

# A Non-Linear Eddy Viscosity Model for Turbulent Natural Convection in Geophysical Flows

J. P. Panda, K. Sasmal, H. V. Warrior

**Abstract**—Eddy viscosity models in turbulence modeling can be mainly classified as linear and nonlinear models. Linear formulations are simple and require less computational resources but have the disadvantage that they cannot predict actual flow pattern in complex geophysical flows where streamline curvature and swirling motion are predominant. A constitutive equation of Reynolds stress anisotropy is adopted for the formulation of eddy viscosity including all the possible higher order terms quadratic in the mean velocity gradients, and a simplified model is developed for actual oceanic flows where only the vertical velocity gradients are important. The new model is incorporated into the one dimensional General Ocean Turbulence Model (GOTM). Two realistic oceanic test cases (OWS Papa and FLEX' 76) have been investigated. The new model predictions match well with the observational data and are better in comparison to the predictions of the two equation k-epsilon model. The proposed model can be easily incorporated in the three dimensional Princeton Ocean Model (POM) to simulate a wide range of oceanic processes. Practically, this model can be implemented in the coastal regions where trasverse shear induces higher vorticity, and for prediction of flow in estuaries and lakes, where depth is comparatively less. The model predictions of marine turbulence and other related data (e.g. Sea surface temperature, Surface heat flux and vertical temperature profile) can be utilized in short term ocean and climate forecasting and warning systems.

**Keywords**—Eddy viscosity, turbulence modeling, GOTM, CFD.

## I. INTRODUCTION

**T**URBULENCE modeling in computational fluid dynamics and geophysical modelling can be classified into four major approaches as eddy viscosity model, Reynolds stress model, large eddy simulation, and direct numerical simulation. Eddy viscosity models are simple and require less computational resources as compared to other mentioned approaches and are based on the Reynolds averaged Navier Stokes (RANS) equations in which Reynolds stresses appear as a result of time averaging of momentum conservation equations. The transport equations for mean velocity with the Reynolds stress term can be written as

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (1)$$

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$$\frac{DU_i}{Dt} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial U_i}{\partial x_j} \right) - \frac{\partial}{\partial x_j} (\overline{u_i u_j}) \quad (2)$$

where  $U$  and  $P$  are the velocity and pressure, respectively. Boussinesq [1] was the first to postulate the assumption that the Reynold stress tensor is proportional to the strain rate tensor and can be written as

$$\overline{u_i u_j} = -2\nu_t S_{ij} + \frac{2}{3} k \delta_{ij} \quad (3)$$

The strain rate tensor  $S_{ij}$  is defined as

$$S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \quad (4)$$

$\nu_t$  is the eddy viscosity which takes into account the enhanced momentum transfer because of turbulence and  $k$  is the turbulent kinetic energy.

In the standard  $k-\varepsilon$  model [2],  $\nu_t$  is defined as

$$\nu_t = c_\mu \frac{k^2}{\varepsilon} \quad (5)$$

To obtain  $\nu_t$ , the transport equations of turbulence kinetic energy  $k$  and dissipation rate  $\varepsilon$  are needed to be solved in the  $k-\varepsilon$  model [2], [3]. Mellor and Yamada [4] solved equations for  $k$  and  $kl$ , where  $l$  is the length scale.  $c_\mu$  is the structure parameter. Wilcox [5] replaced dissipation  $\varepsilon$  by  $\omega$  (which is the ratio of dissipation and kinetic energy). In  $k-\varepsilon$  two equation model, the structural parameter is a constant value that can be defined by referring to local equilibrium shear layers, but in the geophysical turbulence model of Mellor and Yamada the structural parameter was taken as function of shear and buoyancy. The effects of buoyancy, vorticity and Reynolds stress anisotropy were included in the structural parameter [6].

In the general ocean turbulence model [7], the transport equations of turbulence kinetic energy and dissipation are modelled as follows:

$$\frac{\partial k}{\partial t} - \frac{\partial}{\partial z} \left( \nu_k \frac{\partial k}{\partial z} \right) = P + B - \varepsilon \quad (6)$$

$$\frac{\partial \varepsilon}{\partial t} - \frac{\partial}{\partial z} \left( \nu_\varepsilon \frac{\partial \varepsilon}{\partial z} \right) = \frac{\varepsilon}{k} (c_{\varepsilon 1} P + c_{\varepsilon 3} B - c_{\varepsilon 2} \varepsilon) \quad (7)$$

where,  $P$  and  $B$  are the production terms due to shear and buoyancy, respectively. The detailed description of the model constants and the terms used in the above equations are available in [7].

Reynolds stress anisotropy can be defined as

$$b_{ij} = \frac{\overline{u_i u_j} - \frac{2}{3} k \delta_{ij}}{2k} \quad (8)$$

Maity and Warrior [8] proposed an eddy viscosity model based on a transport equation of second invariant of Reynolds stress anisotropy and studied the natural convection flow and mixing in a vertical water column. For non-equilibrium shear flows Craft et al. [9] proposed a non linear eddy viscosity model and took Reynolds stresses as more general function of vorticities and mean velocities. Considering the above formulation of anisotropy tensor, Sasmal et al. [10], [11] proposed an eddy viscosity formulation for geophysical turbulent flows and validated the model against various realistic test cases.

In this paper, a constitutive equation for Reynolds stress anisotropy tensor is adopted which accounts the effects of streamline curvature and swirl effects [9], [12]. The transport equation of Maity and Warrior [8] for the second invariant of Reynolds stress anisotropy tensor is solved. By assuming velocity variations only in the vertical direction, a formulation for eddy viscosity is developed, which can tackle complex geophysical turbulent flows by taking into consideration of the bed roughness and curvature.

## II. FORMULATION OF THE PROBLEM

The one dimensional form of the Reynolds-averaged Navier-Stokes equations, energy conservation and salt conservation equation are used for the study of natural convection flow and heat transfer in a vertical water column. Effects of the advection, internal pressure gradients and horizontal transport are neglected.

$$\partial_t U - \nu \partial_{zz} U + \partial_z \langle u'w' \rangle - fV = -g \frac{\bar{\rho}}{\rho_0} \partial_x \zeta \quad (9)$$

$$\partial_t V - \nu \partial_{zz} V + \partial_z \langle v'w' \rangle + fU = -g \frac{\bar{\rho}}{\rho_0} \partial_y \zeta \quad (10)$$

where  $U$  and  $V$  are the velocities,  $g$  is the acceleration of gravity,  $\zeta$  is the elevation of the free surface,  $\bar{\rho}$  is the

averaged density, and  $\rho_0$  is a constant reference density resulting from Boussinesq approximation [13].  $f$  is the Coriolis frequency and is taken as:

$$f = 2\Omega \sin \phi \quad (11)$$

where,  $\Omega$  is the earth's angular velocity and  $\phi$  is the latitude [14].

The energy and salinity conservation equations can be written respectively as

$$\partial_t T - \nu' \partial_{zz} T + \partial_z \langle w'T' \rangle = \frac{\partial_z I(z)}{c_p \rho_0} \quad (12)$$

$$\partial_t S - \nu'' \partial_{zz} S + \partial_z \langle w'S' \rangle = 0 \quad (13)$$

where  $T$  and  $S$  are the temperature and salinity respectively.

$\nu'$  denotes the molecular diffusivity,  $c_p$  is the heat capacity.

$I$  denotes the short wave radiation and its vertical divergence is taken as the source term in the energy conservation equation [12].

The net shortwave radiation,  $I_0$  at the surface is used as a source of heat and follows the exponential formula of Paulson and Simpson [15],

$$I(z) = I_0 \left( A e^{z/\eta_1} + (1-A) e^{z/\eta_2} \right) B(z) \quad (14)$$

where  $\eta_1$  and  $\eta_2$  are the absorption coefficients and they

depend on water type [16],  $B(z)$  is a damping term which accounts for bioturbidity and the value of the source term in the transport equation of temperature depends on type of water [15]. For OWS Papa and FLEX76, Jerlov type II (turbid water) and type I (clear water) are used respectively.

The heat fluxes at the air-sea interface were calculated using the bulk formulae of Kondo [17]. Following Burchard [18], boundary conditions for the surface heat fluxes can be expressed as

$$\nu'_t \frac{\partial T}{\partial z} = \frac{Q_s + Q_l + Q_b}{c_p \rho_0} = \frac{Q_{tot}}{c_p \rho_0} \text{ at } z = \zeta \quad (15)$$

where  $\nu'_t$  is the eddy diffusivity,  $\rho_0$  is the density of sea water and  $T$  is the average temperature,  $c_p$  specific heat of sea water at constant pressure ( $\text{J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$ ) and  $\zeta$  represents the sea surface elevation.  $Q_s$ ,  $Q_l$  and  $Q_b$  are the sensible heat flux, the latent heat flux, and the long wave back radiation, respectively and  $Q_{tot} = Q_s + Q_l + Q_b$  is the total surface heat flux. The boundary condition for the surface fresh water flux

(salinity flux) can be written using difference between evaporation and precipitation fluxes ( $Q_E, Q_P$ ) as

$$v'_t \frac{\partial S}{\partial z} = \frac{S(Q_E - Q_P)}{\rho_0(0)} \text{ at } z = \zeta \quad (16)$$

where,  $\rho_0(0)$  is the density of fresh water at sea surface temperature. The salinity flux is often neglected in short term calculation since their values are relatively small compared to the heat flux [18].

TABLE I  
MODEL CONSTANTS

$C_1$	$C_2$	$C_2$	$C_4$	$C_5$	$C_6$	$C_7$
-0.04	0.1	0.02	0.1	-0.8	-0.6	0.6

#### A. New Formulation of Eddy Viscosity

The Reynolds stress in terms of Boussinesq eddy viscosity can be written as

$$\overline{u_i u_j} = -2v_t S_{ij} + \frac{2}{3} k \delta_{ij} \quad (17)$$

The constitutive equation [12] for the Reynolds stress anisotropy tensor is considered for the present eddy viscosity formulation:

$$\begin{aligned} b_{ij} = & \frac{-v_t}{k} S_{ij} + c_1 \frac{v_t}{\varepsilon} (S_{ik} S_{kj} - \frac{1}{3} S_{kl} S_{kl} \delta_{ij}) + c_2 \frac{v_t}{\varepsilon} (\Omega_{ik} S_{kj} + \Omega_{jk} S_{kl}) + \\ & c_3 \frac{v_t}{\varepsilon} (\Omega_{ik} \Omega_{jk} - \frac{1}{3} \Omega_{lk} \Omega_{lk} \delta_{ij}) + c_4 \frac{v_t k}{\varepsilon^2} (S_{kl} \Omega_{ij} + S_{kj} \Omega_{li}) S_{kl} + \\ & c_5 \frac{v_t k}{\varepsilon^2} (\Omega_{il} \Omega_{lm} S_{mj} + S_{il} \Omega_{lm} \Omega_{mj} - \frac{2}{3} S_{lm} \Omega_{mn} \Omega_{nl} \delta_{ij}) + \\ & c_6 \frac{v_t k}{\varepsilon^2} S_{ij} S_{kl} S_{kl} + c_7 \frac{v_t k}{\varepsilon^2} S_{il} \Omega_{kl} \Omega_{kl} = v_t \mathfrak{R}_1 \end{aligned} \quad (18)$$

A similar expression can be written for  $b_{ji}$ , by interchanging the indices  $i$  and  $j$

$$\begin{aligned} b_{ji} = & \frac{-v_t}{k} S_{ji} + c_1 \frac{v_t}{\varepsilon} (S_{jk} S_{ki} - \frac{1}{3} S_{kl} S_{kl} \delta_{ji}) + c_2 \frac{v_t}{\varepsilon} (\Omega_{jk} S_{ki} + \Omega_{ik} S_{kl}) + \\ & c_3 \frac{v_t}{\varepsilon} (\Omega_{jk} \Omega_{ik} - \frac{1}{3} \Omega_{lk} \Omega_{lk} \delta_{ji}) + c_4 \frac{v_t k}{\varepsilon^2} (S_{kj} \Omega_{li} + S_{ki} \Omega_{lj}) S_{kl} + \\ & c_5 \frac{v_t k}{\varepsilon^2} (\Omega_{jl} \Omega_{lm} S_{mi} + S_{jl} \Omega_{lm} \Omega_{mi} - \frac{2}{3} S_{lm} \Omega_{mn} \Omega_{nl} \delta_{ji}) + \\ & c_6 \frac{v_t k}{\varepsilon^2} S_{ji} S_{kl} S_{kl} + c_7 \frac{v_t k}{\varepsilon^2} S_{ji} \Omega_{kl} \Omega_{kl} = v_t \mathfrak{R}_2 \end{aligned} \quad (19)$$

The vorticity tensor  $\Omega_{ij}$ , is defined as

$$\Omega_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right) \quad (20)$$

Multiplying (18) and (19), an expression for the second

invariant of Reynolds stress anisotropy can be obtained,

$$II_b = b_{ij} b_{ji} = v_t^2 \mathfrak{R}_1 \mathfrak{R}_2 \quad (21)$$

After rearrangement of the terms,

$$v_t = \frac{II_b^{(1/2)}}{(\mathfrak{R}_1 \mathfrak{R}_2)^{(1/2)}} \quad (22)$$

In the real oceans, because of the strong disparity between the horizontal and vertical dimensions, the strain and vorticity take a simplified form. Thus, by considering only vertical gradients of velocity and neglecting variations of  $U$  and  $V$  in other directions, the strain and vorticity tensors acquire the form [19]

$$S_{ij} = \frac{1}{2} \begin{pmatrix} 0 & 0 & \frac{\partial U}{\partial Z} \\ 0 & 0 & \frac{\partial V}{\partial Z} \\ \frac{\partial U}{\partial Z} & \frac{\partial V}{\partial Z} & 0 \end{pmatrix} \text{ and } \Omega_{ij} = \frac{1}{2} \begin{pmatrix} 0 & 0 & \frac{\partial U}{\partial Z} \\ 0 & 0 & \frac{\partial V}{\partial Z} \\ \frac{\partial U}{\partial Z} & -\frac{\partial V}{\partial Z} & 0 \end{pmatrix} \quad (23)$$

#### B. Transport Equation for the Second Invariant of Stress Anisotropy

An equation for the second invariant developed by Craft et al. [12] is taken into consideration. A transport equation for Reynolds stress anisotropy can be written as,

$$\frac{Db_{ij}}{Dt} = \frac{1}{k} (d_{ij} + P_{ij} + \phi_{ij} - \varepsilon_{ij}) - \frac{b_{ij}}{k} (d_k + P_k - \varepsilon) \quad (24)$$

The transport equation for  $II_b$  is derived by multiplying the above equation by  $2b_{ij}$ . The resulting equation for the stress invariant is written as

$$\frac{DII_b}{Dt} = -2 \frac{II_b}{k} (d_k + P_k - \varepsilon) + 2 \frac{b_{ij}}{k} (d_{ij} + P_{ij} + \phi_{ij} - \varepsilon_{ij}) \quad (25)$$

where  $d_{ij}$  represents the diffusive transport,  $P_{ij}$  is the shear production,  $\phi_{ij}$  is the pressure strain correlation which is the summation of slow and rapid term, and  $\varepsilon_{ij}$  is the dissipation rate of Reynolds Stress and  $d_k$ ,  $P_k$  and  $\varepsilon$  are the corresponding contractions.

In order to model the pressure strain correlation, the Poisson equation for fluctuating pressure should be solved for determining the pressure fluctuations [20]

$$\frac{1}{\rho} \nabla^2 p' = -2 \frac{\partial U_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} - \frac{\partial^2}{\partial x_i \partial x_j} (u'_i u'_j - \overline{u'_i u'_j}) \quad (26)$$

The above equation can be solved through the decomposition of the pressure fluctuation as

$$p' = p'_{slow} + p'_{rapid} \quad (27)$$

Slow and rapid pressure fluctuations satisfy the following equations.

$$\frac{1}{\rho} \nabla^2 p'_{slow} = - \frac{\partial^2}{\partial x_i \partial x_j} (u'_i u'_j - \overline{u'_i u'_j}) \quad (28)$$

$$\frac{1}{\rho} \nabla^2 p'_{rapid} = -2 \frac{\partial U_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} \quad (29)$$

The slow term represents the turbulence-turbulence interactions and is known as the return to isotropy term, when a system is excited into turbulence through the effect of mean shear, strain or buoyancy, the turbulence develops anisotropies in Reynolds stresses [21], dissipation rates [22] and in length scales [23], [24]. Most of the two-equation eddy viscosity formulations assume that the return to isotropy process is instantaneous, this erroneous assumption can be overcome through the incorporation of the slow term into eddy viscosity formulation. For the representation of interaction turbulence and mean flow gradient, a formulation of rapid pressure term is adopted in this paper.

The second moment closure model of Craft et al. [12] for the pressure strain correlation is written as

$$\phi_{ij} = \phi_{ij}^S + \phi_{ij}^R \quad (30)$$

$$\phi_{ij}^S = -c_1 \varepsilon [b_{ij} + c_1^* (b_{ik} b_{kj} - \frac{1}{3} II_b \delta_{ij})] - (A^*)^{0.5} \varepsilon b_{ij} \quad (31)$$

$$\phi_{ij}^R = -0.6 (P_{ij} - \frac{1}{3} \delta_{ij} P_{kk}) + 0.3 b_{ij} P_{kk} \quad (32)$$

In the expression of  $\phi_{ij}^S$

$$c_1 = 3.1 [1 - \exp\{-(-R_t/80)^2\} \sqrt{A^*} \min(\sqrt{II_b}, 0.5)], c_1^* = 1.2 \quad (33)$$

$$P_{ij} = -(\overline{u'_i u'_k} \frac{\partial U_j}{\partial x_k} + \overline{u'_j u'_k} \frac{\partial U_i}{\partial x_k}) \quad (34)$$

$$P_k = 0.5 P_{ii} \quad (35)$$

$b_{ij} \phi_{ij}^S$  and  $b_{ij} \phi_{ij}^R$  work out to be

$$b_{ij} \phi_{ij}^S = II_b (-c_1 \varepsilon) + III_b (-c_1 c_1^* \varepsilon) - II_b \varepsilon (A^*)^{0.5} \quad (36)$$

$$b_{ij} \phi_{ij}^R = -0.6 b_{ij} P_{ij} + 0.3 II_b P_k \quad (37)$$

Equation (27) can be simplified as:

$$\frac{DII_b}{Dt} = -\frac{2}{k} (P_k - \varepsilon) + \frac{2}{k} \left( \frac{-1.6k}{3} b_{ij} S_{ij} + 0.3 II_b P_k - c_1 \varepsilon II_b - c_1 c_1^* \varepsilon III_b - II_b \varepsilon \sqrt{A^*} \right) \quad (38)$$

After suitable modifications and assumption for the geophysical flows, equations for  $II_b$  [8] can be written as:

$$\frac{DII_b}{Dt} = \frac{2}{k} \varepsilon II_b (1 - c_1 - \sqrt{A^*}) - S \sqrt{II_b} (2.8 II_b - 1.067) - \frac{2}{k} c_1^* c_1 \varepsilon III_b \quad (39)$$

where  $III_b$  is the third invariant of Reynolds stress anisotropy tensor

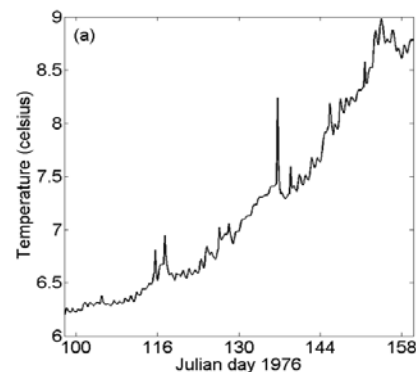
$$II_b = b_{ij} b_{ji} \text{ and } III_b = b_{ij} b_{jk} b_{ki} \quad (40)$$

$$II_b = 0.22 + 2 III_b \quad (41)$$

$A^*$  is the Lumley's stress flatness parameter [25] which is zero at the wall where turbulence goes to two-component limit.

For preventing the model from blowing up during numerical simulations, realizability constraints for the second invariant were considered. The values of second invariant can be larger than one, near the walls because of higher values of turbulent stresses at those regions, those were not considered in this study.

$$0 \leq II_b \leq 1 \quad (42)$$



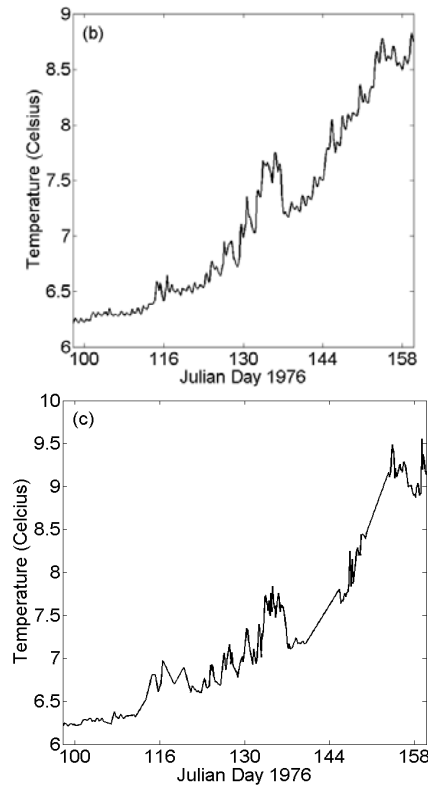
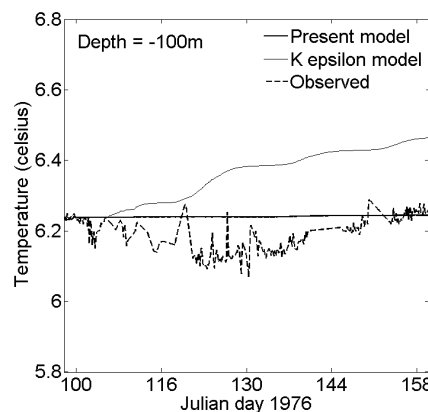


Fig. 1 Time series of sea surface temperature (SST) profiles during Flex 76: a. present model, b. k-epsilon model and c. observed data

### III. NUMERICAL MODELING AND RESULTS

The temperature decreases with depth in the ocean and an upward and downward movement of water occurs as a result of temperature difference between the fluids, which can be termed as free or natural convection flow, is dependent on the temperature, salinity and depth of the water. A one-dimensional water column model "General Ocean Turbulence Model" [7] is used to study the natural convection flow in a vertical water column. The newly developed formulation is used to simulate the flow, and the results obtained from the simulation are compared with the observational results of ocean weather station papa (OWS Papa) and a realistic ocean test case of the Fladenground experiment 1976 (FLEX 76).



The discretization of the domain is achieved by dividing the domain into required number of intervals. The vertical discretization was refined at the surface and bottom. The discrete values for the mean flow quantities such as  $x$  and  $y$  components of velocity, temperature and salinity represent interval means and are located at the centers of the interval, and the turbulent quantities are positioned at the interfaces of the intervals. The staggering of the grid allows for a second order approximation of the vertical fluxes of momentum and tracers without averaging. Averaging of the eddy diffusivities is required for the vertical fluxes of kinetic energy, length scale and dissipation. Because of absence of advection and fully implicit treatment of diffusion, the time stepping is equidistant, based on two time levels. For momentum and tracers, a fully implicit discretization scheme is used, which results in a system of linear equations with tri-diagonal matrix for each transport equation. The resulting tri-diagonal matrix is solved by means of simplified Gaussian elimination [7], [26].

The Fladenground experiment was performed at the northern North Sea at a water depth of 145 meter and a position  $58^{\circ}.55'N$  and  $0^{\circ}.55'E$ . Measurements of meteorological forcing and temperature profiles were carried out in spring 1976. Various turbulence modelers have validated their models and compared the performance of various turbulence models against FLEX 76 data (e.g. [10] and [14]). Fig. 1 represents the time series of temperature profiles during the Fladenground experiment 1976. From the present model predictions, it is observed that there is little improvement over the model predictions of the k-epsilon model for the sea surface temperature profiles. Time series of temperature profiles at a depth of 100 meter are shown in Fig. 2. Since the present model properly represents the complex flow fields because of the addition of cubic nonlinear terms in the formulation of the Reynolds stress anisotropy, an improved prediction of temperature profiles is observed at a depth of 100 meter. On the Julian day 133, a storm occurred on that site, the storm can be noticed from the vertical temperature profiles from Fig. 3. Figs. 3 (a) and (b) represents Julian day 124 and 136 respectively. Both before and after storm, predictions of temperature are better than k-epsilon model and are matching with the trends of observational data.

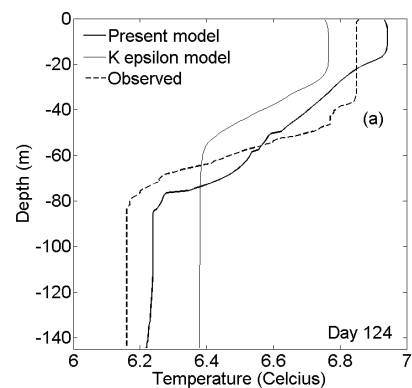


Fig. 2 Time series of temperature profiles during Flex 76

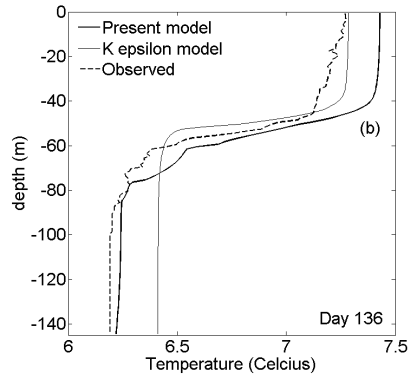


Fig. 3 Vertical temperature profiles during FLEX 76, before and after the storm

The station PAPA is located in the North Pacific, at  $145^{\circ}W$  and  $50^{\circ}N$ , where sea temperature profiles and meteorological data have been collected from 1940s to the early 1980s. The current simulations of OWS Papa have been performed for the year 1961. For OWS Papa, meteorological data for sea surface temperature, air pressure, wind speed and direction are available. In station OWS Papa, horizontal advection of heat and salt is assumed to be small. Time series of SST profiles of OWS Papa are shown in Fig. 4. In the three separate figures, the different model predictions are compared with the observed data. OWS papa observational results are available up to 250-meter depth. In Figs. 5 and 6, time and vertical variations of temperature profiles are shown. The model predictions of temperature profiles are better than k-epsilon model predictions. Fig. 7 show the variation of eddy viscosity with depth. K-epsilon model predicts almost zero viscosity in lower layers, but the present model shows nonzero viscosity in those layers, which can be considered as an improvement of the result because of the non linear terms in the formulation of the eddy viscosity.

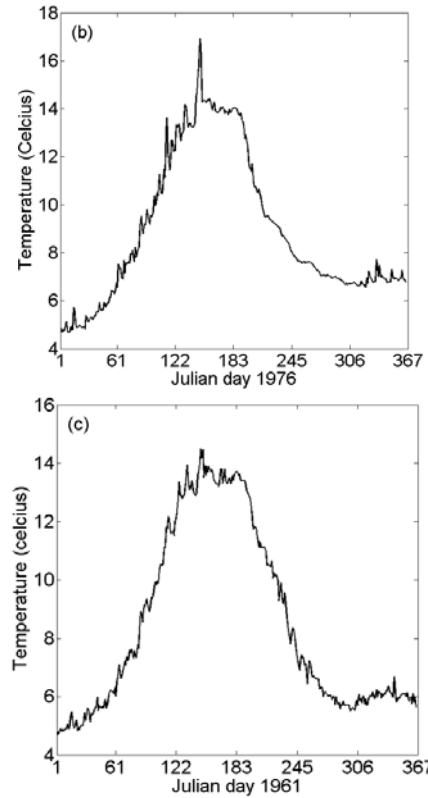


Fig. 4 Time series of sea surface temperature (SST) profiles for OWS papa: a. Present model, b. k-epsilon model and c. observed data

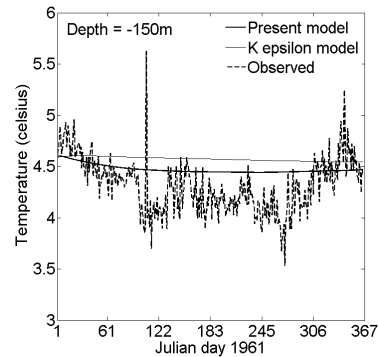
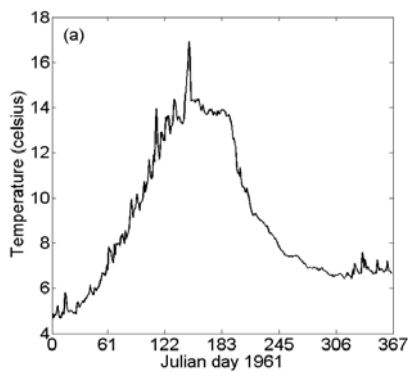


Fig. 5 Time series of temperature profiles for OWS papa: comparison of the model predictions with observed data

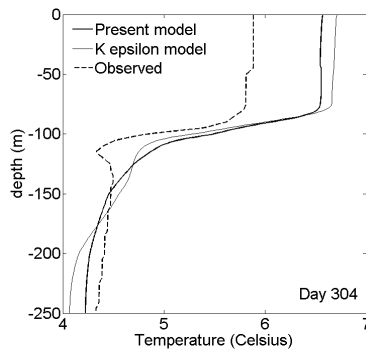


Fig. 6 Vertical profiles of temperature: comparison of the model predictions with the observed data

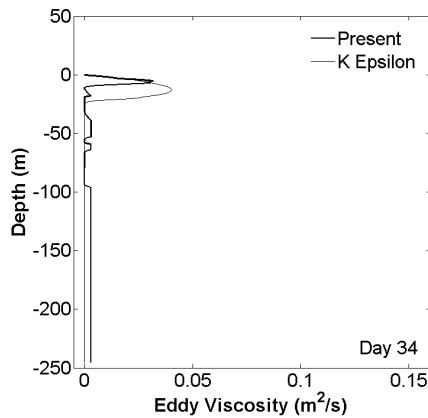


Fig. 7 Profile of eddy viscosity: comparison of the present model and k-epsilon model predictions for OWS papa

#### IV. CONCLUSIONS

Effects of complex strain fields can be tackled by using nonlinear terms in the constitutive equation as done by other researchers. Addition of cubic terms to the Reynolds stress anisotropy constitutive equation ensures proper representation of flow field by mimicking streamline curvature and swirl effects in geophysical flows. Other eddy viscosity models predict almost zero viscosity in the lower layers, but the present model shows nonzero viscosity in those layers which is an improved prediction of the eddy viscosity field. Because of addition of swirling and curvature effects, also there is a marked improvement of the predicted temperature profiles. This model along with other related (weather prediction) models can be utilized for environmental impact assessment of power plants and in short term ocean and climate forecasting and warning systems. In future course of work, focus can be placed on the modelling of the near wall geophysical turbulent flows by considering the wall invariant parameters such as strain and stress invariants in the formulation of eddy viscosity or through incorporation of the pressure strain correlation representing the wall damping effects in the transport equation of the stress invariant.

#### NOMENCLATURE

$U$	Velocity
$\rho$	Mass density
$\nu_t$	Eddy Viscosity
$P$	Pressure
$c_\mu$	Structure Parameter
$T$	Temperature
$S$	Salinity
$\Omega_{ij}$	Vorticity tensor
$S_{ij}$	Strain rate tensor
$\mathcal{E}$	Dissipation rate
$e_{ij}$	Dissipation anisotropy tensor
$\delta_{ij}$	Kronecker delta
$\nu$	Kinematic viscosity
$k$	Kinetic energy

$\phi_j$	Total pressure strain
$t$	Time
$b_{ij}$	Reynolds stress anisotropy
$II_b$	Second invariant of Reynolds stress anisotropy
$III_b$	Third Invariant of Reynolds stress anisotropy

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