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On the Bootstrap P-Value Method in Identifying out of Control Signals in Multivariate Control Chart

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***Abstract*—**In any production process, every product is aimed to attain a certain standard, but the presence of assignable cause of variability affects our process, thereby leading to low quality of product. The ability to identify and remove this type of variability reduces its overall effect, thereby improving the quality of the product. In case of a univariate control chart signal, it is easy to detect the problem and give a solution since it is related to a single quality characteristic. However, the problems involved in the use of multivariate control chart are the violation of multivariate normal assumption and the difficulty in identifying the quality characteristic(s) that resulted in the out of control signals. The purpose of this paper is to examine the use of non-parametric control chart (the bootstrap approach) for obtaining control limit to overcome the problem of multivariate distributional assumption and the p-value method for detecting out of control signals. Results from a performance study show that the proposed bootstrap method enables the setting of control limit that can enhance the detection of out of control signals when compared, while the p-value method also enhanced in identifying out of control variables.

***Keywords*—**Bootstrap control limit, p-value method, out-of-control signals, p-value, quality characteristics.

I. Introduction

CONTROL charting procedures have some similarities with traditional statistical inference procedures like the hypothesis testing and confidence intervals. Most of the procedures are obtained following some defined postulation that the variable(s) under consideration follow some form of multivariate parametric distribution and they are known as parametric statistical inference methods. These methods are more effective and most efficient when the distributional assumption is satisfied. However, the usual practice is that such information is not available to the quality control manager who is interested in finding solution to the problem. In order to solve this issue, statistical inference methods that include hypothesis tests, confidence intervals, and control charts that do not desire any specific parametric distributional assumptions have been introduced and reviewed in the literature. Collectively, these methods are known as the non-parametric or distribution-free methods [1], [2]. Violating the distributional assumptions underlying parametric control charts may result in ineffective control chart method, and a nonparametric control chart may provide a better alternative [3]. It is of this view that the non-parametric methods such as the bootstrap approach of setting control limits and identification of out of control signals shall be looked into in this work.

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II. The Bootstrap Method of Setting Control Limits

Suppose a population with mean vector (μ) and variance covariance matrix (∑), where μ and ∑ are known from a multivariate distribution assumption that is normal, the χ^2 distribution is used to obtain a control limit for setting up Hotelling's χ^2 control charts. When the (μ) and (∑) are not known, and must be obtained from the given data as $\bar{x}$ and S respectively, the f-distribution is used in estimating Hotelling's T^2 control limits [4], [5]. The Hotelling's T^2 statistic of any given set of observation is expressed as:

$$T_i^2 = (x_j - \bar{x})' S^{-1} (x_j - \bar{x});\ i = 1,2,\dots,n; j = 1,2,\dots,d \quad (1)$$

where n is the total number of observations and d is the total number of process quality characteristics and the Hotelling's T^2 control limit is given by:

$$CL_{T^2} = \frac{d(n+1)(n-1)}{n^2 - nd} f_{\alpha,d,n-d} \quad (2)$$

where α represents the specified false alarm rate similar to type I error rate and $F_{\alpha,d,n-d}$ represents the F distribution with parameters d and $n-d$ degrees of freedom.

If multivariate distributional assumption is violated (the usual case in practice), a control limit based on these methods may be inaccurate, thereby increasing the rate of detecting more out of control signals when the process is in control [6]-[12]. To reduce the abnormal behaviors observed when the multivariate distributional assumption is violated, [8] proposed the bootstrap based T^2 multivariate control charts. This method obtained its control limit by bootstrapping the Hotelling's T^2 statistic (i.e. collapsing the multivariate into univariate). However, to address the problem of identifying out of control signals, this study also introduced the p-values method.

A. Algorithm - Proposed Bootstrap Method for Obtaining Hotelling's T^2 Control Limit

Suppose that there are d quality characteristics and each of the quality characteristic contains n set of observations (x_{ij}); $(i = 1,2,\dots,n; j = 1,2,\dots,d)$ as can be summarized in the matrix:

$$\begin{matrix} x_1 & x_2 & \cdots & x_d \end{matrix}$$
$$\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1d} \\ x_{21} & x_{22} & \cdots & x_{2d} \\ & & \cdots & \\ x_{n1} & x_{n2} & \cdots & x_{nd} \end{pmatrix}_{dxn}$$

If the matrix notations of $d\text{x}n$ dimensions can be transposed as expressions below:

$$x_1 = (x_{11}, x_{21}, \dots, x_{n1})'; \; x_2 = (x_{12}, x_{22}, \dots, x_{n2})'; \; \dots \; x_d = (x_{1d}, x_{2d}, \dots, x_{nd})'$$

The proposed bootstrap procedure for obtaining Hotelling's T^2 control limit is as follows:

Step1. Combine the sample sizes of $x_1, x_2, \dots, x_d$ of the sets of observation such that:

$$x = (x_{11}, x_{21}, \dots, x_{n1}; x_{12}, x_{22}, \dots, x_{n2}; \dots; x_{1d}, x_{2d}, \dots, x_{nd})$$

Step2. Draw a bootstrap sample of size $x^* = x_1^*, x_2^*, \dots, x_d^*$ with replacement from Step 1

$$x^* = x_{11}^*, x_{21}^*, \dots, x_{n1}^*; x_{12}^*, x_{22}^*, \dots, x_{n2}^*; \dots; x_{1d}^*, x_{2d}^*, \dots, x_{nd}^*$$

Step3. Repeat Step 2 for number of periods to obtain bootstrap replications as:

$$x^* = x_{11}^{*(i)}, x_{21}^{*(i)}, \dots, x_{n1}^{*(i)}; x_{12}^{*(i)}, x_{22}^{*(i)}, \dots, x_{n2}^{*(i)}; \dots; x_{1d}^{*(i)}, x_{2d}^{*(i)}, \dots, x_{nd}^{*(i)},$$

where $(i^* = 1,2,\dots,B)$, and B is large $(e.g., B > 1000)$.

Step4. Estimate the bootstrap mean vector $(\bar{x}^*)$, bootstrap variance and covariance matrix $(S^*)$from the bootstrap sample variables in Step 3.

Step5. Obtain the bootstrap T_i^{2*} statistic from the data set in Step 4 such that:

$$T_i^{2*} = (x_j^* - \bar{x}^*)' S^{*-1} (x_j^* - \bar{x}^*),$$
$$i^* = 1,2,\dots,B; j^* = 1,2,3,\dots,d.$$

Step6. Repeat the process $B = 3000$ times by changing the values of T_i^{2*} and x_j^* to obtain: $T_1^{2*}, T_2^{2*}, \dots, T_B^{2*}$

Step7. Set the upper control limit such that in each of the bootstrap statistic $(T_1^{2*}, T_2^{2*}, \dots, T_B^{2*})$ arranged from the lowest to highest, determine the position of $B(1-\alpha)^{th}$ value as:

$$CL_{Prop.Boot} = \frac{1}{B} \# \{ (T_1^{2*}, T_2^{2*}, \dots, T_B^{2*}) \le B(1-\alpha) \} \quad (3)$$

Step8. From the control limit established in Step 7, determine those variables that are under control process from those that are out of control process.

B. Proposed P-Values Method in Identifying out of Control Signals

The problem of identifying quality characteristic(s) that is (are) responsible for out of control signal(s) has been an issue in multivariate control charts [13]-[15]. Among the several graphical techniques for interpreting out of control procedures being proposed are the starplots and the multivariate profile charts [16], [17]. A very useful approach in identifying out of control signal is to obtain the p-values of the Hotelling's T^2 statistics that reflect the contribution of each variable. Adopting [14], Step 1-3 were obtained while Step 4-5 were introduced to obtain their p-values.

Step1. For a d-dimensional vector of quality characteristics, the first row is expressed as:

$$T^2 = T^2_{j.i;\,\forall j=1, i=j-1}, T^2_{j.i;\,\forall j=2, i=j-1,j-2}, \cdots, T^2_{j.i;\,\forall j=d, i=j-1,j-2,j-3,\dots,j-d} = T_1^2, T_{2.1}^2, T_{3.1,2}^2, T_{4.1,2,3}^2, \dots, T_{d.1,2,3,\dots,d-1}^2$$

Step2. Obtain f-distribution for each of T_j^2 and $T_{j.i}^2$ terms such that:

$$T_j^2 \sim \frac{c(n+1)(n-1)}{n(n-c)} f_{(c,n-c,\alpha)}, c = 1;$$

and

$$T_{j.i}^2 \sim \frac{c(n+1)(n-1)}{n(n-c)} f_{(c,n-c,\alpha)}, c = 2,3,\dots,j-1$$

are used to check if the jth quality characteristic is conforming to the association with other quality characteristics or not.

Step3. Repeat Steps 1 and 2 for other rows based on the number of quality characteristics (d!) and obtain the distinct terms ($d*2^{d-1}$) for both the unconditional (T_j^2) and conditional $(T_{j.i}^2)$ terms.

Step4. Obtain the bootstrap p-values for each of T_j^2 and $T_{j.i}^2$ terms such that:

$$P_{value(Prop.Boot.)} = \frac{1}{B} \# \{ (T^{2*}_{Prop.Boot.}) \ge (T_j^2) \};$$
$$P_{value(Prop.Boot)} = \frac{1}{B} \# \{ (T^{2*}_{Prop.Boot.}) \ge (T_{j.i}^2) \}$$

where $P_{value(Prop.Boot.)}$ denotes the p-value from the proposed method.

Step5. Use the various P_{values} in Step 4 to assess whether there is a significant difference or not. If $(P_{values(Prop.Boot.)}\ \alpha)$ value, it means that T_j^2or $T_{j.i}^2$ is (are) not responsible for the out of control signal(s). But when $(P_{values(Prop.Boot)} \le \alpha)$value, it means that T_j^2 or $T_{j.i}^2$ is (are) responsible for the out of control signal(s).

III. Application to Numerical Illustration

The set of data used was obtained from the production process of Family Delight Pure Soya Oil produced by Owel Industries Nig. Ltd., a Company located in Ekpoma, Edo State, Nigeria. From the data, four variables namely; phosphoric acid (milliliters), water (liters), caustic soda solution (kg) and industrial salt (kg) denoting X_1, X_2, X_3 and X_4 in that order, resulted in 45 samples as presented in Table I. The main reason for the data used in this study is to show the presence of poor quality of cooking oil sold in local markets in Nigeria. Another reason is the dilemma faced by Quality Control Officers in determining the variable that is responsible for the abnormal control behaviors or the choice to stop the entire production process. Terminating the process will result in a waste of material resources, while continuing with the process without identifying the variable will lead to sub-standard product. Hence, the urge to solve these problems gave rise to this work.

TABLE I
HOTELLING'S T^2 STATISTIC FOR EACH SAMPLE

Sample	X_1	X_2	X_3	X_4	T^2	Sample	X_1	X_2	X_3	X_4	T^2	Sample	X_1	X_2	X_3	X_4	T^2
1	3000	94	30	5.3	2.4020	16	1050	70	20	6.2	15.2622	31	2450	88	24	5.3	1.2222
2	2850	90	28	5.6	0.9290	17	3000	82	30	6	3.3443	32	2680	96	25	4.9	2.4449
3	2300	92	24	5.4	0.9248	18	2850	80	31	5.2	4.1942	33	2750	100	22	6	6.8048
4	2500	80	25	5.2	2.4761	19	2000	95	31	5	5.6474	34	2900	87	29	6.3	3.8612
5	2750	45	27	7.5	22.0536	20	2050	86	25	5.8	1.1140	35	2850	89	30	5.1	2.3115
6	2400	82	25	5.8	0.8169	21	2150	91	25	5.7	0.7655	36	2000	96	25	5.3	1.7741
7	1550	80	20	5.1	9.9768	22	2060	83	28	5.4	2.0744	37	3000	99	27	6.1	5.2394
8	2950	100	30	4.2	8.1484	23	2700	90	25	5.6	0.9296	38	2150	100	28	6	4.1281
9	2850	93	29	6.1	3.2045	24	2800	94	25	5.3	1.6540	39	2300	101	20	5.8	7.1652
10	2300	85	25	5.9	0.7532	25	2950	85	29	5.4	1.8868	40	2400	102	25	5.7	2.6327
11	2250	95	25	5.5	0.7709	26	2250	86	29	5.4	1.4162	41	2600	80	28	5.2	2.2361
12	2900	80	26	5.2	3.4285	27	2005	97	32	5.9	7.7473	42	2015	94	29	5.9	3.4720
13	2550	87	27	5.7	0.1627	28	2010	100	24	5.6	2.8769	43	2225	90	32	6	5.4879
14	2100	98	28	5.4	2.0305	29	3010	98	23	5	5.8388	44	2450	98	27	5.4	0.8001

A. Test of Normality Assumption and Correlation Coefficient

To apply any non-parametric control chart methodology, there is need to know whether the data satisfy the assumption of normal distribution or not. From the given data, the histogram plots against each of the quality characteristics are shown in Figs. 1 (a)-(d), while the test on the data normality assumption using Chi-Square method at alpha level of 0.05 is depicted in Table II.

TABLE II
TEST OF NORMALITY USING THE CHI – SQUARE (χ^2) METHOD

Quality Characteristics	χ^2 Computed	P-values	Significance Level (α)
X_1	17.2738	0.0017	0.05
X_2	347.4387	0.0000	0.05
X_3	10.4187	0.0339	0.05
X_4	10.8679	0.0280	0.05

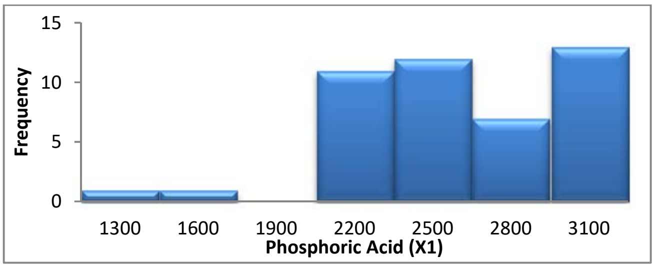

Fig. 1 (a) Variable (X_1) phosphoric acid (milliliters)

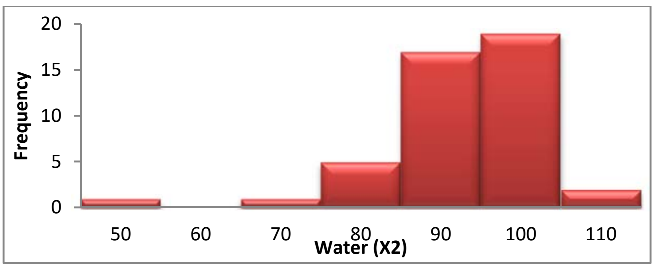

Fig. 1 (b) Variable (X_2) water (liters)

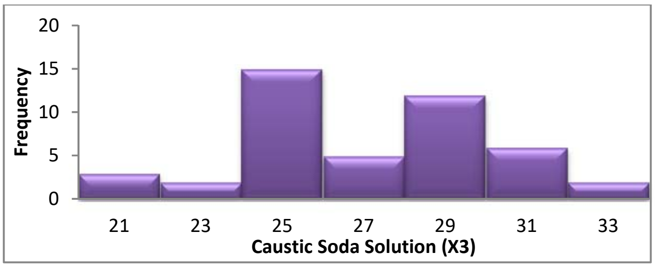

Fig. 1 (c) Variable (X_3) caustic soda solution (kg)

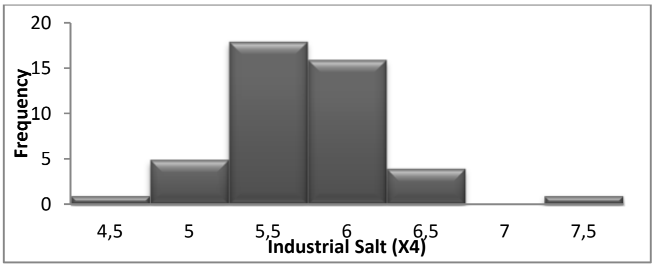

Fig. 1 (d) Variable (X_4) industrial salt (kg)

From Table II, since the p-values < 0.05, the null hypothesis is rejected (that the data are not different from a normal distribution). This assertion is also supported by the histogram plots, providing the basis in delivering the bootstrap method that does not depend on any form of assumption. Furthermore, to apply any multivariate control chart methodology, there is need to know whether there is association among the four variables. From the data, the correlation matrix (r) is given as:

$$r = \begin{bmatrix} 1.0000 & 0.040 & 0.309^{*} & -0.072 \\ 0.040 & 1.000 & 0.020 & -0.388^{**} \\ 0.309^{*} & 0.020 & 1.0000 & -0.068 \\ -0.072 & -0.388^{**} & -0.068 & 1.0000 \end{bmatrix}$$

* Significant at 0.05 (i.e. p-value 0.039 < 0.05), ** Significant at 0.01(i.e. p-value 0.009 < 0.01).

The association matrix denotes that there is relationship among the variables, thus informing the proposed method. Adopting (1) and (2), the values of the Hotelling's T^2 statistic are computed on behalf of every observation as summarized within the final column of Table I and the control limit is estimated to be 11.4089 at α = 0.05 respectively.

Similarly, the proposed bootstrap procedures presented in the algorithm were translated to Multivariate Bootstrap Control System. Bootstrap samples were replicated 3000 times starting with the initial set of observation and Hotelling's T^2 value is computed for each sample as shown in Table III. Implementing Step 7 as represented by (3) of the algorithm, the control limit was determined to be 8.587.

TABLE III
BOOTSTRAP SAMPLE REPLICATED FROM ORIGINAL DATA AND HOTELLING'S T^2 STATISTIC

Sample	X1	X2	X3	X4	T^2	T^2 Sorted
1	2,521.11	85.444	27.6	5.624	12.607	0.06
2	2,454.11	88.844	26.778	5.538	0.142	0.08
3	2,508.89	91.378	27.378	5.629	6.978	0.122
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
2849	2,476.56	89.533	26.4	5.613	1.581	8.581
2850	2,445.67	87.422	27.133	5.433	4.955	**8.587**
2851	2,465.22	89.6	25.644	5.553	6.014	8.628
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
2998	2,475.33	87.822	26.044	5.711	6.947	21.813
2999	2,485.11	91.133	26.956	5.467	3.655	22.015
3000	2,414.67	89.822	26.4	5.671	3.362	22.041

Summary of results of control limits obtained from the methods at α = 0.05 is shown in Table IV

TABLE IV
CONTROL LIMITS FOR THE TWO METHODS AT A LEVEL OF 0.05

Alpha level (α)	Existing F-Distribution Method	Proposed Bootstrap Method
0.05	11.4089	8.5870

IV. IDENTIFICATION AND INTERPRETATION OF OUT OF CONTROL SIGNALS

Samples 5, 7 and 16 are out of control as shown in Table I. Therefore, the need to identify the quality characteristic(s) accountable for the out of control signals and this we propose to resolve through the use of the p-value method. Focusing on Sample 5 by repeating Steps 1-5, Table V shows all the unconditional and conditional T^2 values and compared with their various p-values.

Control limits obtained from the proposed method performed well when compared with the existing method as shown in Table IV, i.e. $\mathrm{CL}_F = 11.4089, \mathrm{CL}_{Boot} = 8.587$. From Table V A, the value of T_2^2 and T_4^2 of the four unconditional T^2 terms associated with Sample 5 are significant, which means X_2 (water in liters) and X_4 (industrial salt in kg) are responsible for the out of control signals individually. From Table V A, it was observed that FCL and BCL are less than T_2^2 and T_4^2 (i.e. 11.4089, 8.587 < 19.1183, 14.1516), hence the next step. A similar interpretation of results from Tables V B and C also shows that $T_{2.1}^2$, $T_{2.3}^2$, $T_{2.4}^2$, $T_{4.1}^2$, $T_{4.3}^2$ of the 1st conditional T^2 terms and $T_{2.14}^2$, $T_{2.34}^2$ and $T_{4.13}^2$ of the 2nd conditional T^2 terms respectively are significant. However, Table V D shows no significant difference because FCL and BCL are greater than the entire 3rd conditional terms (i.e. 11.4089, 8.587 > 0.8898, 5.0433, 0.0023, 5.2888).

TABLE V A
UNCONDITIONAL T^2 TERMS WITH P-VALUES (NUMBER OF $T_{Sortd}^2 \geq T_j^2$ IN PARENTHESIS)

T_j^2 Component	Computed T_j^2 Value	Manson Critical values	Bootstrap P-Value
T_1^2	0.4790	4.1519	0.9723 (2917)
T_2^2	19.1183*	„	0.0017*** (5)
T_3^2	0.0137	„	1.0000 (3000)
T_4^2	14.1516*	„	0.0063*** (19)

*Out of Control Signals ***Significant at 0.01

TABLE V B
1ST CONDITIONAL T^2 TERMS WITH P-VALUES (NUMBER OF $T_{Sortd}^2 \geq T_{j.i}^2$ IN PARENTHESIS)

$T_{j.i}^2$ Component	Computed $T_{j.i}^2$ Value	Manson Critical values	Bootstrap P-Value
$T_{1.2}^2$	0.7498	6.7247	0.9393 (2818)
$T_{1.3}^2$	0.4757	„	0.9723 (2917)
$T_{1.4}^2$	0.9328	„	0.9127 (2738)
$T_{2.1}^2$	19.3891*	„	0.0017*** (5)
$T_{2.3}^2$	19.1470*	„	0.0017*** (5)
$T_{2.4}^2$	9.9952*	„	0.0263** (79)
$T_{3.1}^2$	0.0104	„	1.0000 (3000)
$T_{3.2}^2$	0.0424	„	1.0000 (3000)
$T_{3.4}^2$	0.1393	„	0.998 (2994)
$T_{4.1}^2$	14.6054*	„	0.005*** (15)
$T_{4.2}^2$	5.0285	„	0.2595 (779)
$T_{4.3}^2$	14.2773*	„	0.005*** (15)

*Out of Control Signals **Significant at 0.05 ***Significant at 0.01

TABLE V C
2ND CONDITIONAL T^2 TERMS WITH P-VALUES (NUMBER OF $T^2_{Sortd} \geq T^2_{j.i}$ IN PARENTHESIS)

$T^2_{j.i}$ Component	Computed $T^2_{j.i}$ Value	Manson Critical values	Bootstrap P-Value
$T^2_{1.23}$	0.7115	9.0824	0.9453 (2836)
$T^2_{1.24}$	1.0120	,,	0.9003 (2701)
$T^2_{1.34}$	0.8002	,,	0.9303 (2791)
$T^2_{2.13}$	19.3829*	,,	0.0017*** (5)
$T^2_{2.14}$	10.0744*	,,	0.0253** (76)
$T^2_{2.34}$	9.9802*	,,	0.0263** (79)
$T^2_{3.12}$	0.0041	,,	1.0000 (3000)
$T^2_{3.14}$	0.0068	,,	1.0000 (3000)
$T^2_{3.24}$	0.1244	,,	0.999 (2997)
$T^2_{4.12}$	5.2907	,,	0.238 (714)
$T^2_{4.13}$	14.6018*	,,	0.005*** (15)
$T^2_{4.23}$	5.1105	,,	0.2523 (757)

*Out of Control Signals **Significant at 0.05 ***Significant at 0.01

TABLE V D
3RD CONDITIONAL T^2 TERMS WITH P-VALUES (NUMBER OF $T^2_{Sortd} \geq T^2_{j.i}$ IN PARENTHESIS)

$T^2_{j.i}$ Component	Computed $T^2_{j.i}$ Value	Manson Critical values	Bootstrap P-Value
$T^2_{1.234}$	0.8898	11.4088	0.9183 (2755)
$T^2_{2.134}$	5.0433	,,	0.259 (777)
$T^2_{3.124}$	0.0023	,,	1.0000 (3000)
$T^2_{4.123}$	5.2888	,,	0.2387 (716)

V. CONCLUSION

This study specifically considered the bootstrap method as a means of determining control limits from multivariate control charts. Procedures that can carry out a systematic generation of bootstrap replications for two or more quality characteristics have been proposed. Nevertheless, this paper has also introduced the p-value technique as a means of identifying the variable(s) that is (are) responsible for the out of control signal(s). Due to the signals at X_2 and X_4, the practice in the univariate case is to terminate the procedure, and this will lead to misuse of available resources or abnormal ouput [18]. With the multivariate method, one variable being conditioned on the other(s) as shown in Table V, is the advantages of multivariate control charts; (i.e. from Table V D, it should be noted that the process was under control when simultaneously, quality characteristic X_2 or X_4 were imposed on any other quality characteristics). This outcome will improve the method of production in addition to prevent misuse of available resources [18] as well as improve the quality of product.

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