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Pressure Angle and Profile Shift Factor Effects on the Natural Frequency of Spur Gear Tooth Design

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***Abstract*—**An (irregular) case relating to base circle, root circle and pressure angle has been discussed and a computer programme has been developed to simulate and plot spur gear tooth profile, including involute and trochoid curves based on the formulation of rack cutter using different values of pressure angle and profile shift factor and it gave the values of all important geometric parameters. The results showed the flexibility of this approach and versatility of the programme to draw many different cases of spur gear teeth of any module, pressure angle, profile shift factor, number of teeth and rack cutter tip radius. The procedure developed can be extended to produce finite element models of heretofore intractable geometrical forms, to exploring fabrication of nonstandard tooth forms also. Finite elements model of these irregular cases have built using above programme and modal analysis has been done using ANSYS software and natural frequencies of these selected cases have been obtained and discussed.

***Keywords*—**Involute, trochoid, pressure angle, profile shift factor, natural frequency.

TABLE I
SYMBOLS, QUANTITIES AND UNITS

Symbol	Quantity	SI Unit
r_a	outer radius	mm
r_p	pitch radius	mm
r_b	base radius	mm
r_f	root radius	mm
r_c	cutter tip radius	mm
m	module	mm
Z	number of teeth	
C_f	profile shift factor	
θ	involute angle	degree
CTT	circular tooth thickness at pitch circle	mm
hd	tooth depth	mm
Zoom	programme zoom factor	
Ø or (Phy)	pressure angle	degree

I. INTRODUCTION

THE action of two mating gears tooth is similar to two curved surfaces are in contact, when two curved surfaces are in contact, the point of contact occurs where the two surfaces are tangent to each other, and the forces at any instant are directed along the common normal to the two curves the line which representing the direction of action of the forces is called the line of action,. The mating gears teeth acting against each other will produce rotary motion [1]. In order to make the gear teeth to transform uniform angular velocity, they must satisfy the condition that the line of action (the common normal at the point of contact of two teeth) must pass through the pitch point. There are a few standard profiles namely conjugate gears, Involute, cycloidal and Novikov are used to simulate gear teeth action [2]. Gears having conjugate teeth can successfully use for transmitting motion but they are difficult to manufacture, special devices has been used for this purpose which are costly, conjugate teeth are not much common in use. The common forms of teeth used widely are cycloidal tooth profile and involute tooth profile [3].

There were attempts to use the base circle as a foundation for generating involute gear theory, and there is an alternative definition of the involute without the use of rack (gear forms gear). At a glance, gear shaping or gear rolling process, both gears are made without the use of rack parameters; this makes difference in their geometry and characteristics [4]. Shyue-Cheng Yang [5] has considered when the gear tooth profile is defined, then its parameters used for tool design in other words; the gear tooth parameters are primary and the tool parameters are secondary and he has chosen a traditional way for gear design based on rack-generation method. Gitin M. Maitra [6] has explained four methods to construction of involute gear tooth; in the fourth one he used the principle of gear cutting by generation method, the specialty of this method is that not only the face and flank comprising the tooth profile can be drawn, but the fillet curve portions of the teeth are also represented realistically, this curve is a trochoid and not a circular arc. By this method, the trochoid is automatically generated along with other portions of the tooth profile.

The relative motion of the rack form with respect to the gear blank in the plane of the spur gear is defined by the centrode motion of the pitch line of the rack and the pitch circle of the gear, this relative motion is produced by rolling the pitch line of the basic rack form on the pitch circle of the gear. The cut tooth is the envelope of the successive positions of the rack form as this motion is produced. The generation of the gear tooth fillet is produced by the rack form tip; this tip can take on a sequence of different shapes for the same gear dedendum [7]. When the rack tooth represents the form of the generating tool, then the trochoid gives the form of the fillet of the gear tooth.

When no undercut is present, this trochoid will be tangent to the generated gear tooth profile [8]. Tooth profile modifications for specific purpose are frequently applied on gear cutter such as racks, hobs, and shapers, among which, semi-topping and protuberance are two common modifications [9]. Specifying the pressure angle as some arbitrary parametric functions of the rotation displacement of gears, a large amount of parametric tooth profiles in the dedendum part and addendum part of gears

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and rack can be logically developed and analyzed by the model respectively [10]. It is clear that the pressure angle of the gear is equal to that of the basic rack, there is a more direct proof that the two pressure angles are equal[11]. In this paper we study the profile of spur gear tooth based on rack cutter formulation and variety of pressure angle, and a computer programme is developed to simulate the proposed approach considering the standard values of pressure angle.

II. Construction of Involute Gear Tooth

The involute of a circle is a curve that can be generated by unwrapping a taut string from a cylinder that always tangent to the base circle and the center of involute curvature is always at the point of tangency. A tangent to the involute is always normal to the string, which is the instantaneous radius of curvature of the involute curve. The involute curve is started from the base circle and it can be generated by using the following relations of tooth radii which is shown in Fig. 1 [1], [5]:

$$r_p = 0.5(mZ) \tag{1}$$

$$r_a = r_p + m + C_f m \tag{2}$$

$$r_b = r_p \cos \phi \tag{3}$$

$$r_f = r_p - m(1.25 - C_f) \tag{4}$$

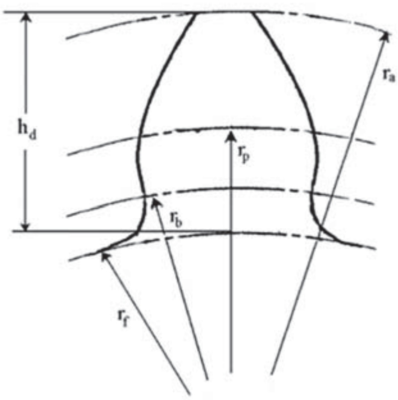

Fig. 1 Spur gear tooth radii's

III. Gear Tooth Generation

When the cutting tool transverse and the work rotate, an involute is generated on the gear tooth flank and a trochoid in the root fillet, as shown in Fig. 2. Fig. 3 is a closer view at a hob tooth and it shows a hob of pressure angle f and diametral pitch π / (TH+TP) [12].

The distance (B + Rrt) on the hob tooth is equal to the dedendum of the generated gear tooth, Rrt is the hob tip radius with its center at point o. TP is the hob tooth space, is equal to the tooth thickness of generated gear at gear pitch diameter.TH is the hob tooth thickness. When the hob traverses a distance (TP + TH), through an angle (TP + TH)/r_p , where (TP + TH) is the circular pitch of the gear, the involute and trochoid can be plotted on a Cartesian coordinate system emanating at the center of the gear tooth as shown in Fig. 4 [12]. The following paragraphs equations are developed to generate this plot, these equations are easily programmed and the coordinates can be plotted automatically:

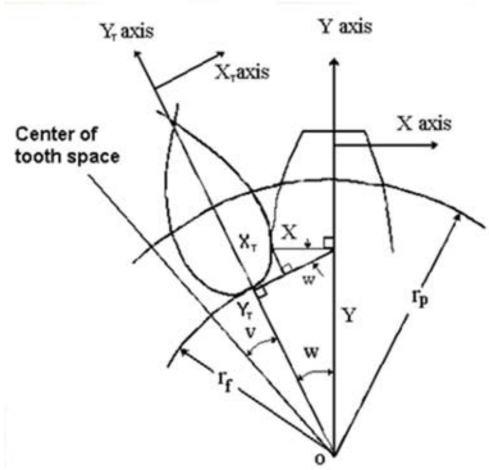

Fig. 2 Trochoid and involute curves

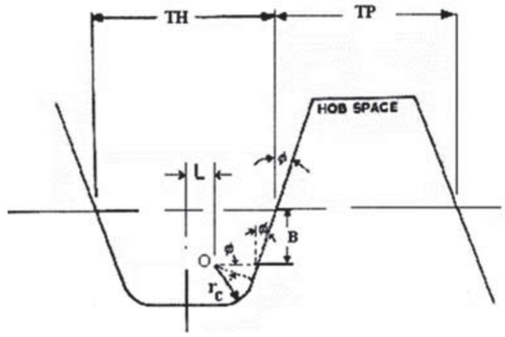

Fig. 3 Hob geometry

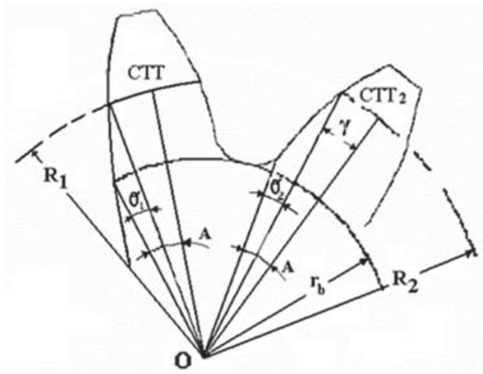

Fig. 4 Tooth thickness calculation

IV. Involute Coordinates:

It is shown that if the base circle radii and the involute angle θ are known, the radii to the curve can be found for any assumed pressure angle. Therefore, to find the coordinates with respect to the center of the tooth, the tooth thickness at any radii must be known. So, let us start at the pitch diameter with a pressure angle ϕ , pitch radii r_p, and a circular tooth thickness CTT_1, the involute angle is:

$$\theta_1 = \tan \phi_1 - \phi_1 \tag{5}$$

It can be seen From Fig. 4 that the angle (A) is:

$$A = \theta_1 + \frac{CTT}{2r_p}$$

$$\beta = A - (\theta_2) = \theta_1 + \frac{CTT}{2r_p} - \theta_2$$

To find the circular tooth thickness at any other radii R2:

$$\phi_2 = \cos^{-1} \frac{r_b}{R_2} \tag{6}$$

$$\theta_2 = \tan \phi_2 - \phi_2 \tag{7}$$

$$\therefore CTT_2 = 2\left(\frac{CTT}{2r_p} + \theta_1 - \theta_2\right)$$

$$\gamma = \frac{CTT_2}{2R_2} \tag{8}$$

To find the X and Y coordinates of the involute at any radii R_2:

$$X = R_2 \sin \gamma \tag{9}$$

$$Y = R_2 \cos \gamma \tag{10}$$

V. Trochoid Coordinates

To find the trochoid coordinates on the desired X-Y system through the center of the tooth, they must be shifted through the angle (w) as in Fig. 2. When the hob traverses a distance (TH + TP) as in Fig. 3, the gear rotates through an angle (TH + TP)/rp; therefore, the angle (w + v) between the center of the gear tooth and the center of the tooth space is [12]:

$$w + v = \frac{(TH + TP)}{2r_p}$$

Angle ν can be calculated as ($v = L/r_p$) where L is the distance between center hob tooth and point O in Fig. 4:

$$L = \frac{TH}{2} - B \tan \phi - \frac{r_c}{\cos \phi}$$

$$w = \frac{0.5(TH + TP) - L}{r_p}$$

Fig. 5 shows the trochoid generated by point O at its starting point and after the hob has moved a distance ($r_p.\beta$) and the gear has rotated through an angle β the coordinate are [8]:

$$X_O = R_O \sin(T - \beta) = R_O(\sin T \cos \beta + \cos T \sin \beta)$$

$$Y_O = R_O \cos(T - \beta) = R_O(\cos T \cos \beta + \sin T \sin \beta)$$

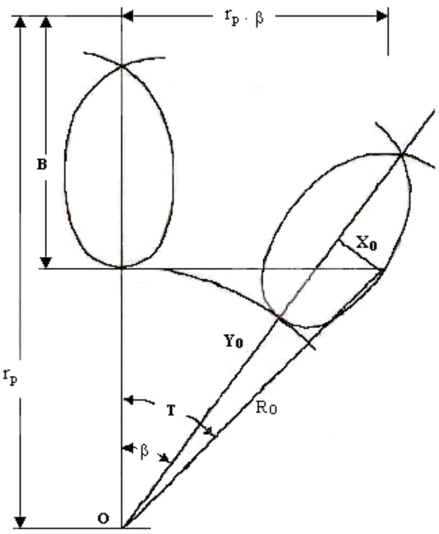

Fig. 5 Trochoid generated by point O

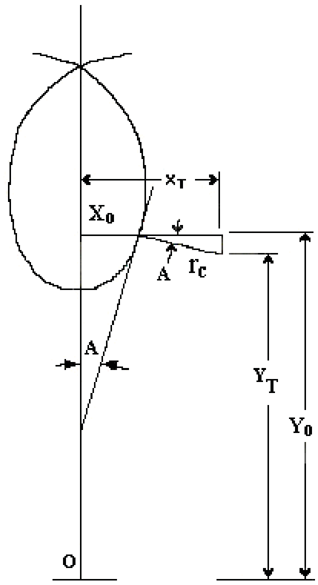

Fig. 6 Trochoid coordinates

where:

$$\cos T = \frac{r_p - B}{R_O} \Leftrightarrow \sin T = \frac{r_p.\beta}{R_O}$$

Therefore:

$$X_O = (r_p \beta)\cos \beta - (r_p - B)\sin \beta \tag{11}$$

$$Y_O = (r_p - B)\cos \beta + (r_p \beta)\sin \beta \tag{12}$$

Fig. 6 shows how to calculate the actual trochoid coordinates by adding rc to the trochoid generated by point O [12]. And r_c can be calculated from the following equation [11]:

$$r_c = 0.3\,m$$

$$X_T = X_o + r_c \cos A \tag{13}$$

$$Y_T = Y_o - r_c \sin A \tag{14}$$

where (A) is the angle formed by a line normal to the trochoid generated by point O and YT axis which can be found as follows:

$$\tan A = \frac{dX_O}{dY_O}$$

To find (dXo / dYo):

$$\frac{dX_O}{d\beta} = -(r_p \beta)\sin\beta + r_p\cos\beta - r_p\cos\beta + B\cos\beta$$

$$\frac{dY_O}{d\beta} = r_p\sin\beta + B\sin\beta + (r_p\beta)\cos\beta + r_p\sin\beta$$

$$\frac{dX_O}{dY_O} = \frac{-(r_p\beta)\sin\beta + B\cos\beta}{B\sin\beta + (r_p\beta)\cos\beta}$$

Finally, to obtain the trochoid coordinates with respect to the system through the gear tooth center, refer to Fig. 2.

$$\sin(w) = \frac{X_T + X\cos(w)}{Y}$$

$$\cos(w) = \frac{Y_T - X\sin(w)}{Y}$$

$$\frac{X_T + X\cos(w)}{\sin(w)} = \frac{Y_T - X\sin(w)}{\cos(w)}$$

$$\therefore\quad X_T\cos(w) + X\cos^2(w) = Y_T\sin(w) - X\sin^2(w)$$

$$X = Y_T\sin(w) - X_T\cos(w) \tag{15}$$

$$Y = Y_T\cos(w) + X_T\sin(w) \tag{16}$$

It is able to add the trochoid and involute curves in one programme to obtain spur gear tooth generation. This programme was operated for 5 mm module and zero profile shift factor and varying the pressure angle values (14.5°, 20°, 25°and 30°).

The programme outputs for the spur gear tooth profiles for each pressure angle are plotted as shown in Figs. 7-10. The root and base radii values are presented as programme output data in these figures.

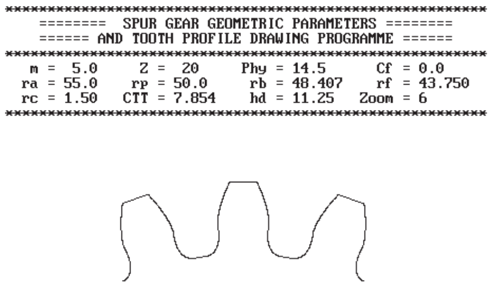

Fig. 7 Spur gear tooth profile at Ø =14.5°

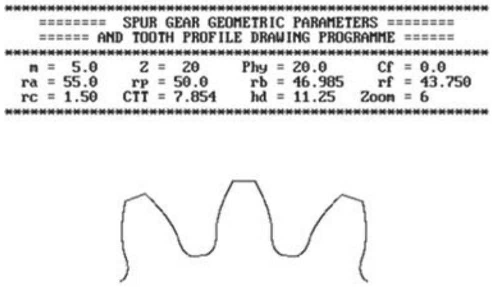

Fig. 8 Spur gear tooth profile at Ø =20°

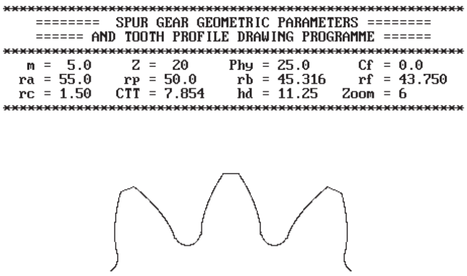

Fig. 9 Spur gear tooth profile at Ø =25°

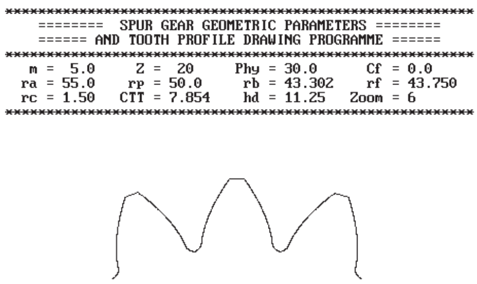

Fig. 10 Spur gear tooth profile at Ø =30°

Also, this programme was applied for module 1mm, 20° pressure angle and 20 teeth with profile shift values (-0.2, 0.2, 0.4, 0.6). The programme outputs for the spur gear tooth profiles for each profile shift factor selected values are plotted as shown in Figs. 11-14 where root and base radii values are also presented in these figures.

It is clear to notice from the outputs that the values of base radii are decreases when the pressure angle values increase, which is shown by the curves of Fig. 15. But here the values of root radii are increases when profile shift factor values increase which is shown by the curves of Fig. 16.

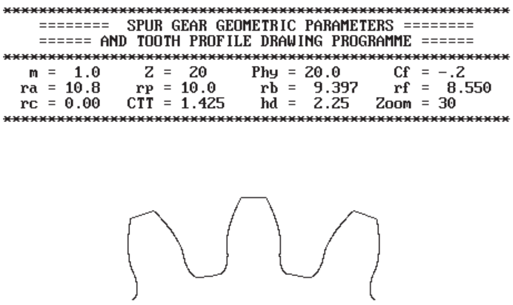

Fig. 11 Spur gear tooth profile at C_f = -0.2

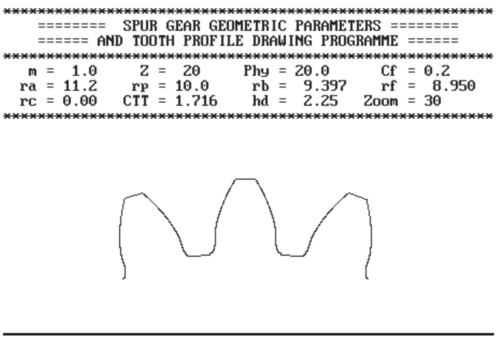

Fig. 12 Spur gear tooth profile at C_f = 0.2

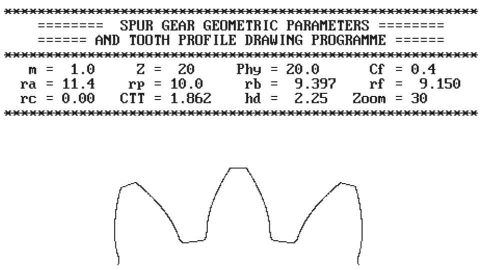

Fig. 13 Spur gear tooth profile at C_f = 0.4

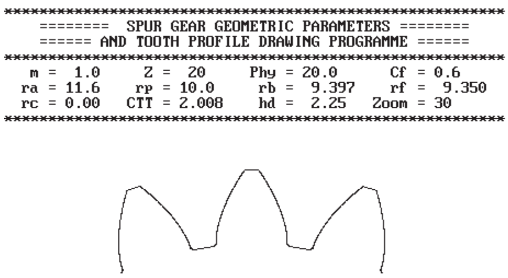

Fig. 14 Spur gear tooth profile at C_f = 0.6

Gitin M. Maitra [6] has been considered that the mutual position of the base circle and the root circle will depend upon the number of teeth for any particular basic rack and it is wrong to presume that the root circle is the smallest circle in a gear, and he proved when the number of teeth exceeding 41, the root circle becomes greater than the basic circle (at 20° pressure angle). The increasing of pressure angle value and profile shift value will cause root circle bigger than base circle too, even when number of teeth is less than 41, which also an anther fact found from this work.

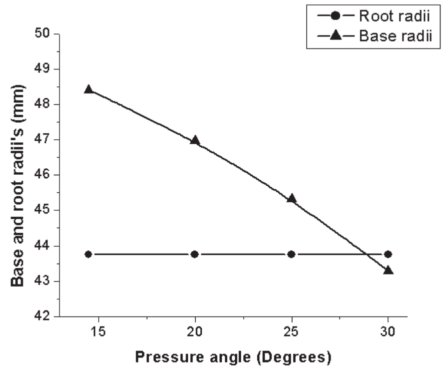

Fig. 15 Pressure angle effects curve

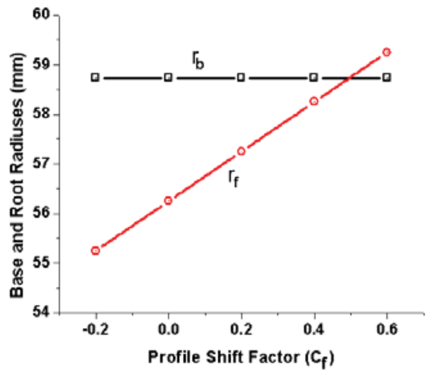

Fig. 16 Profile shift factor effects curve

V. Modal Analysis

For above cases a model has been built for each case using Finite Element Technique, an eight node iso-parametric plane stress quadrilateral quadratic element has been used to operate modal analysis by ANSYS software. A sample of the meshed models of selected cases for spur gear tooth showed in Figs. 17 and 18. Fig. 17 presents case of 1mm module, and pressure angle 14.5˚ and profile shift factor of -0.2 and Fig. 18 presents the finite analysis model of case of 5mm module, and pressure angle 30˚ and profile shift factor of 0.

The resulted natural frequencies from the modal analysis on selected models by ANSYS software have been listed in the following tables:

TABLE II
Natural Frequency Values for Different Pressure Angle Values

Pressure angle (Degree)	Natural Frequency (cycle/sec)	Operating Conditions	
14.5	31917	Module (mm)	5
20	32872	Profile shift factor	0
25	34582	Number of teeth	20
30	37567		

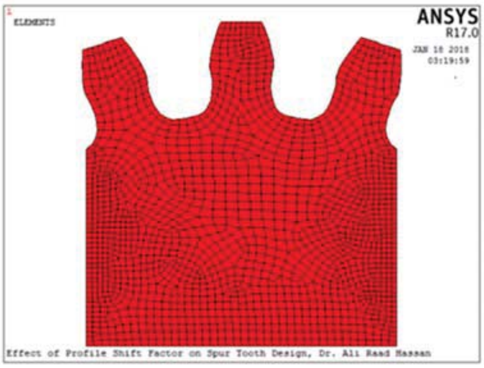

Fig. 17 Model mesh for spur tooth module 1mm of pressure angle 14.5˚ and C_f = -0.2

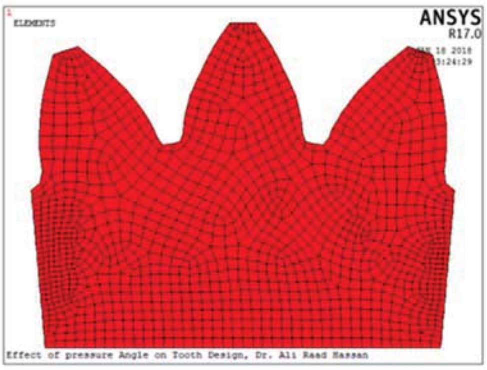

Fig. 18 Model mesh for spur tooth module 5mm of pressure angle 30˚ and C_f = 0

TABLE III
NATURAL FREQUENCY VALUES FOR DIFFERENT PRESSURE ANGLE VALUES

Profile shift factor	Natural Frequency (cycle/sec)	Operating conditions	
-0.2	1.52E+05	module (mm)	1
0	1.63E+05	Pressure angle	14.5˚
0.2	1.72E+05	Number of teeth	20
0.4	1.79E+05		
0.6	1.83E+05		

The results presented at Tables I and II are plotted to find the curve relations shown in Figs. 19 and 20 respectively.

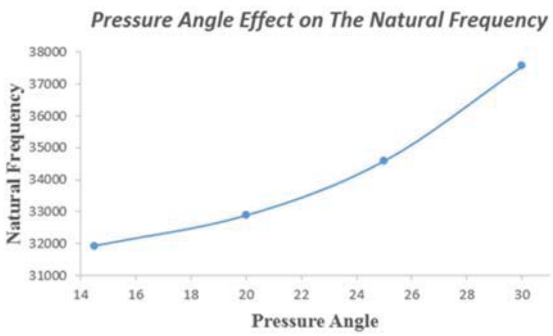

Fig. 19 Pressure angle- natural frequency curve

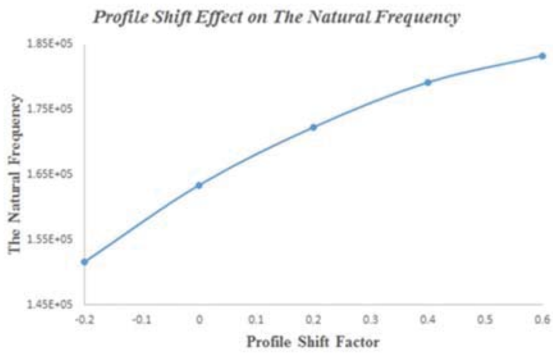

Fig. 20 Profile shift factor- natural frequency curve

VI. CONCLUSION

The paper shows the exact curve for the spur gear tooth profile without using any approximating methods to find the profile points' locations, because each point has been determined from approved theoretical equations. **QBASIC** programme has been developed to plot this profile based on rack cutter formulation. The fillet part of tooth profile curve has been drawn by mathematical analytical curve (trochoid) not by circle arc. The obtained results of this work have been shown in Figs. 7-10 for standard pressure angle values (14.5°, 20°, 25° and 30°) at (5 mm) module, 20 teeth and zero profile shift factor, also it have been shown in Figs. 11-14) for standard profile shift factor values (-0.2, 0.2, 0.4 and 0.6) at (1 mm) module, 20 teeth and 20° pressure angle. The programme is very flexible and any spur gear tooth profile can be found for any other pressure angles or profile shift factors and for any standard or non-standard cases.

This work shows how it is easy to find the spur gear tooth model by using the above drawing programme in order to use this model later in any stress analysis software like **ANSYS**, **ABAQUS**, etc. The programme outputs include the values of the most important geometric parameters for design of spur gear tooth. This work has found very important facts; firstly it has proved that the increasing in pressure angle value will decrease the base radius while the increasing in profile shift factor (correction factor) value will increase the root radius, secondly it has found that the increasing in values of: pressure angle, profile shift factor or both will certainly cause root circle bigger than base circle not only by the increasing in number of teeth.

The modal analysis presented an increase in the natural frequency values due the increase in the pressure angle values as Table II and Fig. 19 shows, meanwhile the increase in the profile shift factor value caused an increase in the natural frequency values as Table III and Fig. 20 shows. The high values of natural frequency is not healthy and causes dynamic problems, stresses, noise and effects other connected mechanical members such as bearings. Hence, this work found that the value of pressure angle from (14.5˚) to (20˚) and the value of profile shift factor from (0) to (0.1) give a better operation conditions and supply a good stability of gear systems.

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