World Academy of Science, Engineering and Technology
International Journal of Aerospace and Mechanical Engineering
Vol:12, No:4, 2018

Steady State Rolling and Dynamic Response of a Tire at Low Frequency

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Abstract**—**Tire noise has a significant impact on ride quality and vehicle interior comfort, even at low frequency. Reduction of tire noise is especially important due to strict state and federal environmental regulations. The primary sources of tire noise are the low frequency structure-borne noise and the noise that originates from the release of trapped air between the tire tread and road surface during each revolution of the tire. The frequency response of the tire changes at low and high frequency. At low frequency, the tension and bending moment become dominant, while the internal structure and local deformation become dominant at higher frequencies. Here, we analyze tire response in terms of deformation and rolling velocity at low revolution frequency. An Abaqus FEA finite element model is used to calculate the static and dynamic response of a rolling tire under different rolling conditions. The natural frequencies and mode shapes of a deformed tire are calculated with the FEA package where the subspace-based steady state dynamic analysis calculates dynamic response of tire subjected to harmonic excitation. The analysis was conducted on the dynamic response at the road (contact point of tire and road surface) and side nodes of a static and rolling tire when the tire was excited with 200 N vertical load for a frequency ranging from 20 to 200 Hz. The results show that frequency has little effect on tire deformation up to 80 Hz. But between 80 and 200 Hz, the radial and lateral components of displacement of the road and side nodes exhibited significant oscillation. For the static analysis, the fluctuation was sharp and frequent and decreased with frequency. In contrast, the fluctuation was periodic in nature for the dynamic response of the rolling tire. In addition to the dynamic analysis, a steady state rolling analysis was also performed on the tire traveling at ground velocity with a constant angular motion. The purpose of the computation was to demonstrate the effect of rotating motion on deformation and rolling velocity with respect to a fixed Newtonian reference point. The analysis showed a significant variation in deformation and rolling velocity due to centrifugal and Coriolis acceleration with respect to a fixed Newtonian point on ground.

Keywords**—**Natural frequency, rotational motion, steady state rolling, subspace-based steady state dynamic analysis.

I. Introduction

NOISE has an adverse environmental effect on urban life. Though there has been tremendous development in automotive technology in recent years, noise generated from vehicles is still a big concern for the auto industry for environmental reasons and because it impacts ride quality. Many moving parts of vehicle generate noise. A significant portion of the noise is generated by the tire/road interaction. The two main sources of tire noise are 1) structure-borne noise and 2) the noise generated by the horning effect in the tire grooves during tire /road interaction [1].

In the horning effect, air is sucked in and pushed out of the contact area at the tire tread during each tire revolution, which creates an air pumping effect, which generates high frequency noise [2]. In the case of tires without the block pattern found in current tire models, the main source of noise is restricted to structural-borne noise.

Previous experiments and simulations have mostly concentrated on eliminating high frequency noise (above 200 Hz) [3]. But the behavior of tire noise varies substantially with frequency due to the complex structure of tires. For structural noise, tension and bending becomes predominant at low frequency and torsion becomes important at higher frequency [4]. So the radial and lateral deformation of any point on tire is not similar at low and higher frequency range. It is also convenient to follow the noise at higher frequency. But at lower frequency, the dynamic response of tire is not easily tractable. For this reason, the current paper only investigate the effects of low frequency noise (below 200Hz) on tire deformation and rolling motion.

To understand the effect of low noise, employing correct prototypes of the tire is very important. To reduce the complexity of analysis, ring or plate/shell shape analytical models [5]-[7] were proposed earlier. These simplifications have reduced the computational effort but limited the importance of complex structural configuration of modern tire. Now for simulation, similar simplified models have been used earlier and only radial part of contact force was considered by neglecting tangential force of moving tire . To remove the constrained mentioned above, we need a proper FE model which can incorporate correct material behavior and forces developed during tire rotation. This detailed finite element models is also necessary to study dynamic response of a tire at different frequency through Eigen value analysis [8]. Now the dynamic response of tire is not similar for static and rolling tire [9]. But most of the previous analysis have been conducted on static tire and neglected centrifugal, Coriolis effects and the dependency of Eigen frequency on rolling motion. So it is necessary to study and compare the dynamic response of a static and rolling tire at low frequency range.

Now for the Eigen value analysis, the cost depends on matrices provided by the finite element model and large-scale systems of linear equations. Generally direct solution analysis is used to calculate linearized response of the dynamic system. Though the result from the analysis is very accurate, the method is computationally expensive for complex FE model of tire [10]. So we need an alternative method which will solve the problem with almost similar accuracy but at less computational cost. The subspace-based steady state dynamics is such an effective solution for the problem mentioned above. In this method, steady-state dynamic equations are projected

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on a subspace of selected modes of the undamped system for a frequency range and the system is excited at Eigen frequency extracted prior to such analysis. The projection of dynamic equilibrium equations into a subspace of selected modes reduce the system of complex equations and computational cost.

Static and steady state transport analysis have to be performed prior to dynamic analysis. Modeling rolling motion of a deformable body, in contact with a surface is difficult for Lagrangian analysis. Because the reference frame is in motion and for observer, the problem becomes transient. But if we are able to attach the reference frame to the axle of rolling tire, the motion of any particle of tire will be fixed with respect to the observer on axle but will only move with deformation of tire. This optimal approach is known as mixed Eulerian-Lagrangian method in which the rigid body rotation is described in Eulerian (spatial) manner and the deformation is described in Lagrangian manner. A short overview on steady state rolling tire analysis can be found in literature review [11], [12]. Now the problem arise when rotational motion is introduced in analysis. Rotational motion enforces centrifugal and Coriolis acceleration which were absent in most of the previous computations. Besides, velocity and acceleration of any point on tire will not be similar with respect to reference point on tire and fixed Newtonian reference point on ground. Here the tire is not only rotating on its own reference axis but also rotating against a fixed reference frame on ground. So the calculation of any parameter with respect to fixed reference point on ground should include change of local parameter on rotational body and relative motion of local reference point with respect to fixed point on ground.

To remove the constraints discussed above, first part of the current paper deals with steady state analysis of a rolling tire with respect to fixed Newtonian fixed point on ground.As a consequence,centrifugal and Coriolis force have been included in the Eulerian-Lagrangian analysis. Before steady state rolling, static foot print analysis of the tire has been performed with different loading conditions and the corresponding deformations have been analyzed for half and full tire model The second part of the paper deals with dynamic analysis of tire. At first, dynamic analysis has been conducted on static tire and the natural frequencies and mode shapes of a deformed tire have been calculated by using finite element package (Abaqus). Then the FE model of tire was excited with 200N harmonic load at low frequency (20-200 Hz) and the corresponding response of road and side node of tire have been obtained at low frequency range. For rolling tire, similar analysis have been performed and the results have been compared with static tire analysis. The results of current study will help us to understand steady state and dynamic response of a static and rolling tire at low frequency range.

II. Model Generation

For correct computational analysis, the elements of FE model should be selected carefully. The tire model can be either discretized with equal number of elements or can be discretized non uniformly with more elements near the region

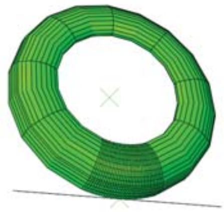


Fig. 1 FE partial model of a tire

of interest. The non uniform distribution of elements makes the computation faster and more elements near the region of interest reduce computational error. Couple of test analysis have been performed for both uniform and non uniform mesh models to check the accuracy of the results and the out come of computations were consistent and similar for both the cases. The axisymmetric model was discretized with bilinear elements. A partial three-dimensional model was generated by revolving the axisymmetric model about the rotational axis. The generated partial model is shown in figure 1. This partial three-dimensional model is also composed of bilinear elements except for footprint region where more linear elements have been utilized to increase resolution near contact patch. The full three-dimensional model was generated by reflecting the partial three-dimensional model. One thing should be noted that all the elements used for the analysis is hybrid in nature. The hybrid element is suitable for large deformation and avoid shear and volume locking. The construction of FE model was very similar to real tire except for the absence of any specific tread pattern. The tread pattern was not introduce in the model to restrict the source of noise only from structure of tire. The tread and sidewalls are made of rubber, and the belts and carcass are constructed from fiber-reinforced rubber composites. The viscoelastic rubber was modeled as an incompressible hyper-elastic material by 1-term Prony series. For viscoelastic model, the value of relaxation modulus was $g^p = 0.3$ and its associated relaxation time was $\tau^p = 0.1$. For accurate representation of FE model, property of material should be similar to real tire values.

III. Static Analysis of Tire

Static analysis of a tire was performed prior to steady state rolling analysis. Then the final results from static analysis was transferred as the initial results for steady state analysis. During static analysis,tire was kept in static condition and load was applied to the tire over several steps. In the current simulation, the tire was inflated with a pressure of 200.0 kPa and a vertical load of 1.7 kN was applied to the rigid body reference node for partial three-dimensional model to represent a 3.4 kN load in the full three-dimensional model. The contact between road and tire was modeled as hard contact. The penalty method approximates hard pressure-overclosure behavior. As the tire was static, the friction between road and tire was considered zero and the contact between two surfaces were modeled using contact pair algorithm. The deformation of final step is shown in Fig. 2 and contact area and pressure is shown in Fig. 3.

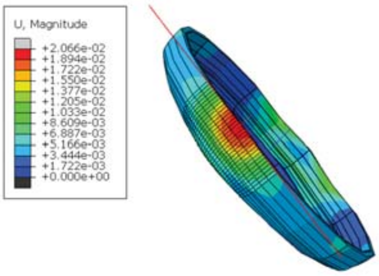
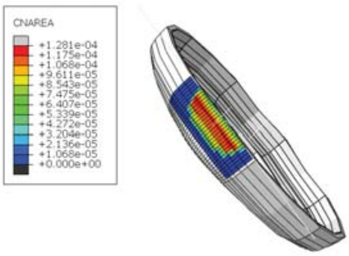
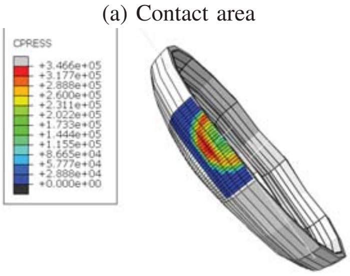


Fig. 2 Deformation of static tire

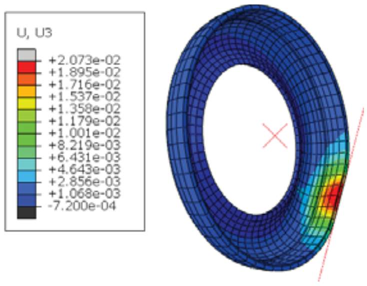


(a) Contact area

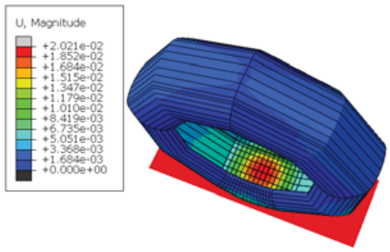


(b) Contact pressure

Fig. 3 Contact area and pressure of a static tire at footprint region



(a) Deformation of equal grid model



(b) Deformation of full tire model

Fig. 4 Deformation of half and full static tire model

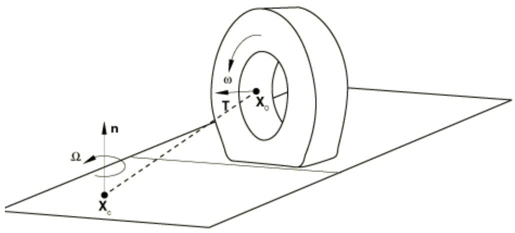


Fig. 5 Steady state analysis model

In the previous case, the footprint region was discretized with more elements than the remaining circumferential element of tire. In the second case, the tire was modeled with equal number of bilinear elements over the partial tire model. For the second simulation, similar steps were followed as discussed in previous case with identical applied loads and boundary conditions and the interaction region between road and tire displayed almost similar deformation. Fig. 4 (a) shows the component wise deformation of partial model tire. Once the static footprint solution for the partial three-dimensional model has been established, symmetric results transfer was used to transfer the solution to the full three-dimensional model and the footprint solution was brought into equilibrium in a single static increment. The deformation of full tire is shown in Fig. 4 (b).

IV. Steady State Transport Analysis

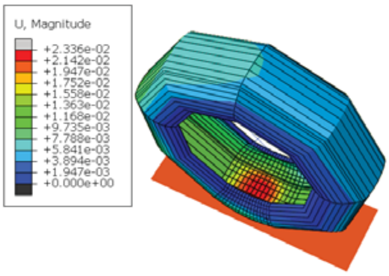
For steady state analysis in current study, a mixed Eulerian/ Lagrangian method has been used where the rigid body rotation is describer in an Eulerian manner and deformation of tire as an Lagrangian manner. Thus the kinematics of problem has been converted to transient to spatial dependent problem. In addition, the analysis requires less time due to fine meshing only in the region of interest. In the analysis, frictional effects, inertia effects, and history-dependent material models were included during the study. The purpose of the steady state analysis analysis was to obtain free rolling equilibrium solutions of a 175 SR14 tire traveling at a ground velocity of 10.0 km/h (2.7778 m/s). The model is shown in Fig. 5 [13] where the ground velocity of body is described in terms of a constant cornering motion.

For steady state condition, velocity and acceleration in the reference frame tied to the body can be expressed by following equations [13]:

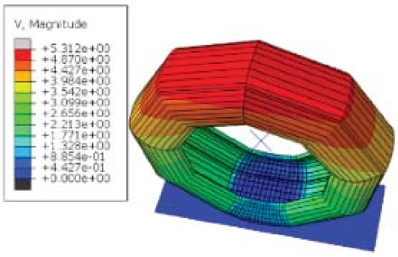
$$V_r = \Omega n \times (x - X_c) + \omega R \frac{\partial \lambda}{\partial S} \qquad (1)$$

$$a_r = \Omega^2 (nn - I).(x - X_c) + 2\omega \Omega R n \times \frac{\partial X}{\partial S} + \omega^2 R^2 \frac{\partial^2 \lambda}{\partial^2 S} \qquad (2)$$

Here V_r and a_r is the velocity and acceleration of reference point on tire respectively. Ω is the rotational velocity with respect to fixed point on ground and ω is the local angular velocity. λ is used for mapping between Eulerian and Lagrangian frame. s represents distance along circumferential direction of tire and R is the radius of a point on the reference body. The velocity expression depends only on local rotational velocity of tire with respect to own axis and rotational velocity with respect to Newtonian fixed frame on

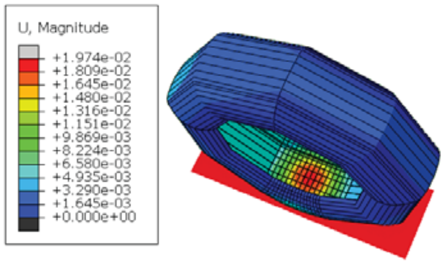


(a) Straight line deformation

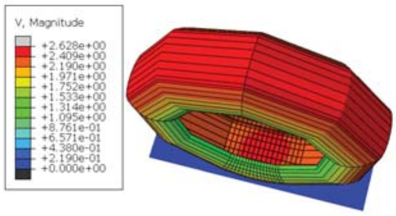


(b) straight line velocity

Fig. 6 Steady state straight line analysis of tire

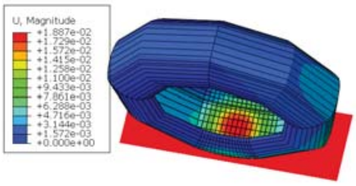


(a) Deformation

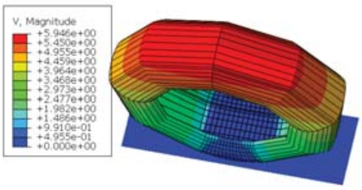


(b) Rolling velocity

Fig. 7 Deformation and rolling velocity only for rotating motion



(a) Deformation



(b) Rolling velocity

Fig. 8 Linear and rotating velocity with respect to ground fixed point

ground. For acceleration, the first term gives rise to centrifugal forces. The second term represents acceleration for Coriolis forces. The last term combines the acceleration for Coriolis and centrifugal forces resulting from local rotation about own axis. If the deformation is uniform along the circumferential direction, this Coriolis effect vanishes so that the acceleration gives rise to centrifugal forces only. For straight line motion, as shown in Fig. 5, the first term of velocity can be neglected. The centrifugal and Coriolis acceleration due to rotational velocity with respect to Newtonian fixed point becomes zero . So the final expression for straight line rolling velocity and acceleration can be expressed as follows:

$$V = V_0 + V_r = V_0 + \omega R \frac{\partial \lambda}{\partial S} \quad (3)$$

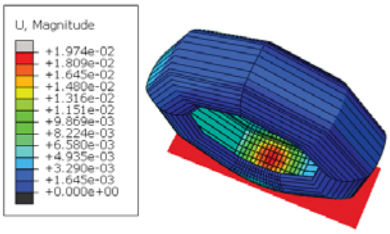
$$a = a_r = \omega^2 R^2 \frac{\partial^2 \lambda}{\partial^2 S} \quad (4)$$

Here V_0 is the ground velocity at the axle of tire. For straight line motion, the magnitude of deformation and straight line velocity is shown in Fig. 6.

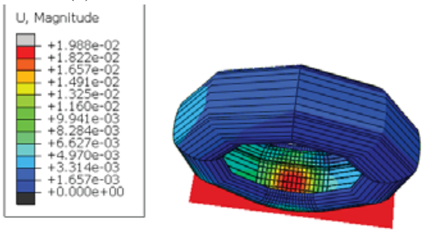
During the simulation, only straight line rolling velocity of tire has been considered and ignored the rotating motion with respect to fixed Newtonian reference point on ground. Because of this assumption, centrifugal and Coriolis acceleration due to rotation with respect to fixed reference point become zero and the expression becomes very simple which include only local centrifugal and Coriolis acceleration. Now in the second case, we have only considered rotating velocity of tire with respect to fixed reference point on ground and ignored the local ground velocity. In this case is tire is not only rotating around fixed Newtonian reference point but also spinning on its own axis. The velocity profile for this case is shown in Fig. 7. As we are considering only rotational velocity of the tire, the magnitude of rolling velocity is less than straight line velocity case and also there is difference in deformation in both the cases.

In the third case, both the ground velocity of tire and rotational velocity with respect to reference point on ground have been considered. So all the terms of velocity and acceleration of expression have to be considered in the computation. The results of rolling velocity and deformation is shown in Fig. 8. If we compare the three steady state cases, we will find significant difference in the steady state rolling velocity and deformation due to change in running parameter. In case of only linear ground velocity, maximum straight line velocity was 5.34 m/s. In only rotating velocity case without ground velocity, the maximum rolling velocity was 2.6 m/s and in the final case which include both ground velocity and rotational motion exhibited highest rolling velocity of 5.94 m/s. So there is significant effect of these parameters on tire rolling motion.

The effect of material property of tire was also studied during steady state analysis. A simple 1-term Prony series



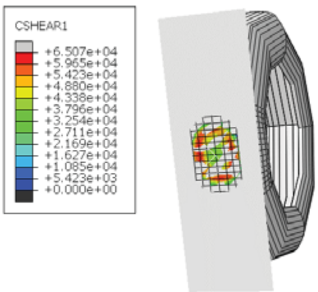
(a) Visco-elastic transient model



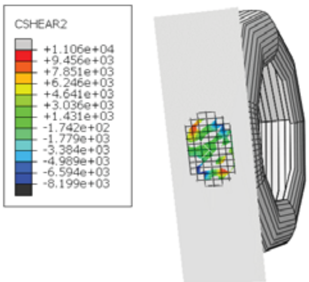
(b) Long term model

Fig. 9 Effect of Material property on deformation of a tire

model was used for analysis. In the current analysis, relaxation modulus $g^p = 0.3$ and its associated relaxation time, $\tau^p = 0.1$ have been used for calculation.Time dependent viscoelastic material property and also the long-term behavior of the material have been investigated during steady state analysis. The results of effect of material property on deformation is shown in Fig. 9. For long term analysis, the deformation was predicted higher than viscoelastic transient model.



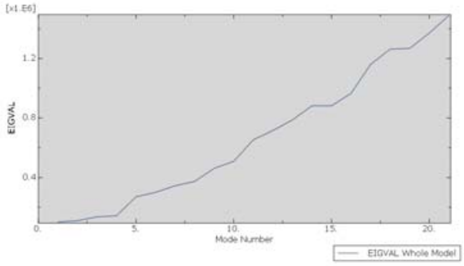
(a) Shear stress along the road



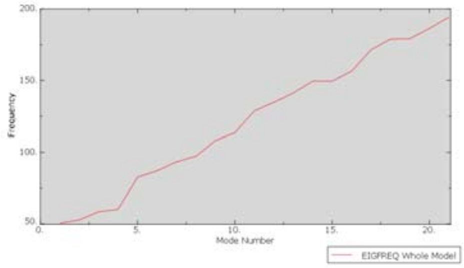
(b) Shear stress transverse of road

Fig. 10 Shear stress in footprint region during steady state analysis

In the static analysis,the value of friction coefficient was zero and only normal force was acting on contact point at road node. For the steady state case, the value of friction co-efficient was not zero and not only contact pressure but

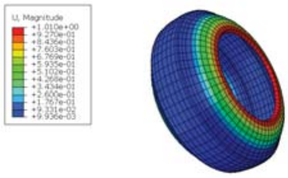


(a) Eigen value

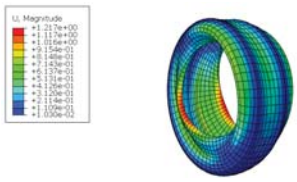


(b) Eigen frequency

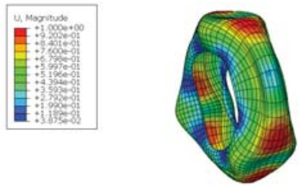
Fig. 11 Eigen value and frequency of static tire



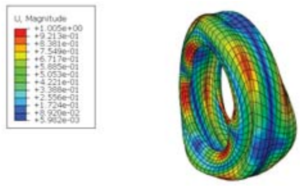
(a) Eigen frequency 42.77



(b) Eigen frequency 72.09



(c) Eigen frequency 113.26



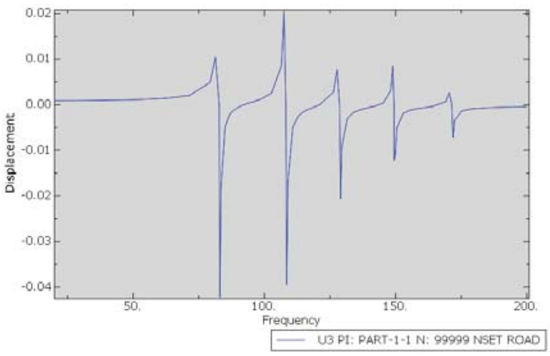
(d) Eigen frequency 176.51

Fig. 12 Mode shapes of tire at different Eigen frequency

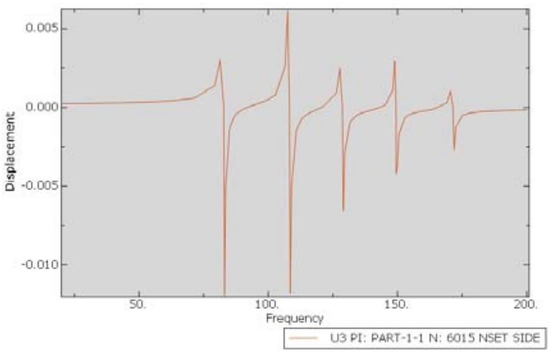
shear stress developed due to continuous interaction of tire with road surface in the footprint region. Fig. 10 shows shear stress developed near the footprint region along the road and transverse direction of travel with friction co-efficient 0.2 during straight line motion study.

V. Subspace-Base Dynamic Analysis of Tire at Low Frequency

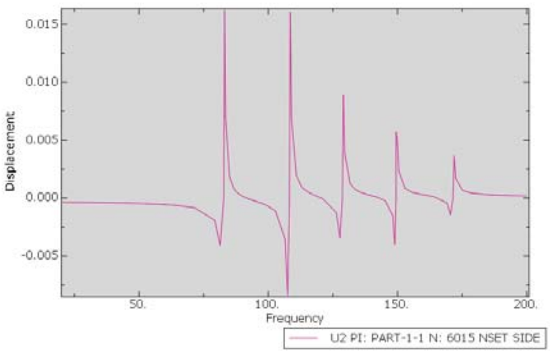
Subspace-based steady-state dynamic study is an analysis procedure to determine dynamic response of a system subjected to harmonic excitation by directly projecting the solution into a reduced-dimensional subspace of undamped system. Prior to analysis, Eigen frequency has been extracted within the frequency domain of interest. Figs. 11 and 12 display Eigen value and frequency of FE tire model at different modes.



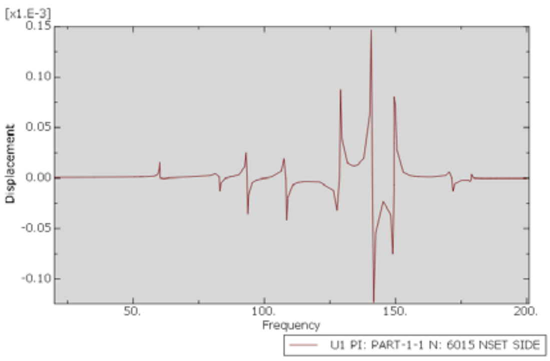
(a) Deformation (u3) at road node



(b) Deformation (u3) at side node



(c) Deformation (u2) at side node



(d) Deformation (u1) at side node

Fig. 13 Deformation of static tire at low frequency

The subspace-based steady-state dynamic is a linear method but nonlinear response from static and steady state rolling analysis can be included in the study. The static tire was

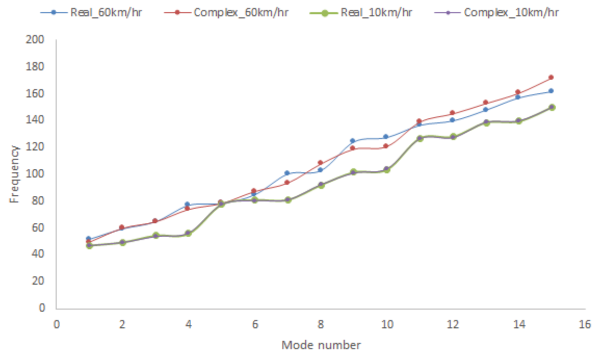


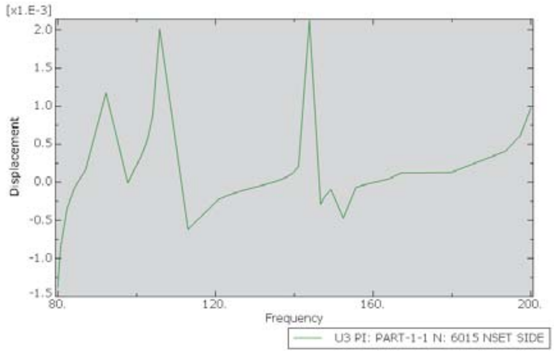
Fig. 14 Eigen frequency at rolling velocity

excited with a vertical load of 200 N and the frequency was swept from 20 Hz to 200 Hz. The boundary condition was kept similar to the static footprint analysis of tire. The objective of the analysis was to observe frequency response of static tire in low frequency range at the road node (contact point between road and tire)and side node (point located on lateral side of tire).

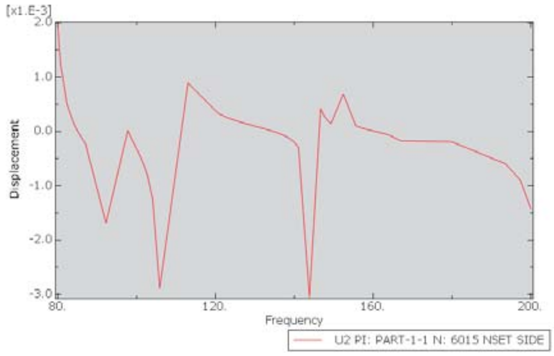
The response at road node for vertical displacement (U_3) is shown in Fig. 13 (a). From the plot, it can be observed that almost nothing happens until 80 Hz. But between 80 Hz to 200 Hz, there was periodic fluctuation which decreased with the increasing frequency. The frequency until 80 Hz was too small to effect the displacement of tire. For side node, similar frequency response was observed in the vertical direction but smaller in magnitude. Now for road direction, one single fluctuation was observed at 60 Hz and the magnitude of oscillation increased between 80 to 140 Hz. The displacement magnitude was smaller for road direction with respect to displacement in other two directions for side node. From the dynamic response it can be concluded that the displacement of tire dies with frequency for static tire at lower frequency range.

The dynamic response of a rolling tire was also studied for rolling velocity of 60 km/hr. Direct solver was used to solve the systems of equations. Before the dynamic analysis, real and complex Eigen values of tire were extracted for rolling motion to understand the effect of centrifugal and Coriolis forces. Fig. 14 shows the comparison of first 15 pairs of Eigen frequency between 0 to 200 Hz at two different rolling velocity of 10 and 60 km/hr.

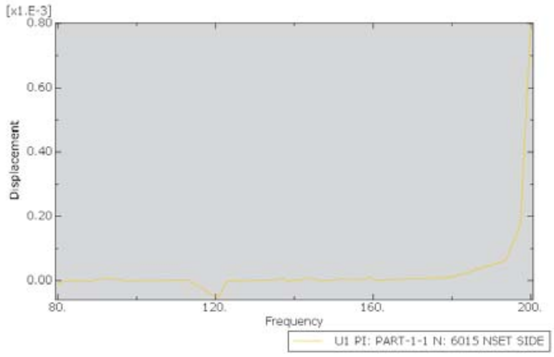
At low rolling velocity, the value of real and complex Eigen frequency was close in magnitude. But as the velocity increased, the real and complex Eigen frequency changed and displayed dependency on rolling motion. The Eigen frequency was for rolling tire was also different from static analysis. After calculating the Eigen frequency, the rolling tire was excited with 200 N vertical load and the response was obtained at Eigen frequency between 80 to 200 Hz. Displacement for side node (point located at lateral side of tire)is shown in Fig. 15. In contrast to static analysis the displacement was small in magnitude and periodic in nature for vertical and lateral direction and continuously increased between 150 to 200 Hz. For road direction, the displacement was almost zero until 180



(a) Deformation (u3) at side node



(b) Deformation (u2) at side node



(c) Deformation (u1) at side node

Fig. 15 Deformation at side node for rolling tire at low frequency

Hz and then increased very sharply between 190-200 Hz. From the dynamic response it can be concluded that displacement becomes small for rolling tire but does not die out at lower frequency range in contrast to dynamic analysis of static tire.In contrast to static analysis the displacement was small in magnitude and periodic in nature for vertical and lateral direction and continuously increased between 150 to 200 Hz. For road direction, the displacement was almost zero until 180 Hz and then increased very sharply between 190-200 Hz. From the dynamic response it can be concluded that displacement becomes small for rolling tire but does not die out at lower frequency range in contrast to dynamic analysis of static tire.

VI. Conclusion

Static footprint and steady state transport analysis have been performed prior to dynamic analysis of tire at different rolling conditions. For steady state analysis, three cases have been studied to understand the effect of rotating motion with respect to fixed Newtonian point on ground. The velocity and acceleration of reference point on the body were also different for all the three cases. Then the dynamic response of the tire was recored for static and rolling tire between 20 Hz to 200 Hz. The results show difference in Eigen frequency for static and rolling tire at different rolling conditions. The results also display that the deformation was negligible until 80 Hz and decreased with increasing frequency between 80 to 200 Hz for static tire. In contrast, the displacement of rolling tire was small and periodic in nature but did not die out at lower frequency range. Results obtained in the study will be helpful to understand steady state rolling motion and dynamic response of a tire at low frequency range.

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