Radiative Reactions Analysis at the Range of Astrophysical Energies

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Abstract**—**Analysis of the elastic scattering of protons on ^{10}B nuclei has been done in the framework of the optical model and single folding model at the beam energies up to 17 MeV. We could enhance the optical potential parameters using Esis88 Code, as well as SPI GENOA Code. Linear relationship between volume real potential (V_0) and proton energy (E_P) has been obtained. Also, surface imaginary potential W_D is proportional to the proton energy (E_P) in the range 0.400 and 17 MeV. The radiative reaction $^{10}B(p,\gamma)^{11}C$ has been analyzed using potential model. A comparison between $^{10}B(p,\gamma)^{11}C$ and $^{6}Li(p,\gamma)^{7}Be$ has been made. Good agreement has been found between theoretical and experimental results in the whole range of energy. The radiative resonance reaction $^{7}Li(p,\gamma)^{8}Be$ has been studied.

Keywords—Elastic scattering of protons on 10B nuclei, optical potential parameters, potential model, radiative reaction.

I. Introduction

OPTICAL model still is a good tool to reproduce cross sections for nuclear reactions. Optical potential parameters (OMPs) provide us information about the interacting nuclei. The choice of the optical parameters as starting parameters is very important as potential depth, radius, and diffuseness. The electron scattering is a reliable method to choose the radius of interacting nuclei where diffuseness could be taken as standard value 0.65 fm. The potential depth (real part) is taken depending on the incident particle. For example, in the case where the incident particles are protons, the potential depth is located between 40-60 MeV. The global optical potential parameters are the best choice for the study, if available. The volumetric integral is given by the relation:

$$J_R(E)=-(1/A_pA_{li})\int V(r)4\pi r^2 dr, \quad (1)$$

where A_p and A_t are the mass values of the incident particle and the target nucleus. Hodgson discussed such idea many years ago [1]. The volume integral was found to be energy independent. In our published paper [2], we have discussed such idea in details and the result was that $J_R(E)$ is slightly energy dependent. The imaginary volume integral is determined by:

$$J_W(E)=(1/A_pA_{li})\int[W_V(E.r)+W_S(E.r)]dr, \quad (2)$$

The values of J_R and J_W should be independent on the projectile and target and are reliable tool to compare between different sets of optical potential parameters for different nuclei [3]. The volume integral per nucleon (J_R) has been discussed before in details in [3] and it was shown that J_R value located between 450-360 MeV.fm^3 in energy range up to 40 MeV. Our mission is to find such parameters which could analyze the experimental data and have physical meaning, by calculating J_R(real part) and J_W (imaginary part). The choice of optical potential parameters is very important in sense of physics.

There are two methods to calculate the direct capture cross sections. One of them is the solution of many-body problem for the bound and continuum states. The second one is to use the potential model [4]. For simplicity, the second method is our choice to reproduce the cross sections with a reliable accuracy.

The spectroscopic amplitudes for the $^{10}B+p$ configuration ^{11}C nuclei were fixed at the values taken from [5]. But, their signs have been changed according to the FRESCO convention:

$$S_{\text{FRESCO}}=(-1)^{J_C+j_x-J_A}S_x, \quad (3)$$

where J_C, J_A and j_x are the spins of the core, composite, and transferred nuclei, respectively. The value of the SF for the $^{11}C \rightarrow {}^{10}B + p$ configuration was S = 0.806 [6] if the standard geometry of the Woods-Saxon potential is used. So, in our calculations, we could only extract the spectroscopic amplitudes for the ground and first excited states of ^{7}Be in the $^{6}Li + p$ configuration.

Low energy proton capture reaction study has a special importance in the astrophysics. Measuring the cross section at very low energies is so difficult because it is extremely small. The extrapolation of the cross section from higher energies to lower one is still one solution, may be not completely true but is somewhat fair, which is done using astrophysical S-factor. The measurements available are always between 0.2 to 1 MeV which provide more difficulties to make an extrapolation in astrophysical region [7]. Theoretical predictions can provide missing information of thermonuclear reactions.

Astrophysical S-factor has a rate of change with energy at the astrophysical energy range which should be known to obtain reliable cross section extrapolation. There are many reasons that have negative effect on the extrapolation process such low-energy resonances or sub-threshold states [8]. In contrary with those data published in [9] calculated by Cecil et al., we have obtained negative slope for the reaction $^{6}Li(p,\gamma)^{7}Be$ in [7]. Astrophysical S-factor energy dependence was assumed to be linear as:

$$S(E) = S(0) + AE + BE^2 + \ldots \quad (4)$$

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The results of a measurement of the slope of the astrophysical *S*-factor for the ^{6}Li(p,γ)^{7}Be reaction are reported in [8], and a new mechanism is introduced to explain the observed slope. The slope was determined from the relative yields at five incident proton energies. The slope of the astrophysical S-factor was found to be negative. Cecil et al. [9] measured the branching ratio of ^{6}Li(p,γ_0)^{7}Be and ^{6}Li(p,γ_1)^{7}Be with respect to ^{6}Li(p, α)^{3}He from 45 to 170 keV and deduced the astrophysical *S*- factors for ^{6}Li(p,γ_0)^{7}Be and ^{6}Li(p,γ_1)^{7}Be as a function of energy. Their results gave a positive slope for the *S* factor. Barker's analysis [10] had a negative astrophysical *S*-factor slope for ^{6}Li(p,γ_0)^{7}Be and ^{6}Li(p,γ_1)^{7}Be at energies below the range of the data.

The present work will be divided into two parts. Obtaining optical parameters in wide range of energy is the first step, extracting the spectroscopic factors of the reaction under consideration, if it is not available from literature. The second step is calculation astrophysical S-factor using cross sections calculated by FRESCO program [11].

II. Analysis of ^{10}B(p,γ)^{11}C Reaction

A. Optical Model Analysis

The description of experimental data, obtained in the present work, on the protons elastic scattering on the ^{10}B nucleus is given in Fig. 1. The obtained optical potentials parameters are presented in Table I. The optical parameters for protons scattering on ^{10}B nuclei can be represented by:

$$V_0=56.68-1.15E_p,\ W_D=-0.58+0.56E_p,\ J_w=8.91+1.3E_p,\ \text{and}\ J_R=724-11.24\ E_p. \tag{5}$$

B. Single Folding Analysis

The real part of the optical potential for the nucleon–nucleus elastic scattering is given for the single folding model, in the following form:

$$U_F(R) = \int dr_1\, \rho_1(r_1)\, V(r), \tag{6}$$

where $r = R - r_1$, $\rho_1(r_1)$ is the matter density distribution of the target nucleus, V (r) is the effective NN-interaction. In the present calculation, the effective NN-interaction is taken in the form of M3Y-interaction [12]:

$$V(R) = 2999\left(\frac{\exp(-4R)}{4R}\right) - 21342999\left(\frac{\exp(-4R)}{4R}\right) - 276(1 - \frac{0.005E}{A})\delta(R) \tag{7}$$

The density of the ^{10}B target nucleus is considered in the form [13]:

$$\rho(r) = \rho_0(1 + \alpha\left(\frac{r}{a}\right)^2 \exp(-\left(\frac{r}{a}\right)^2) \tag{8}$$

where for harmonic oscillator a=1.71 fm and α=0.837 fm. The analytical form of the real part of the optical potential is obtained by substituting (7) and (8) into (6) and carrying out the required integrations over r_1.

III. Results and Discussions

Fig. 1 shows theoretical (optical model) solid lines and experimental as points for angular distribution at different energies for proton scattering on ^{10}B. A good agreement between experimental and theoretical data has been obtained using optical model, but in case of 5.3 MeV a deep minimum is obtained, which describes other mechanism of nuclear reactions, i.e. compound nucleus is there. Using SPI GENOA Code, we could calculate a set of optical parameters as the volume integral per nucleon pair for the real and imaginary potentials, J_R and J_w (MeV. fm^3). As mentioned before and depending on our calculations using Ecis88 and Spi Genoa, we could enhance the parameters. The parameters dependence on the incident particle energy is given in the simplest form. Since the well-depth for the nucleon scattering is roughly 50 MeV, this leads to central potential of protons which is about 50 MeV. While experimental cross sections can be described with several discrete values (e.g. 50 MeV, 100 MeV, 150 MeV), the above argument leads to one to prefer 50 MeV deep potential. The analysis of elastic scattering of protons on ^{10}B nuclei has been published before by us at [14]. The cross sections for elastic scattering were calculated by using Eciss88 [15], and SPI-GENOA code [16].

Angular distributions of the elastic scattering of protons on ^{10}B nuclei were calculated at energies E_p = 0.4, 0.6, 0.8, 1, 1.15, 5.3, 8, 13, 17.9, 33.6, 49.5 MeV, experimental data were taken from [14], [17]-[20]. Optimal parameters of the optical potential (OP) are required for further calculation of the cross sections of the reaction ^{10}B(p,γ); the resulting sets of parameters are given in Table I. To minimize discrete ambiguity potential, the starting parameters used geometrical parameters obtained for the elastic scattering p+^{10}B at E_p = 49.5 MeV [21]. Fig. 1 shows the comparison between calculated and experimental data for energies above. The figure shows that the calculations for volume W_V (Set A) and surface W_D (set B) absorption differ only $E_{p,\ lab.}$ = 5.3 and 8 MeV at scattering angles larger than 60°. With increasing energy, the calculations with both sets of parameters are the same (see Table I).

The optical potential parameters (have been calculated and published before by us in [14]) and single folding parameters calculated for protons elastically scattering on ^{10}B nuclei are shown in Table II. The analysis, carried out in wide energy range, had shown that, for ^{10}B nuclei, the most suitable parameters values are r_0 = 1.2 fm, r_C= 1.3 fm, r_D= 1.37 fm, a_s = 0.65 fm and $r_{s.o.}$=1.06 fm. Fig. 2 shows the comparison between calculated and experimental angular distributions for protons elastically scattering on ^{10}B nuclei at different energies. Energy dependence for real (V_0) and imaginary (W_D) depths is shown in Fig. 3. The relation between incident particle energy (E_p) and the volume integral per nucleon pair for the real potential, J_R and imaginary J_W MeV.fm^3 are presented in Fig. 4. If the radius and diffuseness are well-adjusted, the potential depth for real at least could be expected. For example, in case of protons, the potential depth will be 40-60 MeV. Single folding potential has been calculated for available experimental data. The numerical calculations have been done using the DFPOT code [24]. The variations of the real potential values according to the radius are directly put into the calculations with the aid of this model, and the imaginary parts are defined by a phenomenological way.

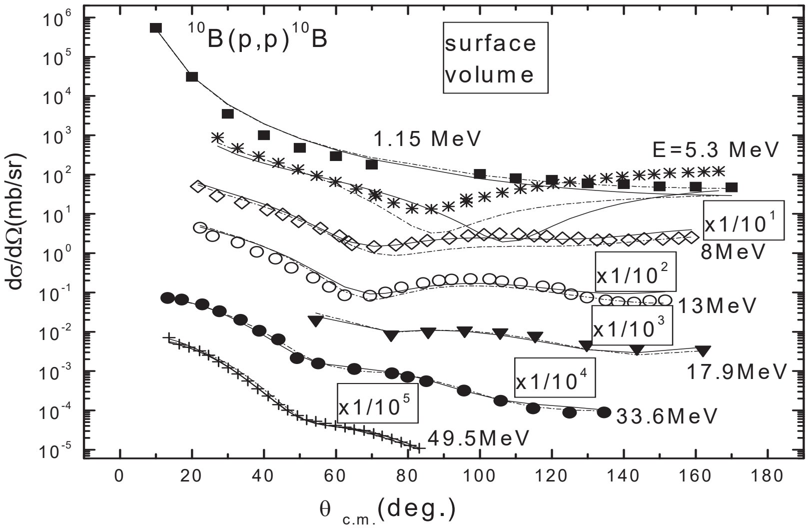
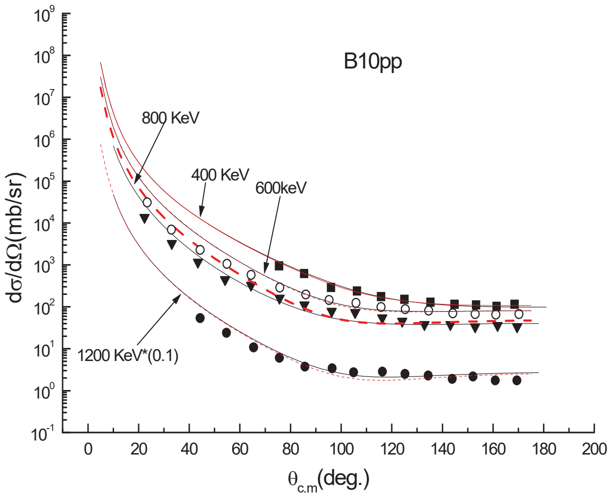
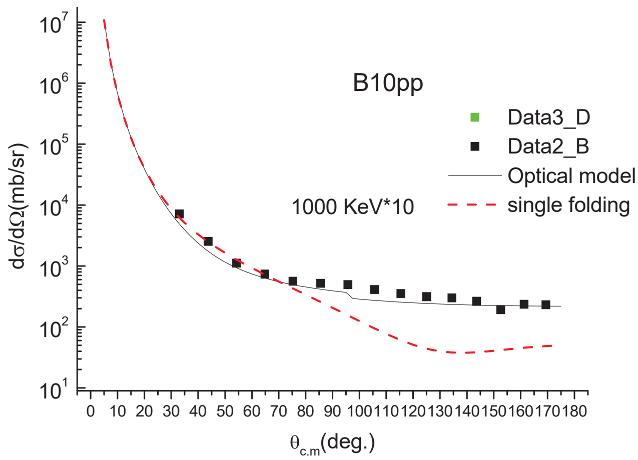


Fig. 1 Angular distribution for protons elastically scattered by ^{10}B at different energies. The solid curves are calculations based on volume absorption W_V (set A of Table I); dashed calculations are based on surface absorption W_S (set B of Table I); experimental data are the points which were taken from [17]-[20]

TABLE I
OPTICAL POTENTIAL PARAMETERS CALCULATED FOR ELASTIC SCATTERING OF PROTONS BY 10B

Energy Ep, MeV	*set*	*VR, MeV*	*rV, fm*	*aV, fm*	*W, MeV*	*rI, fm*	*aI, fm*	*VSO, MeV*	*rSO, fm*	*aSO, fm*	*Exp. data*
1.15	*A*	*53.54*	*1.1*	*0.758*	*5.50*	*1.33*	*0.644*	*6.31*	*1.09*	*0.644*	*[14]*
	B	*53.9*	*1.1*	*0.646*	*6.00*	*1.71*	*0.600*	*6.31*	*1.09*	*0.644*	
5.3	*A*	*53.9*	*1.1*	*0.548*	*6.22*	*1.21*	*0.644*	*6.31*	*1.09*	*0.644*	*[18]*
	B	*53.9*	*1.1*	*0.646*	*5.97*	*1.81*	*0.600*	*6.31*	*1.09*	*0.644*	
8.0	*A*	*53.3*	*1.1*	*0.759*	*5.02*	*1.44*	*0.644*	*6.31*	*1.09*	*0.644*	*[18]*
	B	*54.5*	*1.1*	*0.702*	*6.76*	*0.96*	*0.600*	*6.31*	*1.09*	*0.644*	
13.0	*A*	*56.7*	*1.1*	*0.607*	*9.05*	*1.20*	*0.644*	*6.31*	*1.09*	*0.644*	*[18]*
	B	*56.7*	*1.1*	*0.598*	*6.19*	*1.20*	*0.600*	*6.31*	*1.09*	*0.644*	
17.9	*A*	*53.9*	*1.1*	*0.587*	*6.22*	*1.60*	*0.644*	*8.0*	*0.90*	*0.400*	*[19]*
	B	*53.9*	*1.1*	*0.572*	*6.03*	*1.15*	*0.600*	*8.0*	*0.90*	*0.400*	
33.6	*A*	*54.9*	*1.1*	*0.618*	*8.48*	*1.42*	*0.644*	*8.0*	*0.90*	*0.400*	*[20]*
	B	*54.7*	*1.1*	*0.567*	*6.36*	*1.21*	*0.600*	*8.0*	*0.90*	*0.400*	
49.5	*A*	*40.66*	*1.1*	*0.580*	*5.34*	*1.68*	*0.644*	*6.44*	*0.750*	*0.590*	*[22]*
		40.37	*1.12*	*0.650*	*15.85*	*1.23*	*0.600*	*8.0*	*0.90*	*0.400*	*[23]*
	B	*47.7*	*1.1*	*0.668*	*7.08*	*1.19*	*0.600*	*-*	*-*	*-*	*[22]*
		46.64	*1.1*	*0.680*	*20.93*	*0.78*	*0.530*	*-*	*-*	*-*	*[23]*





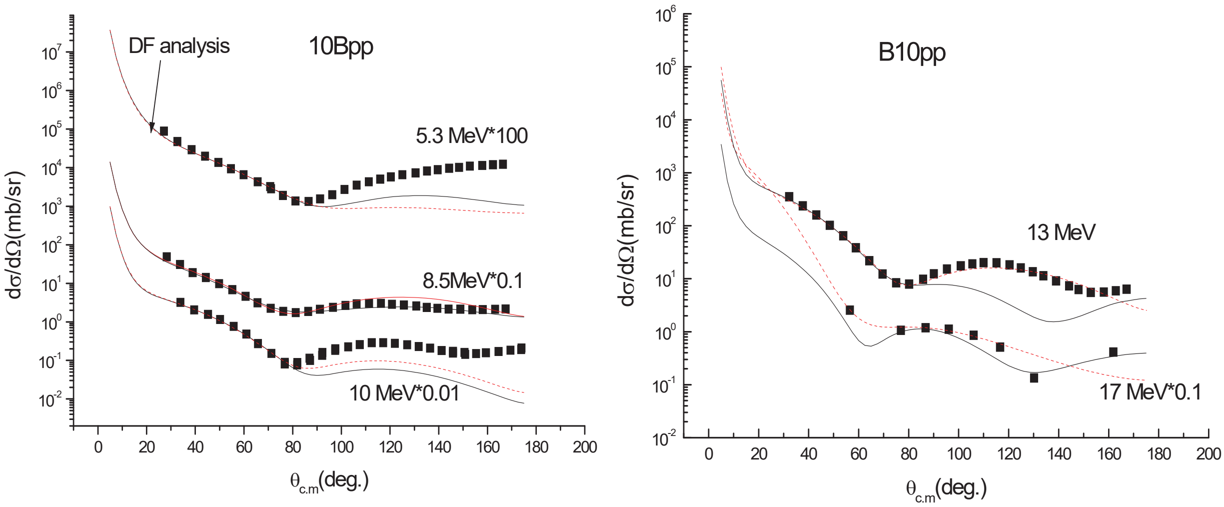


Fig. 2 Angular distribution of protons elastically scattered by [10]B at different energies (points) taken from [14], [17] and calculated differential cross section (curves)

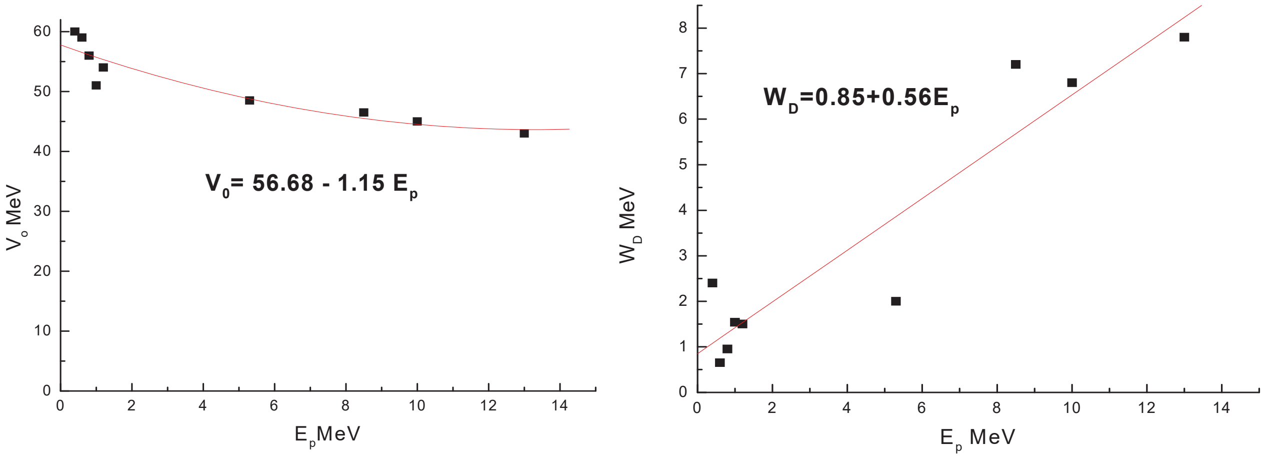


Fig. 3 (a) Linear relation between V_0, W_D and E_p

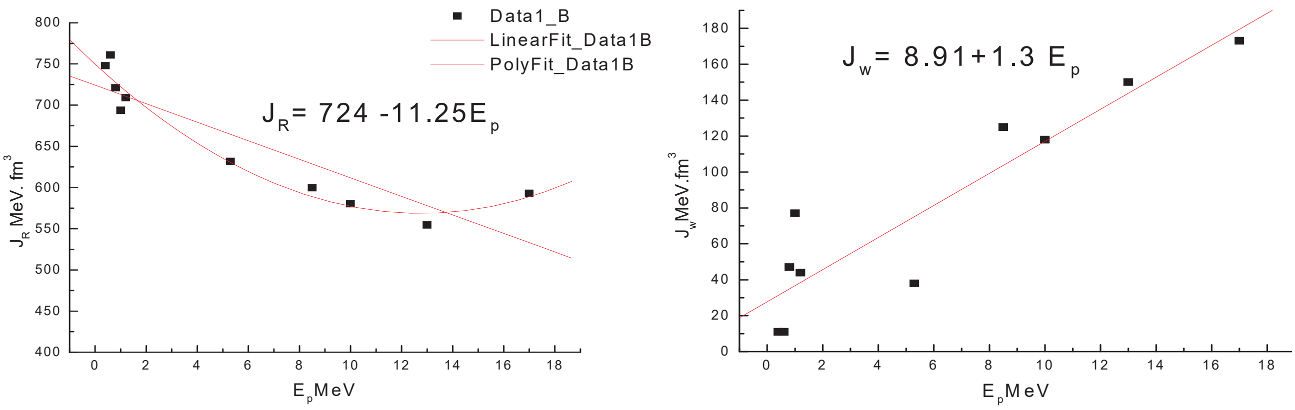


Fig. 3 (b) Relation between proton energy E_p and the volume integral per nucleon pair for the real potential, J_R and J_W imaginary in MeV.fm^3

A. Spectroscopic Factor

There are two methods to obtain the spectroscopic factors of the configuration under consideration. Extracted values of spectroscopic factor from experimental data are the one of these choices and may be the best choice. If not, we have to find the spectroscopic factors from literature. The extraction of spectroscopic factors depends on the optical model parameters (OMPs). The extracted spectroscopic factors should be adjusted by available data from literature. Also, global optical potential parameters should be used in the extraction of the spectroscopic factors. As, we have no experimental data to extract the spectroscopic factor of $^{11}C \rightarrow {}^{10}B+p$ configuration, the literature is our choice to obtain the spectroscopic factor of such configuration. Cohen and Kurath calculated the spectroscopic factors for the 1p-shell many years ago [6]. By using OMPs represented in Table I and spectroscopic factor from literature [6], we could reproduce the cross section of the reaction $^{10}B(p,\gamma)^{11}C$ at such low energies. We have extracted spectroscopic factor of ^{6}Li in [2] from experimental data. Also, spectroscopic factors extracted for ^{7}Be from the radiative reaction $^{6}Li(p,\gamma)^{7}Be$ have been done as shown in Table III.

TABLE II
THE OPTICAL PARAMETERS AND SEMI-MICROSCOPIC PARAMETERS FOR PROTONS SCATTERING ON 10B NUCLEI, SF MEANS SINGLE FOLDING WHERE P. W MEANS PRESENT WORK

Ep MeV	V0 MeV	r0 fm	a0 fm	WD MeV	rD fm	aD fm	Vs MeV	rs fm	as fm	JR MeV.fm3	Jw MeV.fm3	Reference
0.400 OMP	61.30	1.20	0.77	0.22	1.37	0.63	6.05	1.06	0.65	747.97	11	P.W
SF	85.37	0.95	0.749	0.066	1.37	0.630	6.04	1.064	0.733			P.W
0.60 OMP	56.049	1.20	0.85	0.38	1.37	0.85	6.05	1.06	0.65	760.87	11	P.W
SF	78.05	0.95	0.818	0.38	1.37	0.65	6.04	1.064	0.733			P.W
1.0 OMP	60.716	1.20	0.64	0.37	1.37	1.05	6.05	1.06	0.65	721.20	47	P.W
SF	77.20	0.95	0.725	0.45	1.37	0.65	6.04	1.064	0.733			P.W
0.80 OMP	57.98	1.25	0.66	0.718	1.25	0.660	7.50	1.25	0.65	693.64	77	[14]
SF	74.58	0.95	0.818	0.711	1.370	0.630	6.04	1.064	0.733			P.W
1.20 OMP	57.40	1.25	0.578	0.874	1.25	0.65	7.50	1.25	0.65	709.0	44	[14]
SF	74.339	0.95	0.749	0.376	1.370	1.030	6.04	1.064	0.738			P.W
5.3 OMP	48.05	1.25	0.65	0.167	1.25	0.135	7.50	1.25	0.65	631.65	38	[14]
SF	77.369	0.95	0.749	24.169	1.370	0.630	6.04	1.064	0.738			P.W
8.5 OMP	46.50	1.25	0.65	3.085	1.25	0.65	7.50	1.25	0.65	599.67	125	[14]
SF	63.503	0.95	0.749	2.158	1.37	0.63	6.04	1.064	0.738			P.W
10 OMP	45	1.25	0.65	6.80	1.25	0.54	7.50	1.25	0.65	580.32	118	[14]
SF	57.33	0.95	0.76	4.66	1.37	0.63	6.04	1.064	0.738			P.W
13 OMP	34.77	1.25	0.65	6.475	1.25	0.65	7.50	1.25	0.65	554.53	150	[14]
SF	59.35	0.95	0.76	5.326	1.37	0.63	6.04	1.064	0.738			P.W
17 OMP	32	1.25	0.65	3.669	1.15	0.54	12	1.15	50	593.0	173	[14]
SF	47.56	0.95	0.55	6.348	1.37	0.63	6.04	1.064	0.738			P.W

B. Astrophysical S-Factor

A theoretical extrapolation has been performed by Barker [10] within potential model, based on simultaneous fit of $^{6}Li(n,\gamma)^{7}Li$ and $^{6}Li(p,\gamma)^{7}Be$ cross sections. Arai et al. [25] used a four-cluster microscopic model to investigate low-energy ^{6}Li+p and ^{6}Li+n reactions. The derived astrophysical S-factor for the $^{6}Li(p,\gamma)^{7}Be$ in [10] is in a good agreement with the available experimental data. In order to calculate the astrophysical S-factor for direct capture, we employed the standard expression:

$$S(NJ, J_f) = \sigma(NJ, J_f) E_{cm} \exp\left(\frac{31.335\, Z_1 Z_2 \sqrt{\mu}}{\sqrt{E_{cm}}}\right) \quad (9)$$

which was proposed as far back as the 1950s where σ is the total cross section for the radiative capture process (in barn units), $E_{c.m.}$ is the c.m. energy of particles in the entrance channel (in keV units), μ is the reduced mass of the entrance-channel particles (in atomic mass units), and Z are the charges of the particles (in elementary charge units, e) [7]. The astrophysical $S(E)$ factors are related to the cross sections $\sigma(E)$ by [26]:

$$S(E) = E\,\sigma(E)\exp(E_G/E)^{1/2}, \qquad n \quad (10)$$

where the Gamow energy $E_G = 0.978(Z_1Z_2)^2\mu$ MeV, μ is the reduced mass of the system.

The total S -factor is calculated from [27]:

$$S_{tot}(E) = S_{dc}(E) + S_{res}(E) \pm 2[S_{dc}(E)S_{res}(E)]^{1/2}\cos(\delta_r), \quad (11)$$

where δ_r is the resonance phase shift, given by

For resonance calculation using FRESCO program, we have to first find a potential that makes that state weakly bound, say 10 keV. The resonance calculated was 12 keV instead of 9 keV which has been calculated in [28].

C. $^{6}Li(p,\gamma)^{7}Be$ Reaction Analysis

Radiative capture reaction of light nuclei can used for rapid γ diagnostics of the high temperature plasma, by an estimation the elements in astrophysical nucleon synthesis, for quantitative PIGE (proton induced gamma emission) analysis of light elements: Li, Be, B and others. The low energy behavior of radiative capture reaction of protons is important in nuclear astrophysics. In many cases, the cross sections at considered (interested) energies (lower than 1 MeV) could not be measured directly. Such cross sections at low energies can be obtained by the extrapolation of values from those measured at high energies. Usually the extrapolation is carried out for the astrophysical S –factor [29]. Then the cross sections could be obtained from astrophysical S–factor. The $^{6}Li(p,\gamma)^{7}Be$ reaction is experimentally studied in [30] at energies from 200 keV to 1200 keV. The theoretical interpretation is presented by Barker [10] within the framework of the potential model based on the simultaneous analysis of (^{6}Li +n) and (^{6}Li + p), mirror systems.

The properties of mirror reactions of direct capture can be described by the optical potential, in which there are used the same parameters of values for both reactions. The ratio of yields of γ quanta to those of charged particles and angular distributions of γ quanta were measured for reactions of protons radiation capture on ^{6}Li, ^{7}Li, ^{9}Be and ^{11}B nuclei for energies of bombarding protons between 40 and 180 keV in [31]. These measurements were used in order to obtain S- factors and total cross section of the reactions. After obtaining of polarized protons beam, the ^{6}Li(p,γ)^{7}Be reaction was studied in the TUNL (Triangle Universities Nuclear Laboratory, Durham, North Carolina, USA) –laboratory at energies of from 80 to 130 keV [8]. The polarization of the proton beam gave the possibility to measure not only the spectra of γ quanta and yields of the reaction but also, the analyzing power. These data are used in order to determine the slope of the astrophysical S–factor for reactions ^{6}Li (p,γ_0) ^{7}Be and ^{6}Li (p,γ_1) ^{7}Be. The slope was determined from relative yields at five energies of incident protons. The slope of the of the astrophysical S–factor was found to be negative. In such low energy region, the main contribution into the cross section of the ^{6}Li(p,γ)^{7}Be reaction is determined by the direct capture mechanism. The capture of nucleons from the ground state ($J^{\pi} = 1^{+}$) of the ^{6}Li to the ground ($J^{\pi} = 3/2^{-}$) and the first excited state ($J^{\pi} = 1/2^{-}$) of the ^{7}Be takes place in the p-state. The s-wave and d-wave of incident protons can introduce contributions into the E1 transition. M1 and E2 multipoles can give contributions into the total cross section theoretically.

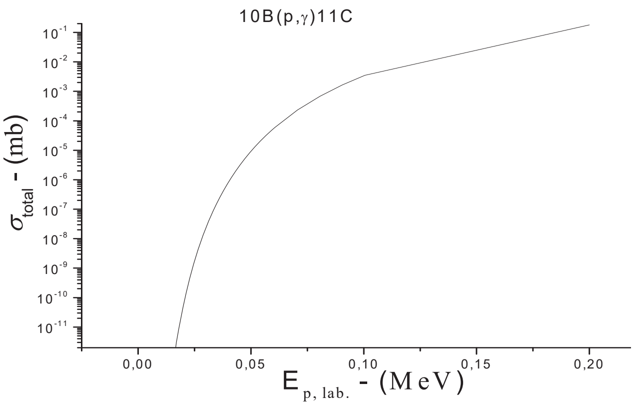


Fig. 4 Calculated cross sections of ^{10}B(p,γ)^{11}C reaction

Total cross sections calculations within the framework of the direct capture, it is necessary to know wave function of input and output channels. In the two-body approach, wave function of input and output channels are generated in the optical parameters besides to know the spectroscopic factor of the ^{7}Be ≡ ^{6}Li + p configuration. One of first calculations of ^{6}Li (p,γ)^{7}Be reaction cross sections was carried out by Barker [10] within the framework of the direct capture in the potential model. The calculation was carried out in order to verify the assumption that properties of direct capture mirror reactions can be described in the optical parameters using the same parameters values for every reaction. The present test includes the mirror reactions: ^{6}Li(n,γ) and ^{6}Li (p,γ). The calculation of the cross sections depended on parameters of the potential and spectroscopic factors. It was shown that standard values of parameters of the potential and spectroscopic factors led to too small values of cross section for both (n,γ) and (p,γ) reactions . And only modified values of these parameters, fitted to the (n,γ) cross section, give also an agreement with the (p,γ) cross section. However, it was noted that the spectroscopic factor for the spin s =3/2 must be multiplied by the factor 5.97, and the total spectroscopic factor by the factor 2.06.

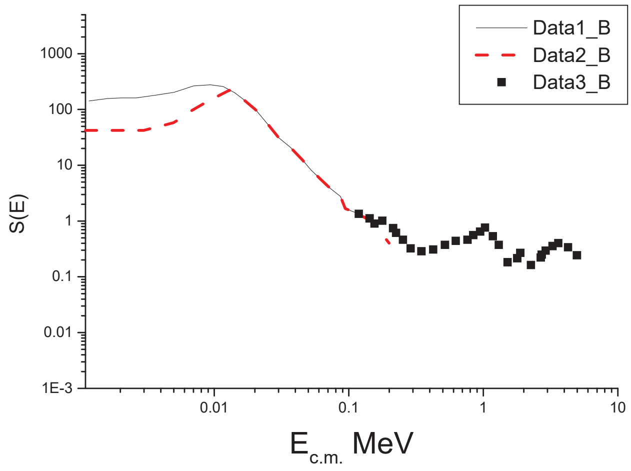


Fig. 5 Astrophysical S (E) factor for the ^{10}B(p, γ)^{11}C reaction calculated (solid and dashed lines) and experimental data (squares) are taken from [28]

The ^{6}Li(p,γ)^{7}Be reaction S-factor is in [25] good agreement with the available experimental data. Knowledge of the rate of change of the S-factor with energy at very low energies is needed to perform a reliable extrapolation. Although this is frequently determined by the use of a direct capture-model calculation, there are cases when this does not suffice. Low-energy resonances or sub-threshold states can affect the extrapolation. In [8], the results of a measurement of the slope of the astrophysical S factor for the ^{6}Li(p,γ)^{7}Be reaction are reported, and a new mechanism is introduced to explain the observed slope. Cecil et al. [9] measured the branching ratio of ^{6}Li(p,γ_0)^{7}Be and ^{6}Li(p,γ_1)^{7}Be with respect to ^{6}Li(p, α)^{3}He from 45 to 170 keV and deduced the S factors for ^{6}Li(p,γ_0)^{7}Be and ^{6}Li(p,γ_1)^{7}Be as a function of energy. Their results gave a positive slope for the S factor. Switkowski et al. [30] measured the ^{6}Li(p,γ)^{7}Be cross section from 160 to 1150 keV. Their data points are all at energies above the present dataset and show an S-factor that increases with increasing energy. Barker's analysis [10] for the data of Switkowski et al. does have a negative S-factor slope for ^{6}Li(p,γ_0)^{7}Be and ^{6}Li(p,γ_1)^{7}Be at energies below the range of the data. The present analysis was undertaken to examine this discrepancy in these previous of Cecil et al. and Switkowski et al.

In our publications [14], [29] for ^{6}Li(p,p)^{6}Li, we have enhanced optical potential parameters at low energies. The reaction ^{6}Li(p,γ)^{7}Be has been used to extract the spectroscopic factors of ^{7}Be≡^{6}Li+p from measured experimental data. FRESCO Code has been used to reproduce the cross section of the ^{6}Li (p,γ)^{7}Be reaction within framework of the direct capture in the potential model. OMPs and spectroscopic factors have been used in our calculations. A group of spectroscopic factors were extracted with only our optical model parameters (OMPs) and the spectroscopic factors were changed to analysis the experimental data and this is shown as *solid line* in figure 6. The spectroscopic factors of ^{7}Be at these low energies are energy dependent so, their values changed from energy to another especially at very low energies. The spectroscopic factors for ground state were extracted $1P_{3/2}$=0.207 and $1P_{1/2}$=0.18 and for excited state were $1P_{3/2}$= 0.306 and $1P_{1/2}$= 0.065 at E_p=387 keV. Spectroscopic factors values have been changed up and down by constant values to analyze the experimental data. The values of spectroscopic factors extracted depend on the choice of OMPs used. The dot line in Fig. 6 was obtained when spectroscopic factor and optical model parameters (OMPs) were taken from [6], [7] respectively. The analysis of the proton data carried out at wide energy range from 0.4 up to 50MeV had shown that the most suitable parameters for ^{6}Li nuclei values are r_0=1.05 fm, r_c=1.3 fm, r_D=1.923 fm, a_s=0.20 fm and r_s=1.20 fm. Our complete analysis for ^{6}Li(p,p)^{6}Li has been published in [14].

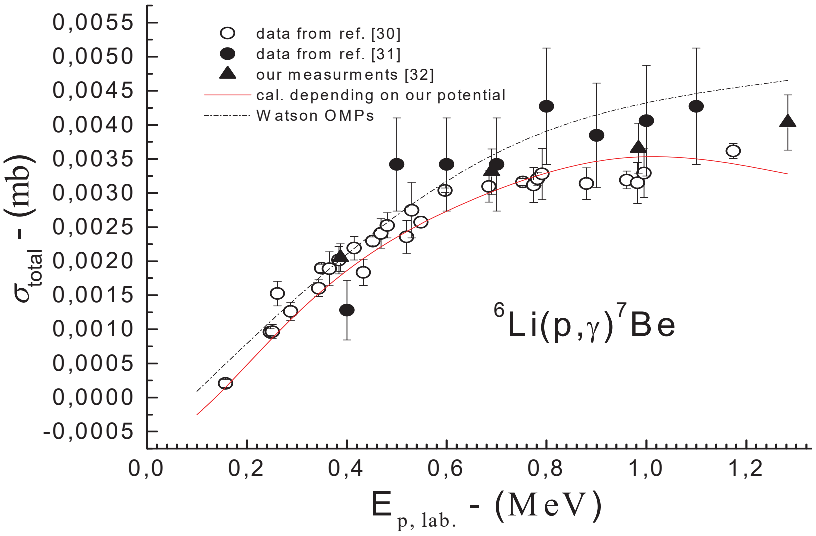


Fig. 6. Cross section of the reaction ^{6}Li(p,γ)^{7}Be, the experimental points are taken from [30] (open circles), [31] (closed circles) and triangle [32]. Solid line is calculated data depending on the OMPs from ^{6}Li(p,p)^{6}Li in ref. [14], [29] where dot line represents the calculations in case of OMPs taken from [17]

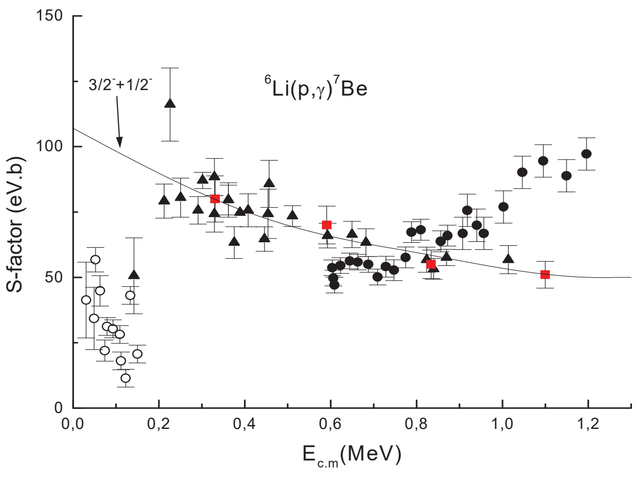


Fig. 7 Astrophysical S-factor calculated using Fresco program (curve), the red points are taken from [29] and the displayed points correspond to experimental data were taken from [25]

D. ^{7}Li(p,γ)^{8}Be Reaction Analysis

The ^{7}Li(p,γ)^{8}Be reaction is considered very important part of pp-chain which takes place in the sun. ^{8}Be unstable nucleus decays to α-particles in 10^{-16} sec [4]. The study of p-wave strength at low energy for ^{7}Li(p,γ)^{8}Be reaction has been done by Barker [10]. He analyzed the cross section at low-energy of ^{7}Li(p,γ)^{8}Be reaction which has been measured and analyzed by Chasteler et al. [33], where their analysis pointed out that p-wave should be added to s-wave in the analysis which reduces the zero-energy astrophysical S-factor obtained by assuming pure s-waves, and suggested that a similar phenomenon might be present in the ^{7}Li(p,γ_0)^{8}Be reaction, which is of importance in the solar neutrino problem.

Our calculations of the cross section of the *^{7}Li (p,γ)^{8}Be* reaction was carried within the framework of the direct capture in the potential model (in the long wave approximation) using FRESCO Code. The calculation was carried out in order to verify the assumption, that properties of mirror reactions of the direct capture can be described in the OMPs, using the same parameters of values for all reactions.

The ^{7}Li(p,γ)^{8}Be reaction has been studied extensively by various investigations in the proton energy range of E_p= 200 keV to 1700 keV. These experiments have confirmed the existence of the well-known proton resonances at E_p= 441 keV and 1030 keV [33]. Two other resonances have recently been proposed by Cavallaro et al. [34] at E_p= 720 keV and 870 keV. The γ radiation resulting from the reaction corresponds to transitions to the ground state and first-excited state of ^{8}Be [33]. Resonances in the reaction ^{7}Li(p,γ)^{8}Be have been observed at proton energies 441 keV and 1030 keV. Above the 441 keV resonance, the cross section is larger than can be explained in terms of neighboring resonances, and the weak 1030 keV resonance is superimposed on a steadily rising non-resonant background [35]. The aim of the present work is to verify the assumption, that properties of mirror reactions of the direct capture can be described in the OP, using the same parameters of values for all reactions.

Our calculations of the cross section of the ^{7}*Li (p,γ)*^{8}*Be* reaction was carried within the framework of the direct capture in the potential model (in the long wave approximation) using FRESCO Code. The calculation was carried out in order to verify the assumption, that properties of mirror reactions of the direct capture can be described in the OMPs, using the same parameters of values for all reactions.

TABLE III
OPTICAL PARAMETERS FOR PROTONS SCATTERING ON ^{7}LI NUCLEI

E_p MeV	V_0 MeV	r_0 fm	a_0	W_D MeV	r_D fm	a_D fm	V_s MeV	r_s fm	a_s fm	J_R MeV.fm^3	J_w MeV.fm^3
0.359	56	1.17	0.65	0.70	1.80	0.504	12.48	1.17	0.50	650.31	11
0.491	62	1.17	0.60	0.30	1.80	0.504	12.48	1.17	0.50	720.49	4.09
0.792	61.3	1.17	0.68	0.56	1.80	0.504	12.48	1.17	0.50	712.52	6.89
0.991	55	1.17	1.04	0.93	1.80	0.87	18.86	1.17	0.74	535.89	32.18
3.1	49.67	1.17	0.84	1.012	1.80	0.80	12.86	1.02	0.51	316.03	30.23
10.3	37.29	1.17	0.527	8.55	1.80	0.545	12.86	1.17	0.8	109.43	140.38

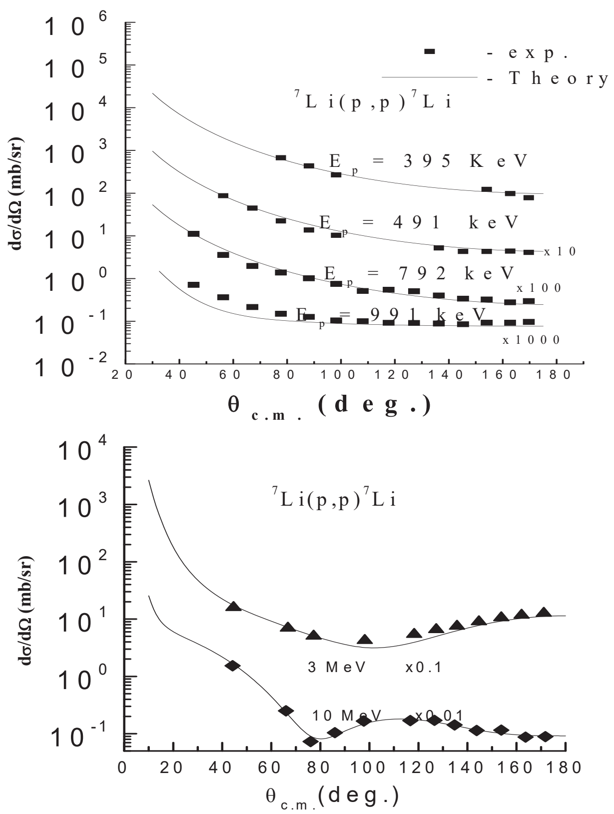


Fig. 8 Angular distribution of protons elastically scattered by ^{7}Li at different energies (squares) and differential cross sections calculated with Optical model (curves); experimental data were taken from [14]

The similar data were obtained in experiments on elastic scattering of protons on ^{7}Li nuclei at energies of 346, 451, 750, 991 and 1030 keV in ref. [14]. The errors of measured differential cross-sections are approximately equal to dimensions of presented dots and do not exceed 5%. The parameters calculated for ^{7}Li is good agreement with those calculated for light nuclei by Watson et al. [17]. This gives us normal starting point to deal with light nuclei and their behaviors in spite of the results in the simplest form, we tried to put a lot relation in linear and others in second order. Fig. 8 shows also a comparison between calculated and experimental for ^{7}*Li+p*. As shown in Fig. 8, the differential cross sections calculated using optical parameters and experimental values are close to each other. The minimum of the peak is obtained at 80° as expected.

Spectroscopic factors for ^{8}Be used are taken from [36], as ^{8}Be (g.s) =3.18 and ^{8}Be*(2.90) =2.80. These calculations have confirmed the existence of the well-known proton resonances at E_p=441 keV and 1030 keV [33]. From these calculations, we confirmed especially proton resonance E_p= 870 keV. Reference [4] used values of spectroscopic factor very small comparing with these published in previous works [36], [6].

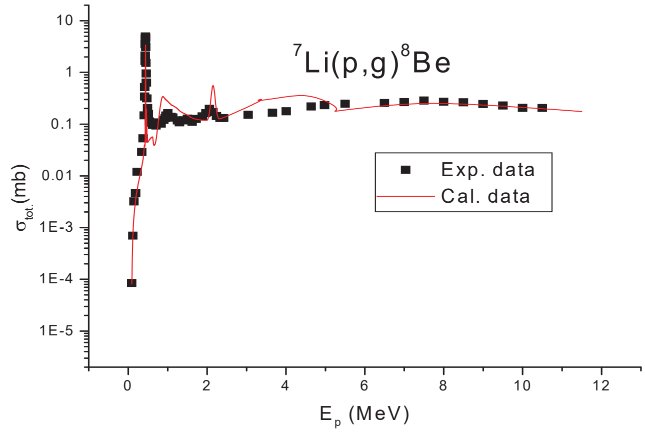


Fig. 9 Potential model calculation for ^{7}Li(p,γ)^{8}Be reaction, points are experimental data and calculated value is solid line, experimental data are taken from [37]

IV. CONCLUSION

Optical model parameters have been calculated for ^{10}B(p,p)^{10}B. The non-resonant cross sections of the ^{10}B(*p*,γ)^{11}C reaction were computed. Astrophysical S-factor has been calculated for the reaction $^{10}B(p,\gamma)^{11}C$. Elastic scattering of $^{6}Li(p,p)^{6}Li$ has been studied. New sets of OMPs have been obtained for the proton elastic scattering from ^{6}Li. The calculations of the cross section of the $^{6}Li(p,\gamma)^{7}Be$ reaction were carried within the framework of the direct capture in the potential model using FRESCO Code. Cross sections of $^{6}Li(p,\gamma)^{7}Be$ reaction were directly obtained from the calculations depending on OMPs from $^{6}Li(p,p)^{6}Li$. Spectroscopic factors have been extracted from our experimental data of the radiative reaction $^{6}Li(p,\gamma)^{7}Be$. Elastic scattering of protons with ^{7}Li has been studied with optical model. The cross section of the reaction $^{7}Li(p,\gamma)^{8}Be$ has been produced from data obtained from elastic scattering of protons with ^{7}Li.

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