

# Constructal Enhancement of Fins Design Integrated to Phase Change Materials

Varun Joshi, Manish K. Rathod

**Abstract**—The latent heat thermal energy storage system is a thrust area of research due to exuberant thermal energy storage potential. The thermal performance of PCM is significantly augmented by installation of the high thermal conductivity fins. The objective of the present study is to obtain optimum size and location of the fins to enhance diffusion heat transfer without altering overall melting time. Hence, the constructal theory is employed to eliminate, resize, and re-position the fins. A numerical code based on conjugate heat transfer coupled enthalpy porosity approach is developed to solve Navier-Stoke and energy equation. The numerical results show that the constructal fin design has enhanced the thermal performance along with the increase in the overall volume of PCM when compared to conventional. The overall volume of PCM is found to be increased by half of total of volume of fins. The elimination and re-positioning the fins at high temperature gradient from low temperature gradient is found to be vital.

**Keywords**—Constructal theory, enthalpy porosity approach, phase change materials, fins.

## 1. INTRODUCTION

THE Latent Heat Thermal Energy Storage (LHTES) using Phase Change Materials (PCMs) is in research drift to serve purpose due to large thermal storage density within small volume. These PCMs are characterized by melting temperature and latent heat of fusion through which charging and discharging are able to accumulate and release thermal energy. However, the drawback of PCMs is low thermal conductivity [1]. This constrained thermal conductivity of PCM delays the quick respond to thermal dissipation and exhilarates very high thermal gradient during transient heat transfer. Many researchers proposed various methods for thermal conduction enhancement of PCMs. These methods are extended surfaces with high conductive material [2], [3], micro-encapsulation [4], [5], high conductivity nanoparticles [6], [7], etc. However, the application of high conductive fins as extended surfaces is found to be prominent due to simple, easy to fabricate and cost economical. Also, fins assist in maintaining uniform temperature field which is difficult to achieve using nanoparticles and micro encapsulation.

A lot of research has been reported on the study of heat transfer characteristics with a given arrangement of fins. The fin improvisation parameters such as fin numbers, geometry of fins, types of fins, dimensions of fins and total fin volume ratio, etc. are primarily concerned [8]. Yang et al. [9]

investigated effect of annular fins in shell and tube LHTES numerically. The study aimed to quantify the effects of number of fins, fins geometry and size on thermal performance. The results suggested that the application of optimum number and size of the fins can help to reduce the overall melting time by 65%. It was also suggested that these fins variables should be appropriately selected, excessive fins number will not enhance thermal performance. Jmal and Baccar [10] investigated the influence of fin numbers on heat transfer enhancement during solidification process. It was found that fins above optimum number do not show significant improvement in the solidification process. Also, the liquid PCM gets confined between fin spacing which results in generation of thermo-convective flow. Similarly, Gharebaghi and Sezai [11] performed thermal and geometrical parametric study for establishing thermal performance of a finned rectangular cavity filled with PCM. A significant reduction in time was observed when fin spacing decrease for melting of vertical and horizontal modules. Also, larger temperature difference across walls, higher heat transfer enhancement can be obtained by inserting fin arrays. It is also mentioned that decrease in PCM module thickness should be preferred over decreasing inter fins distance. Lacroix and Benmadda [12] studied effect of fin number and size of fins in a rectangular vertical cavity filled with n-octadecane. The study advised to increase in the length of fins over increasing the number of fins. The longer fins were found to be more significant in thermal penetration over shorter fins [13].

It can be observed in above literature that the fins are promoted as conductive enhancement medium to improve the heat transfer. These literatures often focus on substantial dependency fins parameters on the performance of the overall LHTES. Thus, shape, size, volume, specific geometry and allocation of fins are highly dependent on thermal behavior of PCM. It is logical that the increment in these parameters enhances the conductive heat transfer but decreases the volume of PCM, hence thermal energy storage capacity. This ambiguity in the creating finned structure for LHTES systems raises optimization opportunities. Also, numerical analysis of fins embedded LHTES requires special analysis using conjugate heat transfer, needed to evaluate large gradient across PCM and fins due to extensive difference in thermal diffusivity. The present study has the following objectives: first, to develop a numerical code to predict the thermal performance of fins assisted LHTES; secondly, to optimize the design of fins with constructal theory without altering overall melting time; finally, to position the fins at optimal location to enhance conduction heat transport.

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## II.SYSTEM DESCRIPTION

The schematic layout of the system consists of a rectangular enclosure with three fins made of aluminum. The fins are mounted to the right side heat source wall as shown in Fig. 1. The enclosure size is  $120(l) \times 50(b) \times 120(h)$  mm. The fin size is  $25(l) \times 4(b)$  mm. The fins were installed ensuring no thermal contact resistance between contact surface and fin base. The right wall is exposed to a constant temperature of  $70^\circ\text{C}$ , while others are well insulated. For model validation purpose, temperature monitor points are selected based on the experimental measurement locations in the experimental setup performed by Kamkari and Shokouhmand [14]. The enclosure with fins is filled with Lauric acid with thermo-physical properties as shown in Table I and uniform initial temperature is maintained at  $25^\circ\text{C}$ .

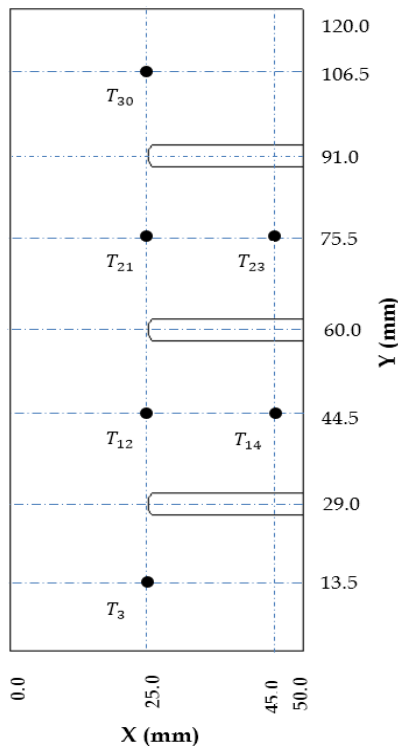


Fig. 1 Schematic of fins integrated LHTES and thermocouples locations [14]

TABLE I  
THERMO-PHYSICAL PROPERTIES OF LAURIC ACID AND FINs [14]

Melting temperature (K)	316.5-321.2
Latent heat (kJ/kg)	187.21
Density (kg/m <sup>3</sup> )	940 (solid) 885 (liquid)
Specific heat (kJ/kg.K)	2.18 (solid) 2.39 (liquid)
Thermal conductivity (W/m.K)	0.16 (solid) 0.14 (liquid)
Kinematic viscosity (m <sup>2</sup> /s)	$6.7 \times 10^{-6}$
Thermal conductivity of aluminum (W/m.K)	130

## III.NUMERICAL APPROACH

The primitive aspect of numerical simulation for transient

thermal transport in LHTES with fins is to track of phase front and evaluate of conjugate heat transfer across interface between fins and PCM. An enthalpy-porosity approach is employed to invoke for tracking of solid liquid interface [15], [16]. Also, a conjugate heat transfer is coupled with enthalpy porosity approach to invoke phase front. Further, with the use of this numerical model, the design optimization of fins assisted LHTES is carried out using the constructal theory.

### A. Governing Equations

The enthalpy-porosity approach for a phase change process invokes the solution of Navies-Stokes and energy transport equations for an incompressible Newtonian fluid whose motion is caused by natural convection. The enthalpy-porosity model involves natural convection-diffusion driven melting process coupled by four variables, i.e. velocity, pressure, temperature, and liquid fraction. The continuity, momentum, and energy transport equations can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + S_v u \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_\infty) + S_v v \quad (3)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + S_E \quad (4)$$

where the dependent variables are liquid velocity vectors ( $u$  and  $v$ ), liquid pressure ( $p$ ), temperature ( $T$ ). The density ( $\rho$ ), time ( $t$ ), kinematic viscosity ( $\nu$ ), volumetric thermal expansion coefficient ( $\beta$ ) and thermal diffusivity ( $\alpha$ ) are assumed to be constant.

The value of momentum source term  $S_v$  can be defined by a non-dimensional permeability term also known as Carmen-Kozeny term [17]. This term matches the physical behavior of a porous medium and eases the numerical computation at melting interface. This can be obtained as follows,

$$S_v = -A_{mushy} \left( \frac{(1-f)^2}{f^3 + q} \right) \quad (5)$$

Here,  $A_{mushy}$  is the computational morphology coefficient often known as mushy constant. The value of  $A_{mushy}$  often ranges from  $10^3$  to  $10^9$  [18], [19]. The value of  $q$  is set to be small number to eliminate division by zero. The source term  $S_E$  in (4) represents determination of the liquid fraction ( $f$ ). The term  $f$  defines the degree of absorption and release of the latent heat.

$$S_E = -\frac{L_h}{c_p} \frac{\partial f}{\partial t} \quad (6)$$

Here,  $L_h$  is the latent heat of fusion. The term  $f$  is liquid fraction which indicates the volume of phase change occurred over period of each iteration of the current time step. The updated liquid fraction at the end of each iteration of current time step enables to discriminate the whole domain: solid, phase change interface and liquid. The term  $f$  is updated according to the enthalpy value [20].

$$f = \begin{cases} 0 & h < c_p (T_m - r) \\ \frac{h - c_p (T_m - r)}{2c_p r + L_h} & c_p (T_m - r) < h < c_p (T_m + r) + L_h \\ 1 & h > c_p (T_m + r) + L_h \end{cases} \quad (7)$$

The temperature interval  $r$  is the half of the phase change temperature range.  $T_m$  is the melting temperature. The term  $h$  is enthalpy, calculated at every current time step.

The conjugate heat transfer is considered at the interface between fin material and PCM due to distinct thermal conductivity. The interface boundary condition at all three surfaces of fins are given by following conjugates.

$$k_{Fin} \frac{\partial T}{\partial n} = k_{PCM} \frac{\partial T}{\partial n} \quad (8)$$

Here,  $k_{Fin}$  and  $k_{PCM}$  are the thermal conductivity of fin and PCM, respectively. The term  $n$  is the vector normal to the interface in the direction from fin to PCM and PCM to fin.

#### B. Numerical Procedure

A volume tracking method (finite volume method) is applied to discretize the governing equations on a uniform Cartesian grid. The Semi Implicit Method for Pressure Linked Equation (SIMPLE) algorithm developed by Patankar and Spalding [21] is adopted to solve momentum equations to derive velocity and pressure field. An enthalpy-porosity formulation is employed to derive temperature field and liquid fraction. The code is fully time implicit and convection terms are handled using a first order upwind scheme. An iterative procedure using Gauss-Seidel Successive Over-relaxation (GS-SOR) algorithm is applied to solve discretized equations. The convergence is checked for velocity, pressure and temperature within each time step. The grid size of  $40 \times 40$ ,  $40 \times 60$  and  $60 \times 80$  are taken for grid independence study. After comparing the effect of grids along time step, a grid size with  $40 \times 60$  is chosen due to achievement of sufficient defined convergence.

#### IV. VALIDATION OF NUMERICAL MODELLING

The thermal performance of discussed numerical approach is compared and validated with an experimental analysis of a rectangular LHTES enclosure with fins [14]. The experiment

results with constant wall temperature of  $70^\circ\text{C}$  are compared to simulation results for validation purpose.

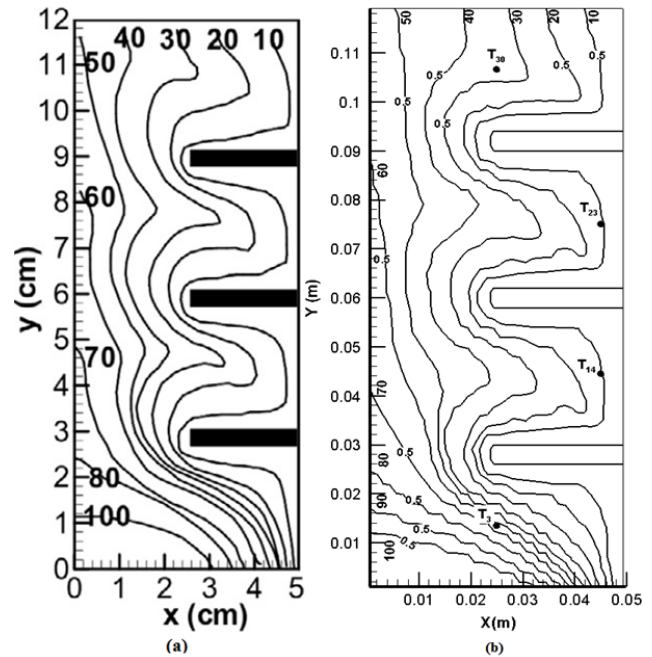


Fig. 2 Comparison of phase front evolution variation between (a) the experimental results [14] and (b) the present numerical study over time

Fig. 2 shows the comparison between the predicted phase front contours and the phase front during experiment along time during melting process. The total melting time taken for complete melting of PCM is found to be 100 min which is well supported by experimental results. Also, the overall position and the shape of phase front agree well with the experimental expedition. Further, to present a detailed temperature comparison with the present study, six thermocouple points of experiment are considered in axial direction as shown in Fig. 3. They are namely  $T_3$ ,  $T_{14}$ ,  $T_{23}$  and  $T_{30}$  as marked at locations in vertical plane. The maximum temperature difference between the present study and the experimental data are found to be  $4^\circ\text{C}$  and  $5^\circ\text{C}$  at  $T_3$  and  $T_{23}$ , respectively. In nutshell, the proposed numerical model agrees well with experimental literature.

#### V. CONSTRUCTAL FINS DESIGN

It is noted from the literature that the increase in the fins parameters above its optimum value does not improve thermal performance and results in reduction in overall volume of PCM and. Hence, it is important to take following objectives into account (i) minimization of fin volume ratio for melting of a fixed volume of PCM and (ii) reduction in total melting time for complete melting of PCM for a given fin volume ratio.

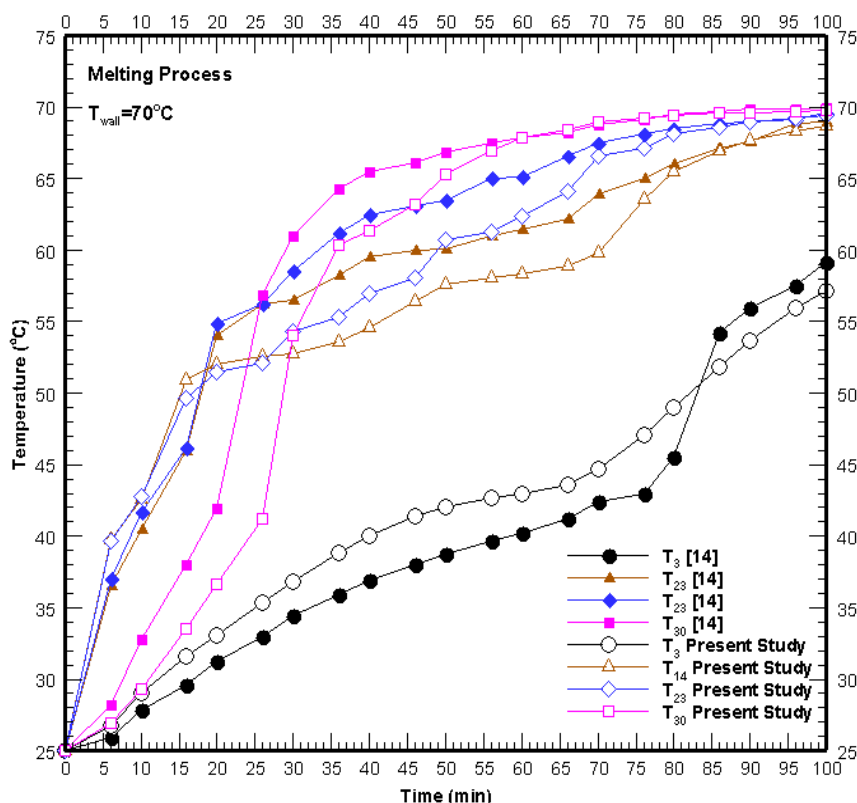


Fig. 3 Comparison of temperature variation between the experimental results [14] and the present numerical study at various points

However, a very few literatures are found which deal with the fins design optimization for heat exchange enhancement. Bejan [22] developed a constructal theory which discusses optimal position of flow path for better heat exchange using fins. The fundamental principle of the theory is to configure limited numbers of fins at optimal position with maximum total heat transport by addition of heat conduction through fins. The principles of this include (i) improvisation of heat transport ability at position where local temperature gradient is maximum and (ii) removal of fins at position with minimum local temperature gradient. The study concluded that the time required for cooling of certain volume by conduction to a concentrated sink can be deduced by appropriate alterations in geometry. Later, Wang et al. [23] applied concept in optimization of fin position subjected to PCM. The PCM-fins configuration was compared with natural growth of 'roots' of a plant in a soil in order to absorb and transport 'nutrition'. The roots of a plants and nutrition are analogue to heat flow path using fins and heat flux, respectively. The fins were designed numerically according to constructal rule where fins were placed at position where local temperature gradient is maximum. The result depicted that the proposed design has much better performance over conventional technique. Also, constructal rule is an effective technique which designs the fins for better performance with enhancement of heat conduction. Kalbasi and Salampour [24], [25] optimized a rectangular enclosure filled with PCM used for cooling of electronics. The fin size and fin numbers were identified as criterion for improvement in thermal performance.

Many literatures claimed dominant presence of natural convection during melting process especially at top portion of enclosure [26]. Also, the heat transport potential is low at top portion of enclosure due to low temperature gradient than that of bottom portion where it is at maximal. Thus, the inclusion of fins at the top portion of cavity does not enhance so much the heat flux transport. The conduction can be observed dominantly at bottom portion. This large temperature gradient raises scope of constructal optimization. The provision of fins at this portion can increase local heat flux transport. Thus, removal of fins and also reducing size of fins from top to bottom portion of enclosure benefits from heat flux transport without altering overall melting time. Also, reduction in fin volume increases total volume of PCM, and hence, thermal storage capacity.

The aforementioned numerical approach is employed to investigate fin assisted LHTES where fins are distributed according to constructal theory. Three constructal strategies are applied. First, eliminating a fin at the top portion out of three fins installed in the enclosure. Second, reducing the length of fins where local temperature gradient is minimal. The length of fin at middle portion is reduced by 50% than that of bottom fin. Finally, re-positioning these fins where temperature gradient is high (towards bottom of enclosure). The distance between two consecutive fins is kept constant with key difference of positioning of fins. The temperature points for comparison are similar to previous study. In the remainder of this article, the previous numerical study, where equal size of fins is mounted, is referred as Case 0 for the

comparison studies.

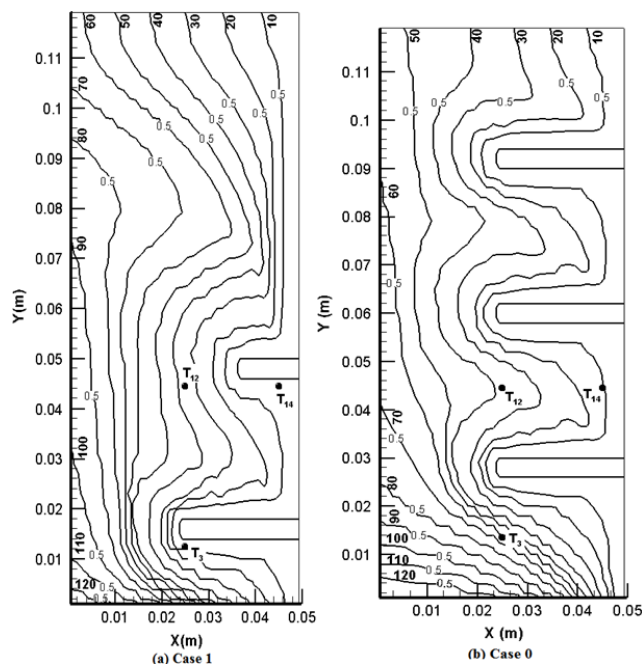


Fig. 4 Comparison between phase front evolutions over time

The phase front contours and temperature evolution for Case 1 and Case 0 during melting process are shown in Fig. 4. It can be observed that the phase front for both the cases

propagates with similar trend. Also, the total melting time in case 1 is found to be almost identical to that of Case 0. Hence, it validates that the eliminating and truncation of fins at the top portion of the enclosure does not affect the overall melting process significantly. Also, benefiting the natural convection at top portion of the enclosure, positioning the fins at bottom portion of the enclosure to enhance conduction is found to be optimized.

Furthermore, it can be seen that, with increasing the depth of fins positions, the phase front propagates more rapidly especially at bottom of the enclosure in Case 1 than that of Case 0. The melting rate is predominant in the Case 1 which indicates uniform fins arrangement. Also, the significant improvement in the phase front is observed after 60 min when the phase front reaches near to bottom surface. For better analysis, the temperatures  $T_3$ ,  $T_{12}$  and  $T_{14}$  located near to bottom of enclosure are compared for all three cases. The comparison is presented in Fig. 5. The desirable difference in temperature over time can be observed when Case 1 is compared with Case 0. This difference is significant especially at bottom points  $T_3$  and  $T_{14}$ , while temperature at point  $T_{12}$  away from bottom surface shows minimal difference. It can be stated that re-positioning the fins according to constructal theory not only improves bottom conduction but also does not affect natural convection at top portion. This shows improvement in the heat flux transport at bottom of enclosure when fins are re-positioned from Case 0 to Case 1.

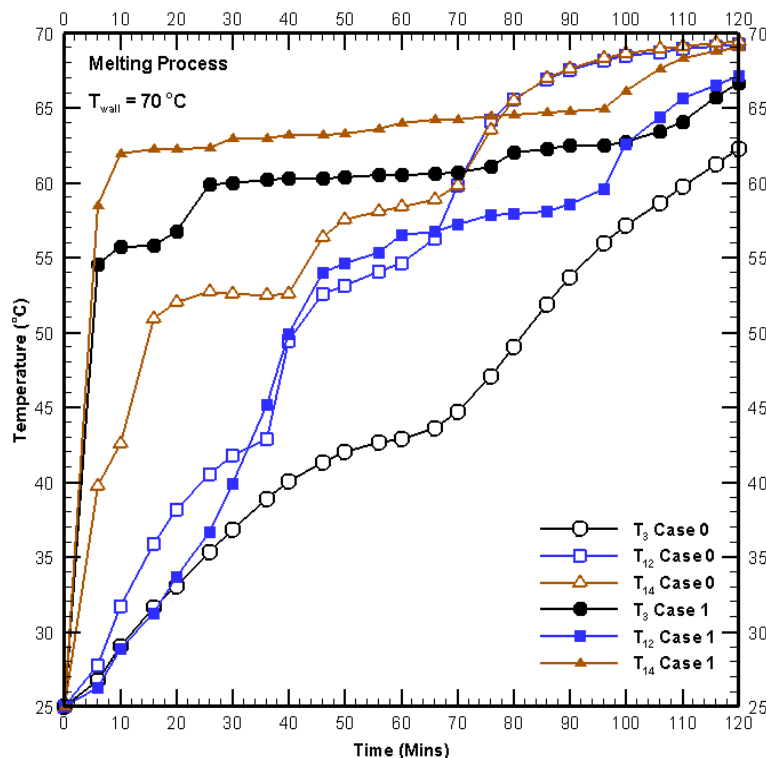


Fig. 5 Temperature comparison for different three cases

Thus, in the present study, the total volume of fins decreased by half of total volume of fins. Also, fins at optimal location in order to enhance bottom conduction without altering overall melting time. In short, the identification of optimized location of fins is equally crucial to move heat flux quickly into PCM. The fin optimization using constructal theory addressed the deficiency of slow melting at the bottom of the enclosure. The further study needs to perform seeking the effect of melting temperature, thermal conductivity ratio and latent heat when constructal theory is employed.

## VI.CONCLUSION

The present numerical study discusses the constructal optimization of thermal transport in fins assisted LHTES. The conjugate heat transfer coupled enthalpy porosity approach is established. The results of developed numerical model are compared with the previous experimental results. The simulation results are found to be in well order.

A fin configuration is identified based on constructal theory. It is found that the elimination and reduction in the volume of fins at top and middle portion of enclosure does not alter the overall melting rate significantly. It is shown that re-positioning the fins according to constructal theory improves the conduction heat transfer at bottom of the enclosure without affecting natural convection at the top. The reduction in fin solid mass can help to increase the total volume of PCM by half of the total volume of fins.

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