that depend on the slope geometry and soil characteristics.  $C_1$  is the cost of the slope if no lateral forces exist (no earthquake), and  $C_2$  is the cost of upgrading the slope so that the failure probability is reduced in an order of “e” the natural log base.

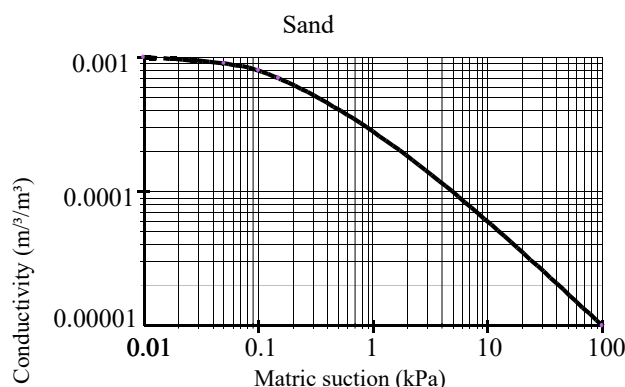


Fig. 3 Conductivity for sand

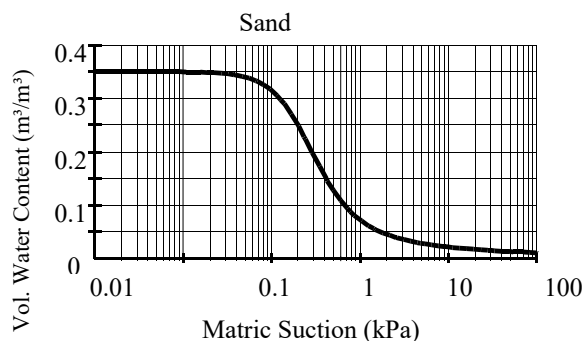


Fig. 4 Water content for sand

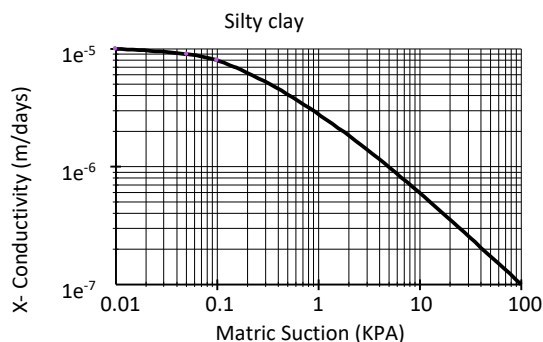


Fig. 5 Conductivity for clay

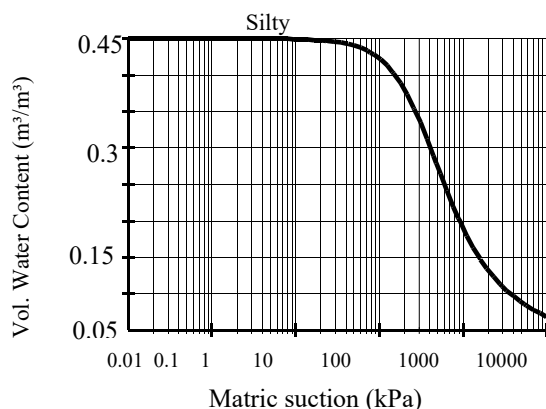


Fig. 6 Water content for clay

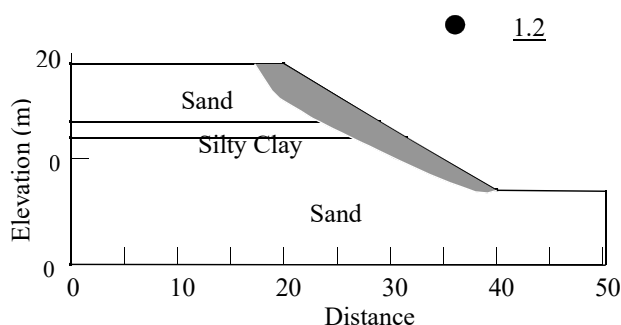


Fig. 7 Slope model (final condition). Dimensions in m

The expected failure cost  $E(C_f)$  is expressed in present value, and they depend on the failure consequences of the slope, which are here calculated according to the concepts and amounts (estimated from worst scenario conditions) shown in Table II.  $P_f$  is the slope annual failure probability.

TABLE II  
 FAILURE CONSEQUENCES FOR THE SLOPE

Concept	Number	Amount (Million USD)
Fatalities	10	1.2
Injuries	10	0.65
Economic loss		5.15

The amount associated to the fatality cost was estimated according to the “human capital approach” by Rosenblueth [17].

The amount of fatalities and injuries is estimated from the number of vehicles and people usually traveling in the highway down the slope. The economic loss is derived from the worst scenario of a landslide: all the trucks and goods being transported in the highway are lost. The total loss  $C_f$  becomes around 7 million USD.

The criterion of minimum life-cycle cost is expressed:

$$\partial E(C_L) / \partial P_f = 0 \quad (5)$$

And, therefore, the acceptable failure probability is:

$$P_f = C_2 / PVF_1(C_f) \quad (6)$$

Therefore, the acceptable failure probability is  $6 \times 10^{-5}$ .

By comparing this value with the one obtained for the slope,  $9 \times 10^{-2}$ , one can realize that the slope is in non-acceptable conditions and requires urgent upgrading or mitigation works.

It is assumed that a concrete cover is applied over the top of the slope, producing a protection against water infiltration. By doing so, and calculating again the slope failure probability, it is obtained  $P_f = 1 \times 10^{-10}$ .

The unit cost of providing the concrete cover is 150 USD/m<sup>2</sup> and, if it is applied over the slope length (considering that the whole area is 2550 m<sup>2</sup>) the total cost becomes 0.38 million USD.

#### IV. CONCLUSION

A procedure to include uncertainties on soil properties and rainfall variabilities was presented to assess the failure probability of a critical slope under strong storms.

It is shown that the procedure may provide a technical basis to decide whether or not the slope is in acceptable conditions and may support the calculations to derive cost-effective mitigation measures.

Cost optimization requires the repetition of the exercise for several mitigation measures, producing different failure probability reductions and different costs. The formulation presented here may be extended to cover these issues.

The formulation may be applied to all the slopes pre-qualified as critical to derive a national or regional program of cost-effective mitigation measures, once they are calibrated against the corresponding costs.

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#### REFERENCES

- [1] Larsen, M. C., Simon, A., 1993. A rainfall intensity-duration threshold for landslides in a humid-tropical environment, Puerto Rico. Geografiska Annaler Series A 75 A (1-2), 13-23.
- [2] Lin, M. L., Jeng, F. S., 2000. Characteristics of hazards induced by extremely heavy rainfall in Central Taiwan-Typhoon Herb. Engineering Geology 58, 191-207.
- [3] Wang Y., C ao Z and Au S-K., 2010, Efficient Monte Carlo Simulation of parameter sensitivity in probabilistic slope stability analysis, Computers and Geotechnics, Vol. 37, 7- 8, pp. 1015-1022.
- [4] Lari S., Frattini P. and Crosta G. B., 2014, A probabilistic approach for landslide hazard analysis, Engineering Geology, Vol. 182 part A, 19, pp. 3-14.
- [5] Zhang J., Huang H. W., Zhang L: M., Zhu H. H. and Shi B., 2014. Probabilistic prediction of rainfall-induced slope failure using a mechanics-based model, Engineering Geology, Vol. 168, 16, Pp. 129-140.
- [6] Lulu Z., 2005, Probabilistic study of slope stability under rainfall condition. Ph.D. Civil Engineering Thesis, Hong Kong University of Science and Technology, Hong Kong.
- [7] Tarolli P., Borga M., Chang K. T. and Chiang S-H., 2011, Modelling shallow landsliding susceptibility by incorporating heavy rainfall statistical properties. Geomorphology, 133 (3-4), pp. 199-211.
- [8] Fredlund D., 2007. Slope stability hazard management systems, Journal of Zhejiang University: Science, Vol. 8, pp. 1879-2040. Zhejiang University Press.

- [9] White J. A. and Singham D. I. 2012. Slope Stability Assessment using Stochastic Rainfall Simulation, Vol. 9, pp. 699–706, Proceedings of the International Conference on Computational Science, ICCS 2012.
- [10] Alcantara-Ayala, I. 2004. Hazard assessment of rainfall induced landsliding in Mexico, *Geomorphology* 61, 19-40.
- [11] Alcantara-Ayala, I. 2008, On the historical account of disastrous landslides in Mexico: the challenge of risk management and disaster prevention. *Adv. Geosci.*, 14, 159-164.
- [12] Rahardjo H. and Fredlund D. G. 1984. General limit equilibrium method for lateral earth force. *Canadian Geotechnical Journal* 21 (1), pp. 166-175.
- [13] Vanapalli S. K., Fredlund D. G., Pufahi D. E. and Clifton A. W. 1996. Model for the prediction of shear strength with respect to soil suction. *Canadian Geotechnical Journal*, 1996, 33(3), pp. 379-392.
- [14] The SoilVision Systems Ltd. Team, 2017, SVOFFICE 5 Help Manual, Canada.
- [15] Ang, A. and De Leon, D. 2005. Modeling and Analysis of Uncertainties for Risk-Informed Decision in Infrastructures Engineering, *Journal of Structure and Infrastructure Engineering*, Vol.1, No. 1, pp. 19-31.
- [16] Lind N. C. y Davenport A. G. 1972. Towards practical application of Structural Reliability Theory”, ACI Publication SP- 31, Probabilistic Design of Reinforced Concrete Buildings, Detroit, Mich., pp. 63-110.
- [17] Rosenblueth, E., (1982). “Information value in certain class of problems” (In Spanish), Internal Report 448, Instituto de Ingenieria, UNAM, Mexico.